Optimal Unemployment Insurance in a THANK Model*

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April 18, 2024

Abstract

We characterize optimal unemployment insurance (UI) in a heterogeneous-agent model with unemployment risk and sticky prices. In the long run, the optimal reform calls for a lower replacement rate that raises vacancies and lowers unemployment. In the short run, the optimal reform *raises* the replacement rate initially to smooth real wage adjustments along the transition and attenuate short-run welfare losses. Once at its optimal level, the replacement rate should vary counter-cyclically in response to demand shocks. Productivity shocks generate quasi-efficient fluctuations and call for a quasi-constant replacement rate. The aggregate welfare gains from an optimal reform are large, around 1% of equivalent consumption. The aggregate welfare gains from an optimal UI policy over the business cycle are smaller, around 0.2%, and essentially vanish with flexible prices because the aggregate demand stabilization motive is muted.

Keywords: Unemployment, Borrowing constraints, Incomplete markets, Unemployment Insurance.

JEL Class.: D52, E21, E62, J64, J65.

*We thank Loukas Karabarbounis, an Associate Editor, and two anonymous referees for their invaluable comments. We thank Julien Albertini, Edouard Challe and many participants in seminars and conferences for their valuable comments. We acknowledge financial support from the French National Research Agency (ANR-20-CE26-0018-02).

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1 Introduction

Unemployment remains a major issue in most developed economies, especially in the event of large economic downturns. The dynamics of the labor market during the recent Great Recession was particularly important in determining the dynamics of aggregate variables. The same has been be true for the macroeconomic dynamics resulting from the Covid-19 pandemic episode and the associated lockdown policies. In face of unemployment risk, how should unemployment insurance be designed? What should be its level? Should it vary along the business cycle, and if so how and by how much? How large are the potential gains from an optimal unemployment insurance?

On the one hand, many contributions on the subject either assume full risk-sharing among a large family, in the spirit of Merz (1995) or Andolfatto (1996), but in this case there is no income risk to insure in the first place. On the other hand, the Baily-Chetty literature recently summarized by Landais, Michaillat, and Saez (2018) does not incorporate important macroeconomic and labor-market assumptions such as sticky prices or real wage inertia, and thus potentially overlooks importance of demand shocks in shaping unemployment risk over the business cycle.

We analyze the design, the macroeconomic effects as well as the redistributive properties of optimal unemployment insurance (UI) in a model with incomplete markets, unemployment risk, sticky prices and real wage inertia. A recent and dense macroeconomic literature burgeoned to incorporate microeconomic heterogeneity and risk in macroeconomic environments featuring otherwise standard New Keynesian features such as sticky prices.[1] Some of these contributions specifically focus on fiscal policy.[2] But to the best of our knowledge, only Kekre (2022) and more recently Gorn and Trigari (2024) investigate the effects of extensions in UI benefits in a heterogeneous-agents model with sticky prices, especially at the zero lower bound. However, as opposed to our paper, their approaches are more positive than normative.

Our model is a tractable heterogeneous-agent model based on Ravn and Sterk (2017) with unemployment risk. It considers a zero-liquidity economy, imperfect insurance, sticky prices and real wage inertia, and search and matching frictions on the labor market. It also features an existing UI system where unemployed workers receive a fraction of the equilibrium real wage on top of home production. The key policy instrument is this fraction, the replacement rate. The UI system is financed by a distortionary tax on labor income, paid both by employed and unemployed workers. The main interest of the zero-liquidity limit is that the distribution of wealth is degenerate and the model boils down to only three types of agents while preserving

[1] Among many others, see Gornemann, Kuester, and Nakajima (2016), Kaplan, Moll, and Violante (2018), Auclert (2019), Bhandari et al. (2021) for environment with non-degenerate wealth distributions, and Bilbiie (2018), Ravn and Sterk (2021), Challe (2019) or Gorn and Trigari (2024) for more tractable environments.

[2] See Challe and Ragot (2011), Albertini et al. (2021), Auclert, Rognlie, and Straub (2018), Bayer, Born, and Luetticke (2020) or Hagedorn, Manovskii, and Mitman (2019).

strong effects of unemployment risk on the aggregate dynamics through precautionary savings.[3] As such, it offers an explicit relation between unemployment dynamics, unemployment risk and aggregate demand, with feedback general equilibrium effects through sticky prices and real wage inertia.

After calibrating the baseline version of our model to match key features of the European economy, we investigate optimal UI policies. We first characterize the optimal reform implying a dynamic transition from the initial steady state to the Ramsey steady state. Once at the optimal steady state, we look at how UI should vary along the business cycle in response to shocks.

In the long run, we find that the optimal UI policy *lowers* the replacement rate. This leads to a lower level of unemployment by raising the outside option of workers and compressing the real wage, to a lower labor income tax rate and to aggregate consumption and output gains. In addition, the composition effect by which more workers are employed and thus consume more is an important driving force of this long-run equilibrium. Further, the consumption losses of unemployed workers are more than compensated by the consumption gains of employed workers and firm owners.

In the short run, the optimal implementation depends on whether the planner chooses the transition path freely, a case we label 'time-0 optimal' policy, or subject to prior optimal commitments, a case known in the literature as the 'timeless perspective'. The timeless perspective implies a monotonic adjustment towards the new steady state, generating short-run welfare losses – especially for employed workers – because the real wage falls. The time-0 optimal policy has an additional degree of freedom and implies a *rise* in the replacement rate on impact. The underlying logic closely follows that of Mitman and Rabinovich (2021). The latter generates a fairly persistent rise in the real wage that generates a smoother transition and additional lifetime welfare gains. Firms being intertemporal optimizers through the marginal value of a job filled, they create more jobs also in the short-run because they take into account the commitment of the planner to lower permanently the real wage in the future.

We find that the optimal reform generates a 1% consumption equivalent rise in lifetime welfare but very unequally distributed. In per-capita terms, employed workers (+0.025%) and firm owners (+0.0132%) gain by a small amount, unemployed workers lose substantially (-0.24%). Those numbers do not add up to 1% because most of the gains stem from the reduced unemployment rate, which raises the number of employed workers who consume more than unemployed workers.

We then look at the optimal UI policy over the business cycle. The literature on optimal UI usually highlights two opposing motives for the planner to vary the replacement rate or the level of UI benefits: the insurance motive and the vacancy creation motive. In our model, as in Gorn

[3] A by-product of the zero-liquidity limit is that all workers end-up being hand-to-mouth: unemployed workers because they are financially constrained, and employed workers as an equilibrium result, since their demand for precautionary savings is not matched with any corresponding supply of asset in equilibrium.

and Trigari (2024), an additional motive arises because of sticky prices: the aggregate demand stabilization motive. When aggregate demand is inefficiently low, raising the replacement rate can boost aggregate demand and optimally stabilize the economy. This motive can usually be handled by an optimal monetary policy, but if monetary policy is restricted to follow a Taylor rule – as in our model – or restricted by the zero lower bound, the level of UI benefits can be used as an alternative instrument.

We consider three shocks: productivity shocks, discount factor shocks and markup shocks. We find little (if any) role for time-varying UI after productivity shocks because they produce quasi-efficient fluctuations – even with sticky prices – once the economy is at its optimal level of unemployment insurance. Discount factor and markup shocks produce inefficient fluctuations because of sticky prices, and imply large and counter-cyclical changes in the replacement rate. Those are akin to the aggregate demand stabilization motive, and disappear almost completely if prices are flexible.

The aggregate welfare gains from an optimal UI policy over the business cycle are of smaller magnitude. In the baseline case with sticky prices, aggregate welfare gains are 0.18%. Whenever prices are flexible, the aggregate welfare gains shrink to 0.01%. In per-capita terms however, welfare effects from an optimal UI policy over the business cycle are larger than the per-capita gains/losses from the transition: in the baseline case, the welfare gains for employed workers and firm owners are 0.08% and 1.07% respectively, and the losses for unemployed workers are 1.4%.

Literature. In the literature, the analysis of optimal UI policies is usually considered to start with Baily (1978) and Chetty (2006). More recent studies such as Landais, Michaillat, and Saez (2018) and Mitman and Rabinovich (2015) find that optimal UI benefits should be counter-cyclical, at least in the short-run and based on US estimations/calibrations. Mitman and Rabinovich (2021) show that the intertemporal dimension of UI policies matters critically and that UI extensions may be optimal after COVID-19 type shocks. Le Grand and Ragot (2022) develop a truncation method and apply it to optimal UI policy in a model with exogenous unemployment risk that abstracts from sticky prices or search and matching frictions, and find it to be counter-cyclical. Birinci and See (2020) study the optimal UI policy over the business cycle using a model with heterogeneous agents, job-search, aggregate risk and incomplete markets. They find that replacement rates should be counter-cyclical, with longer average durations than the current US system. McKay and Reis (2021) analyze the design of optimal automatic stabilizers (and progressive taxation), but only focus on the effects of the average (*i.e.* steady-state) replacement rate on macroeconomic stabilization. In line with our results, they find that a lower steady-state replacement rate helps households better smooth consumption through precautionary savings. Den Haan, Rendahl, and Riegler (2017) show that the interaction between incomplete markets and sticky wages amplifies the effect of productivity shocks through deflationary spirals. They abstract from considering an optimal level or response of the replacement rate though. Other studies, such as Jung and Kuester (2015) and Kekre (2022) focus on crisis environments, where the crisis is either generated by productivity or discount factor shocks. While Jung and Kuester (2015) find little role for changes in the level of UI benefits in a framework with several other instruments, Kekre (2022) focuses on the (positive and large) multiplier effect of a rise in the level of unemployment benefits when agents are heterogeneous and prices are sticky. In a similar vein, Gorn and Trigari (2024) show that the aggregate demand stabilizing channel implied by UI extensions has contributed to stabilize the unemployment rate in US data during the Great Recession and during the Pandemic.

Our paper differs from the above contributions along several dimensions. First, to the best of our knowledge, it is the first to characterize the transition resulting from an optimal reform of UI system. Second, the analysis considers sticky prices and real wage inertia, allowing for a stronger role for the dynamics of aggregate demand induced by unemployment and unemployment risk. Those clearly alter the optimal design of unemployment insurance, especially in response to discount factor and markup shocks. For the optimal transition, we find that wage inertia contributes to the aggregate demand motive and matters for the implementation, *i.e.* for the exact path of the replacement rate. For the optimal UI policy over the business cycle, we find that sticky prices are key in shaping the aggregate demand stabilization motive.

The paper is organized as follows. The baseline model is described and discussed in Section 2. Section 3 calibrates the model and estimates shocks' processes using European data. Section 4 focuses on the effects of shocks to the replacement rate around the pre-reform steady state to understand the key mechanisms involved in optimal UI policies. Section 5 aims at designing a (Ramsey) optimal reform of the UI system and characterizes the optimal transition. Section 6 analyzes the optimal UI policy over the business cycle, and discusses their business cycle and welfare implications. Section 7 offers some concluding remarks.

2 Model

The model structure borrows from Ravn and Sterk (2017) and features three types of households: employed workers, unemployed workers and firm owners. As will be clear, unemployed workers are financially constrained while employed workers hold zero assets as an equilibrium result. Firm owners receive profits, consume and hold government bonds. The rest of the model is a standard search and matching framework with (inertial) Nash-bargained wages. Finally, a government sector is introduced, that levies distortionary taxes on labor income and issues one-period bonds to finance UI benefits.

2.1 Households

The economy is populated with a unit size continuum of households: a proportion $\chi \in [0,1]$ of workers that can either be employed or unemployed, and a proportion $(1-\chi)$ of firm owners that receives profits from intermediate-good producers and retailers. Workers are excluded from the market of government bonds but potentially have access to a private asset to save.

Workers. There is a measure χ of workers in the household sector. Worker $i = \{e, u\}$ maximizes its lifetime log-utility:

$$\mathbb{E}_t \left\{ \sum_{s=t}^{\infty} \beta_s^{s-t} \log \left(c_s^i \right) \right\}, \tag{1}$$

where $\beta_t \in [0, 1]$ is the subjective discount factor, subject to persistent (AR1) shocks, and $c_t^i > 0$ denotes the individual level of private consumption of worker i. Its budget constraint is:

$$a_t^i + c_t^i = (1 + r_{t-1})\, a_{t-1}^i + (1 - \tau_t)\, \varepsilon_t^i w_t + \left(1 - \varepsilon_t^i\right) \left((1 - \tau_t)\, b_t + \mu \right),\ a_t^i \geq 0 \tag{2}$$

where a_t^i is the individual level of private wealth and r_{t-1} its real return between period $t-1$ and t. Variable $\varepsilon_t^i = \{0, 1\}$ defines the employment status of the worker: when $\varepsilon_t^i = 1$, the worker is employed at the real wage w_t; when $\varepsilon_t^i = 0$, the worker is unemployed and receives $b_t = \delta_t w_t$, where δ_t is the replacement rate of UI benefits, and produces (tax-free) home goods μ. Both the wage income and the unemployment benefit are taxed at the rate τ_t. The proportion of employed workers among workers n_t and the rate of unemployment u_t are related by:

$$n_t + u_t = 1 \tag{3}$$

At the beginning of period t, an exogenous proportion s of past employment relationships are destroyed and the pool of unemployed workers within the period is $u_{t-1} + s n_{t-1}$. A fraction f_t of this pool becomes employed before the end of period t. The proportion of employed workers is thus given by:

$$n_t = (1 - s)\, n_{t-1} + f_t \left(u_{t-1} + s_t n_{t-1} \right) = (1 - \sigma_t)\, n_{t-1} + f_t \left(1 - n_{t-1}\right) \tag{4}$$

where we have used $u_t = 1 - n_t$ and defined $\sigma_t = s\,(1 - f_t)$ as the net separation rate, s being the gross separation rate. The matching function is:

$$m_t = \psi \left(u_{t-1} + s n_{t-1} \right)^{\gamma} v_t^{1-\gamma} \tag{5}$$

where ψ is a matching-efficiency parameter. It implies that the job-finding rate $f_t \in [0, 1]$ and the worker-finding rate $q_t \in [0, 1]$ are respectively:

$$f_t = \psi \left(\frac{v_t}{u_{t-1} + s n_{t-1}} \right)^{1-\gamma} \text{ and } q_t = \psi \left(\frac{u_{t-1} + s n_{t-1}}{v_t} \right)^{\gamma} \tag{6}$$

From the perspective of a currently employed worker, the Euler equation on the private asset writes:

$$\mathbb{E}_t \left\{ \beta_t \left(1 + r_t\right) \left(\left(1 - \sigma_{t+1}\right) \frac{c_t^e}{c_{t+1}^e} + \sigma_{t+1} \frac{c_t^e}{c_{t+1}^u} \right) \right\} \leq 1 \tag{7}$$

where $\sigma_t = s(1 - f_t)$ is the transition probability from employment to unemployment at the end of period t, and c_t^e and c_t^u respectively denote the individual consumption level of a worker if employed or not. The above equation holds with equality when employed worker i is not constrained financially, and with inequality when she is constrained. If the private asset is in zero net supply – which is the case in general equilibrium – employed workers hold exactly zero private assets ($a_t^e = 0$) as an equilibrium result, and Equation (7) holds with equality. As a result, the distribution of wealth is degenerate, and all employed workers share the same per-capita level of consumption:

$$c_t^e = (1 - \tau_t)\, w_t \tag{8}$$

Further, given that $\sigma_t > 0$ and $c_{t+1}^e > c_{t+1}^u$ since unemployment benefits are well below the real wage, a precautionary motive arises due to the risk of unemployment. Employed workers face a potentially decreasing future consumption schedule – an increasing schedule for their marginal utility – that pushes them to save to self-insure. However, because they can not precautionary-save since the private asset is in zero net supply, the excess asset demand is entirely reflected in a lower real interest rate. From the perspective of unemployed workers, the Euler equation holds with strict inequality and writes:

$$\mathbb{E}_t \left\{ \beta_t (1 + r_t) \left((1 - f_{t+1}) \frac{c_t^u}{c_{t+1}^u} + f_{t+1} \frac{c_t^u}{c_{t+1}^e} \right) \right\} < 1 \tag{9}$$

which means that they are constrained, and therefore share an identical level of per-capita consumption:

$$c_t^u = (1 - \tau_t)\, b_t + \mu = (1 - \tau_t)\, \delta_t w_t + \mu \tag{10}$$

Firm owners. The household sector also counts $(1 - \chi)$ firm owners. Since they are not exposed to idiosyncratic risk, they hold the same amount of private assets and government bonds. They invest in vacancies, own the retailers and receive the resulting profits denoted Π_t. They maximize their lifetime utility:

$$\mathbb{E}_t \left\{ \sum_{s=t}^{\infty} \beta_s^{s-t} \frac{\left(c_s^f\right)^{1-\rho}}{1 - \rho} \right\} \tag{11}$$

where c_t^f denotes their per-capita consumption level, subject to the following aggregate resource constraint:

$$a_t^f + d_t + (1 - \chi)\, c_t^f = (1 + r_{t-1})\, a_{t-1}^f + (1 + r_{t-1}^d) d_{t-1} + \Pi_t,\ a_t^f \geq 0, d_t \geq 0 \tag{12}$$

where r_{t-1}^d is the real return on government bonds period between period $t-1$ and t, and d_t the aggregate amount of government bonds. The corresponding Euler equations are:

$$\mathbb{E}_t \left\{ (1 + r_t) \Delta_{t,t+1} \right\} \leq 1 \tag{13}$$

$$\mathbb{E}_t \left\{ \left(1 + r_t^d\right) \Delta_{t,t+1} \right\} \leq 1 \tag{14}$$

where $\Delta_{t,t+1} = \beta_t \left(c_t^f / c_{t+1}^f\right)^\rho$ is the stochastic discount factor of firm owners. Because firm owners invest in vacancies with a higher return than r_t, they would like to borrow in private assets but can not due to the borrowing constraint. Their Euler equation thus holds with strict inequality and, as a result, they hold exactly zero private assets in equilibrium, $a_t^f = 0$, which implies:

$$c_t^f = \left((1 + r_{t-1}^d) d_{t-1} - d_t + \Pi_t\right) / (1 - \chi) \tag{15}$$

An additional consequence is that the private asset is in zero net supply in the economy. Finally, firm owners hold government bonds. Therefore, the corresponding Euler equation holds with equality and prices government bonds:

$$\mathbb{E}_t \left\{ \beta_t \left(1 + r_t^d\right) \Delta_{t,t+1} \right\} = 1 \tag{16}$$

2.2 Production and Wage Determination

As in the search and matching literature, each firm is a job. Firms invest in $v_t \geq 0$ vacancies, paying an exogenous unit vacancy cost κ out of which a fraction q_t will be filled to produce goods with a linear technology. The aggregate production function is:

$$y_t = \chi n_t z_t \tag{17}$$

where z_t is the level of productivity that follows an AR(1) process subject to iid shocks. Given that the intermediate good is sold on competitive markets at price φ_t, the marginal value of a filled position is:

$$J_t = \varphi_t z_t - w_t + \mathbb{E}_t \left\{ \Delta_{t,t+1} \left((1 - s) J_{t+1} + s V_{t+1} \right) \right\} \tag{18}$$

where the first argument is the net contribution of the marginal worker, his marginal product less his wage bill, and the second argument is the continuation value. The marginal value of a position remaining vacant is:[4]

$$V_t = -\kappa + q_t J_t + \mathbb{E}_t \left\{ \Delta_{t,t+1} (1 - q_t) V_{t+1} \right\} \tag{19}$$

and we assume that the free entry condition $V_t = 0$ holds, which implies $q_t J_t = \kappa$. Following Challe (2019) and Gorn and Trigari (2024), the real wage is sticky in the sense that the effective

[4] Since vacancies can be filled within the period, the current value of a vacancy depends on the current probability of the vacancy to be filled and the current value of a job filled.

real wage is an average of the past real wage and the (notional) Nash-bargained wage:

$$w_t = w_{t-1}^{\alpha} w_{nt}^{1-\alpha} \tag{20}$$

The notional real wage w_{nt} is determined as the solution to a Nash bargaining problem. It maximizes a geometric average of workers and firm job surpluses:

$$w_{nt} = \max_{w_t} S_t^{\theta} J_t^{1-\theta}, \; S_t > 0, J_t > 0 \tag{21}$$

where θ is the bargaining power of workers, and S_t expresses the marginal value of being employed:

$$S_t = \max \left(\log\left(c_t^e\right) - \log\left(c_t^u\right) + \beta_t \mathbb{E}_t \left\{\left(1 - \sigma_{t+1} - f_{t+1}\right) S_{t+1}\right\}, 0\right) \tag{22}$$

where, remember, $\sigma_t = s\left(1 - f_t\right)$. The solution to this problem implies:

$$w_{nt} = \frac{\theta \kappa / q_t}{\left(1-\theta\right) S_t} \tag{23}$$

which implies that the real wage depends negatively on the worker-finding probability – a tighter labor market favors the bargaining of higher wages – and negatively on the surplus – a better outside option, for instance through more generous unemployment benefits b_t lowers S_t and raises the bargained real wage.

Retailers buy the intermediate good y_t and then differentiate it into varieties ω to sell them at nominal price $p_t\left(\omega\right)$. Let y_t^d denote the total demand for final goods and $y_t^d(\omega)$ the demand for variety ω. Retailer ω sets its price $p_t\left(\omega\right)$ to maximize the discounted sum of its expected dividends:

$$\mathbb{E}_t \left\{ \sum_{s=t}^{\infty} \Delta_{t,s} \left(\frac{p_s\left(\omega\right)}{p_s} - \varphi_s - \frac{\phi}{2}\left(\frac{p_s\left(\omega\right)}{p_{s-1}\left(\omega\right)} - 1\right)^2 \right) y_s^d\left(\omega\right) \right\} \tag{24}$$

where ϕ is a Rotemberg price adjustment cost parameter. The demand for each variety y_t^d is:

$$y_t^d\left(\omega\right) = \left(\frac{p_t\left(\omega\right)}{p_t}\right)^{-\eta_t} y_t^d \tag{25}$$

where η_t is the elasticity of substitution across varieties. Persistent (AR1) shocks hit η_t and we label them markup shocks. Optimal pricing conditions are symmetric in equilibrium and imply the following New Keynesian Phillips Curve:

$$\eta_t - 1 = \eta_t \varphi_t - \phi\left(\pi_t(1+\pi_t) - \mathbb{E}_t\left\{\Delta_{t,t+1}\pi_{t+1}(1+\pi_{t+1}) y_{t+1}/y_t\right\}\right) \tag{26}$$

where $\pi_t = p_t/p_{t-1} - 1$ is the net inflation rate. Finally, total (intermediate and final) profits distributed to firm owners are:

$$\Pi_t = y_t \left(1 - \phi \pi_t^2 / 2\right) - \chi n_t w_t - \kappa v_t \tag{27}$$

2.3 Government, Monetary Policy, Aggregation and Equilibrium

The government provides UI to the unemployed workers representing a fraction χu_t of total population with a replacement rate δ_t. This stream of expenditure is financed using the distortionary labor income tax τ_t levied on workers χn_t. In addition, the UI system is allowed to be financed through public debt, but only in the short run, *i.e.* the UI system is balanced-budget in the long run $d_\infty = 0$. The government's budget constraint writes:

$$d_t + \tau_t \chi \left(n_t + \delta_t u_t\right) w_t = (1 + r_{t-1}^d) d_{t-1} + \chi u_t b_t,\ d_\infty = 0 \tag{28}$$

In the subsequent analysis, the UI replacement rate δ_t is the key policy instrument while the labor income tax rate τ_t responds to the level of public debt to ensure the long-run financial equilibrium of the UI system:

$$\tau_t = \tau + \phi_\tau d_t \tag{29}$$

While the control of the UI system $\{\tau_t, \delta_t\}$ is in the hands of the government, an independent institution is in charge of monetary policy.[5] The central bank controls the nominal interest rate on private assets i_t and commits to the following simple Taylor-type rule:

$$\log \left(\frac{1 + i_t}{1 + i}\right) = d_\pi \log \left(1 + \pi_t\right) \tag{30}$$

The real rate of return on private assets is then determined according to the following Fisher equation:

$$1 + r_t = \mathbb{E}_t \left\{ \left(1 + i_t\right) / \left(1 + \pi_{t+1}\right) \right\} \tag{31}$$

The rule given by Equation (30) shows that the central bank pursues an objective of price stability (implicitly $\bar{\pi} = 0$), and accommodates any potential change in the steady-state level of the nominal rate i. It matters in our framework because potential steady-state changes in the unemployment rate translate into changing unemployment risk, precautionary savings, and thus the steady-state real interest rate, which through the Fisher equation implies changes in the steady-state nominal interest rate. We thus assume that optimal UI does not induce changes in the objective or in the stance of monetary policy.

Finally, the market clearing condition on the market for final goods and services is:

$$y_t \left(1 - \phi \pi_t^2 / 2\right) = \chi \left(n_t c_t^e + u_t \left(c_t^u - \mu\right)\right) + \left(1 - \chi\right) c_t^f + \kappa_t v_t$$

[5] Hence, we abstract from any kind of optimal monetary policy consideration. These assumptions are well suited to analyze the design of optimal UI policies in countries where monetary policy is clearly independent of the government, such as countries belonging to the Euro Area.

A competitive equilibrium in this economy is defined as a situation where, for a given path of the replacement rate of UI benefits $\{\delta_t\}_{t=0}^{\infty}$: (*i*) for a given path of prices, households' first-order conditions and budget constraints hold, firms and retailers optimize, and the government budget constraint holds, and (*ii*) for a given path of quantities, prices adjust – subject to Rotemberg costs – so that all markets clear and the Nash bargaining solution for the notional wage is verified. The full list of equilibrium conditions is reported in Appendix A.

3 Calibration

We calibrate the model for the initial steady state to replicate key features of the average Euro Area economy and estimate the parameters governing shocks' dynamics using Bayesian methods.

Calibration for the households. The model is quarterly. The steady-state discount factor is $\beta = 0.99$. Given the precautionary motive implied by the presence of unemployment risk, employed workers would like to self-insure and therefore demand more private assets than in a perfect-insurance economy. Since private assets are in zero net supply, the resulting excess demand of private assets is reflected in a lower equilibrium real interest rate – $r = 0.85\%$ quarterly, around 3.4% annually – than the interest rate implied by the discount factor alone, that is $r = 0.85\% < 1/\beta - 1 = 1.01\%$. In the initial steady state, we set the value of the replacement rate at $\delta = 0.5468$ based on OECD data for European countries, and adjust the home production parameter μ so that the consumption drop upon job loss is 10%, as in Ravn and Sterk (2021).[6] We consider equilibria where the UI system is at the long term equilibrium, which implies adjusting the labor income tax to balance unemployment benefits distributed to labor income taxes collected, which gives $\tau = 4.31\%$. We follow Challe (2019) for the risk-aversion parameter of firm owners $\rho_f = 0.3$. Finally, as in Challe et al. (2017) who propose a model with a comparable structure of the household sector, we set the share of firm owners to 10%, that is $\chi = 0.9$.

Calibration for prices, wages and policies. We set the steady-state monopolistic competition markup of retailers to 20%, implying $\eta = 6$ a conventional value in the literature.[7] In addition, the Rotemberg parameter is set to $\phi = 60$, a value generating a 0.1 slope of the Phillips curve. Regarding the sticky real wage parameter, we follow Gorn and Trigari (2024) and set a high value for wage inertia $\alpha = 0.98$.[8] For the Taylor-type rule, we set the elasticity of the nominal rate to inflation at $d_\pi = 1.5$. Regarding fiscal policy, we consider that the unemployment insurance system is balanced in the long-run, *i.e.* $b_\infty = 0$ but allow for potential short-run debt financing. In the baseline experiments, the fiscal feedback rule features a very large parameter $\phi_d \to \infty$ akin

[6]This number stands at the lower bound of available estimates, see Saporta-Eksten (2014), the discussion in Den Haan, Rendahl, and Riegler (2017) or more recently Bertheau et al. (2023).

[7]The initial steady state of our economy is distorted since we abstract from a sales subsidy. However, we checked that the level of markups had very little effects on our main results.

[8]Gorn and Trigari (2024) set a comparably high value of $\alpha = 0.9625$ for the US. Challe (2019) set $\alpha = 0.948$, also for the US.

to a balanced-budget rule implying $d_t \to 0$, $\forall t$. As an alternative, we consider a much looser tax reaction, allowing the UI system to adjust through debt in the short run, and assume $\phi_d = 0.1$. This number implies that, whenever debt to annual GDP rises by percentage point, the labor income tax rate adjusts by 0.33 percentage points, for instance from $\tau = 4.31\%$ to $\tau = 4.64\%$, a very mild tax adjustment.

Calibration for the labor market. On the labor market, we also seek to replicate key Euro Area numbers. The elasticity of matches with respect to unemployment is set to $\gamma = 0.7$, which is the upper bound of the range of estimates proposed by Pissarides and Petrongolo (2001). Based on the labor-market transition probabilities estimated by Elsby, Hobijn, and Şahin (2013), we impose a net separation rate of $\sigma = s(1-f) = 0.015$ and adjust the job-finding rate to deliver a 7.6% unemployment rate as reported by the AWM database for the Euro Area in December 2019, implying $f = 0.1823$. We impose a steady-state worker-finding probability of $q = 0.8$ to match the observed job-vacancy rate $v/(v+n)$ in European data of 2.2% in 2019Q4. This probability is close to the number suggested by Christoffel, Kuester, and Linzert (2009) and references therein. This transition probability, together with the targeted unemployment rate, implies adjusting the matching efficiency parameter to $\psi = 0.2841$. Finally, two parameters remain to be pinned down: the vacancy posting cost parameter κ and the bargaining power of workers θ. We target a bargaining power of $\theta = \gamma = 0.7$ so that the Hosios condition is met, and adjust $\kappa = 0.1759w$ to be consistent with our other targets. Vacancy costs represent slightly less than 20% of the quarterly wage, well in line with Hagedorn and Manovskii (2008).

Shocks. The model has three shocks: a shock affecting productivity z_t, a shock affecting the discount factor β_t, and a shock on markups η_t. The persistence and variance parameters are estimated using Bayesian methods, adopting the standard approach of An and Schorfheide (2007). This implies obtaining the posterior distribution of our estimated parameters based on a linear approximation of the model's solution around the initial steady state using the Kalman filter. Our model has three shocks and we use three time series to achieve exact identification: the demeaned log-variation of output $100 \times \log(Y_t/Y_{t-1})$, the demeaned log-variation of the unemployment rate $100 \times \log(u_t/u_{t-1})$, and the demeaned annual inflation rate of the Harmonized Index of Consumer Prices $400 \times \pi_t$.[9] Regarding priors, the persistence of forcing processes are Betas with prior means 0.7 and standard deviations 0.2 and the standard deviations of innovations are Inverse Gammas, with prior means 0.1 and infinite standard deviations. Table 1 summarizes the prior distributions and reports the means of posterior parameter distributions along with 90% confidence intervals.[10]

[9]Real GDP y_t, the unemployment rate u_t and the HICP inflation rate are taken from the AWM database available at `https://eabcn.org/page/area-wide-model`. All variables range from 1999Q1 to 2017Q4.

[10]Confidence intervals are based on 100 000 replications of the MH algorithm where the first 20% were discarded, and where the scale parameter was adjusted to get a 1/3 acceptance rate. Data series and posterior distributions are reported in Appendix B.

Table 1: Estimated parameters

Parameter	Priors			Posteriors		
	Dist.	Mean	Sd.	Mean	Inf.	Sup.
Persistence of productivity shocks (ρ_z)	B	0.70	0.20	0.9962	0.9919	1.0000
Persistence of markup shocks (ρ_η)	B	0.70	0.20	0.9950	0.9900	0.9999
Persistence of discount factor shocks (ρ_β)	B	0.70	0.20	0.3378	0.2113	0.4714
Sd. productivity shocks (σ_z)	IG	0.10	Inf	0.0103	0.0089	0.0116
Sd. markup shocks (σ_η)	IG	0.10	Inf	0.0517	0.0400	0.0625
Sd. discount factor shocks (σ_β)	IG	0.10	Inf	0.0106	0.0091	0.0121

Notes: Results based on 100 000 replications of the MH algorithm. B and IG respectively denote Beta and Inverse Gamma distributions.

4 Exogenous Replacement Rate Shock

Before anything, we want to understand the effects of an exogenous temporary change in the policy instrument, the replacement rate. To this end, we assume that δ_t follows an AR1 process with persistence 0.8 and consider a 5pp drop in the replacement rate. Figure 1 below reports the impulse response functions to this shock in the baseline case (sticky prices and real wages, balanced-budget rule), flexible prices ($\phi = 0$ instead of $\phi = 60$), flexible real wages ($\alpha = 0$ instead of $\alpha = 0.98$), and a looser fiscal rule that relies on debt to adjust the balance of the UI system in the short run.[11]

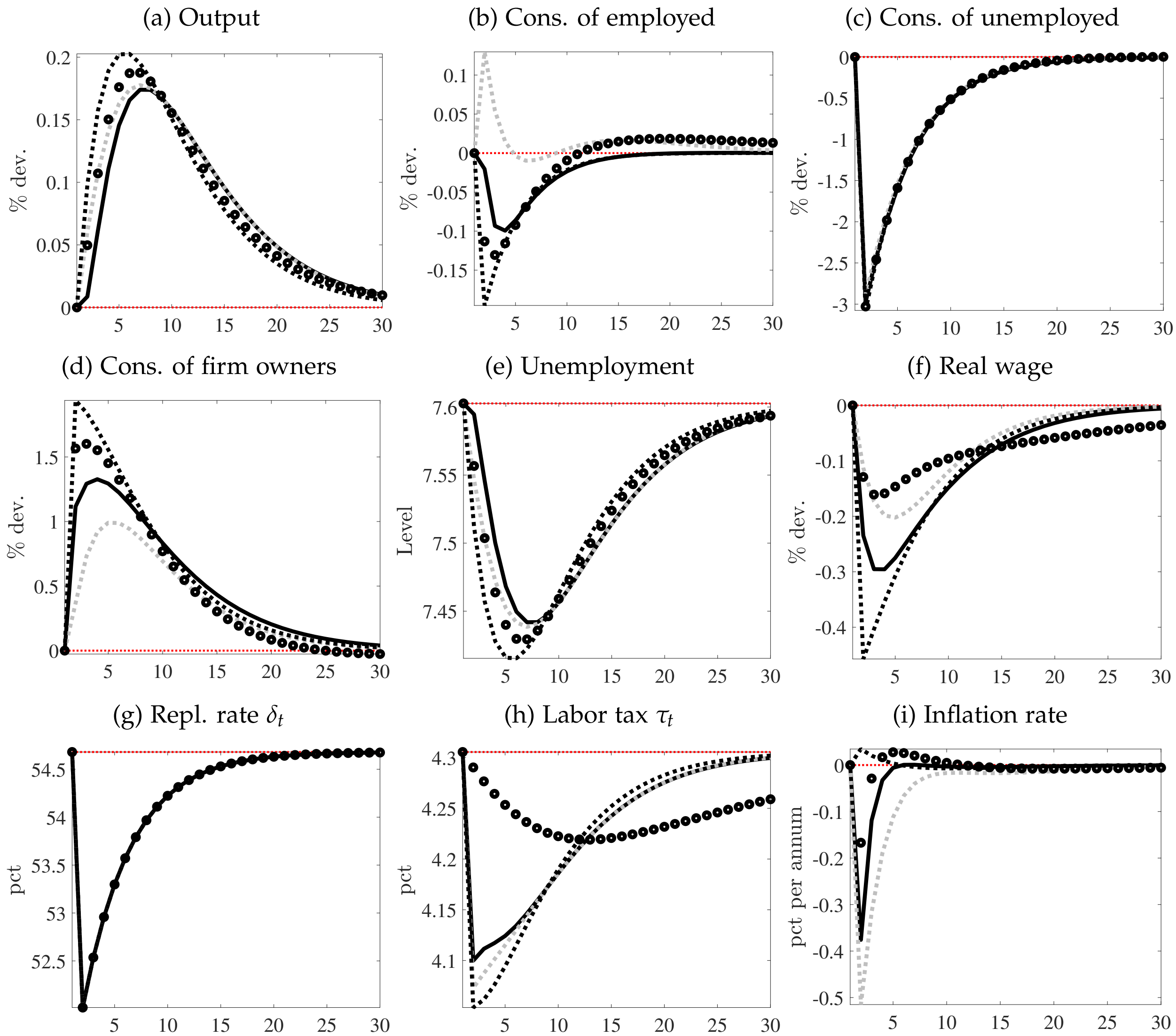
Figure 1 shows that a lower replacement rate δ_t has expansionary effects. The replacement rate basically governs the outside option of workers. As the latter falls – as shown by the drop in the consumption of unemployed workers – so does the equilibrium real wage. This raises the profitability of a match and the creation of new vacancies. It also lowers the unemployment rate and output increases as a result. This mechanism is exactly what the literature coins as the 'vacancy creation' effect of lower unemployment insurance. The consumption of unemployed workers drops equivalently in all case, as it is almost entirely driven by the falling replacement rate.

Even though the shock lowers unemployment, it raises the consumption drop upon job loss because the consumption of unemployed workers falls, inducing a drop in labor-market insurance. This triggers a rise in the precautionary motive and a fall in the real interest rate along with the inflation rate. So unemployed workers lose but other households – employed workers and firm owners – share the resulting output gains. Figure 1 shows that the way this surplus is shared critically depends on the dynamics of the real wage, that is related to assumptions about price and wage stickiness as well as debt adjustment.

In the baseline case or with flexible prices, the real wage falls mildly and the tax rate falls

[11] Since this is a one-time 'MIT' shock, the model is solved under perfect foresight. The algorithm used is a two-point boundary problem using a trust region method and implemented through the Dynare set-up for deterministic simulations (see Adjemian et al. (2011)).

Figure 1: Responses to a persistent shock to the replacement rate δ_t.



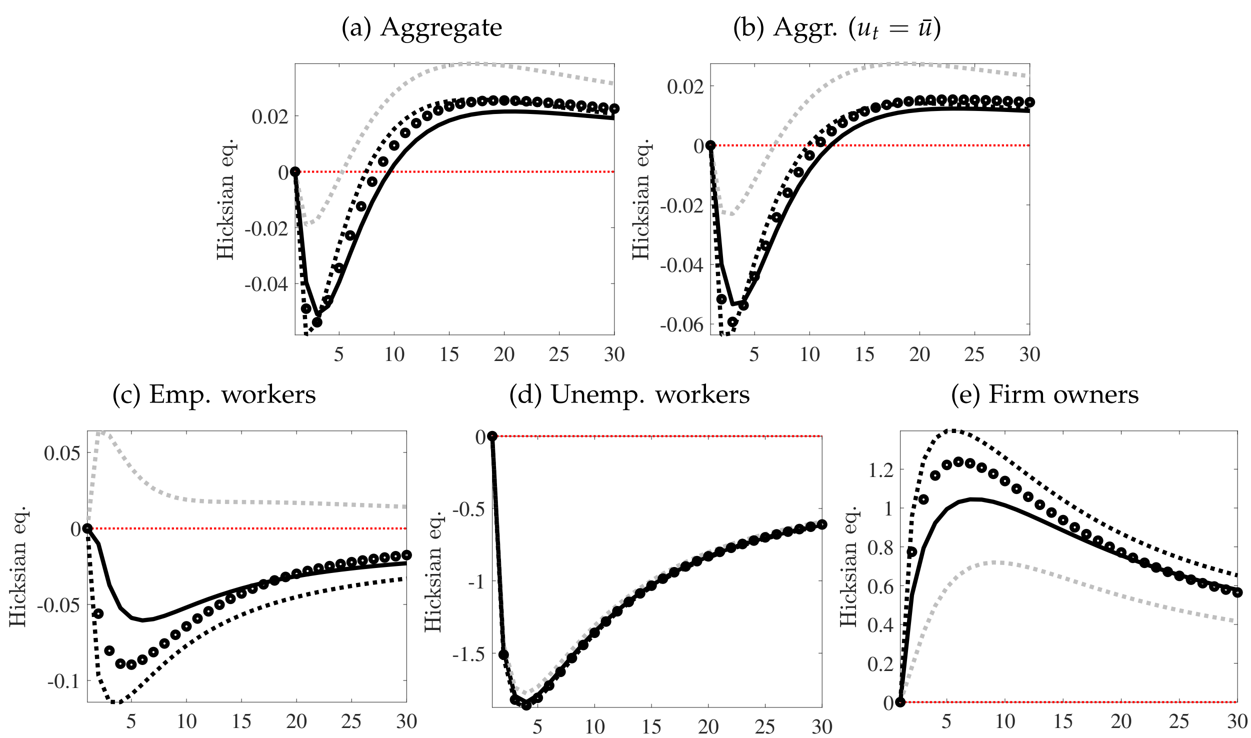
Solid black: Baseline. Dotted grey: Flexible prices. Dotted black: Flexible real wages. Circled: Debt financing substantially – remember the UI system is in equilibrium at all times – so that the consumption of employed workers jumps (flexible prices) or falls mildly (baseline). The consumption of firm owners – which mirrors the dynamics of firms' profits – rises mildly in these two cases. With flexible real wages, the real wage falls much more abruptly. With debt financing the tax rate falls less. As a result, in both cases, the consumption of employed workers drops more substantially – instead of rising or falling mildly – and the consumption of firm owners rise much more. This leads firms to post more vacancies, unemployment to drop more sharply and output to rise more.

The overall welfare effects are indeterminate at this point, and depend on the underlying assumptions regarding prices or real wage rigidity and debt financing. Defining aggregate utility as:

$$\mathcal{U}\left(c_t^e, c_t^u, c_t^f\right) = \chi\left(n_t \log c_t^e + u_t \log c_t^u\right) + (1-\chi)\left(\left(c_t^f\right)^{1-\rho} - 1\right)/(1-\rho_f) \tag{32}$$

we see that the utility of unemployed workers unambiguously shrinks, the utility of firm owners unambiguously rises, and the utility of employed workers can go either way. Further, beyond per-capita changes in welfare, the dynamics of unemployment may affect utility and welfare because employed and unemployed workers have different levels of consumption – and thus welfare. So the aggregate effects are ambiguous. Figure 2 reports the Hicksian equivalent changes implied by the negative shock on the replacement rate at various horizons for the different cases.

Figure 2: Welfare effects of a persistent shock to the replacement rate δ_t.



Solid black: Baseline. Dotted grey: Flexible prices. Dotted black: Flexible real wages. Circled: Debt financing

Figure 2 confirms the ambiguous welfare effects, both among household types but also over time. In all cases, welfare drops in the short run because of the sharp drop in the consumption of unemployed workers, and then rises in the medium run because the profits made by firm owners rise more persistently than the replacement rate falls. The rise or fall in the consumption of employed workers is quantitatively small but as they represent the largest fraction of the household sector, its differential dynamics have significant aggregate effects. Assumptions favoring a rise or a milder drop in their per-capita consumption – baseline and flexible prices – generate smaller short-run aggregate welfare losses, while assumptions leading their per-capita consumption to drop – flexible real wages and debt financing – generate larger short-run aggregate losses.

After 5 to 10 quarters, the shock generates aggregate lifetime welfare gains, driven by firm owners' persistent gains. Most of these gains stem from the creation of new vacancies that improve firms' profits, and only marginally from the composition effect that raises the proportion of employed workers when the unemployment rate drops. From this first-pass analysis we learn that, when choosing the replacement rate optimally, a planner may want to lower it in the long-run because it brings welfare gains. But as lowering it brings short-run welfare losses, the planner may also want to raise the replacement rate in the short-run.

5 Optimal Unemployment Insurance Reform

With the above results in mind, this section asks two questions: Starting from the initial steady state where the replacement rate is $\delta = 0.5468$ and the unemployment $u = 7.6\%$, (*i*) what is the the optimal steady-state replacement rate in the long run? and (*ii*) what is the optimal timing and implementation?

To answer these questions, we solve the dual form of the Ramsey problem. Under Ramsey policies, a benevolent planner credibly commits for an infinity of periods to a sequence of the policy instrument $\{\delta_t\}_{t=0}^{\infty}$ that maximizes the following aggregate lifetime welfare measure:

$$\mathbb{E}_0 \sum_{s=0}^{\infty} \beta_s^s \mathcal{U}\left(c_s^e, c_s^u, c_s^f\right) \tag{33}$$

where $\mathcal{U}$ is defined in Equation (32), subject to the equations defining a competitive equilibrium and given that the economy is in its initial (sub-optimal) steady state in period 0.

The above problem is known to be solvable under two alternative options regarding prior policy commitments. In the first one, the planner adopts a free policy in period 0 and does not take account of any prior commitments. In this case the Lagrange multipliers associated to the dual form of the Ramsey problem are set to zero in period 0. We refer to this policy as 'time-0 optimal policy' in the terms of Woodford (1999a) and Woodford (1999b). In the second case, the planner adopts the 'timeless perspective', by which the Lagrange multipliers of the Ramsey problem are set to their final (optimal) steady-state value. This means that the planner does not take advantage of period 0 to achieve potentially higher welfare levels, because she

Table 2: Initial and Ramsey allocations

Variables ↓	Initial	Ramsey
Output (y)	0.8316	0.8606
Aggregate consumption (c)	0.8235	0.8515
Consumption of unemployed (c^u)	0.7133	0.5120
Consumption of employed (c^e)	0.7925	0.8080
Consumption of firm owners (c^f)	1.3977	1.4694
Real wage (w)	0.8282	0.8186
Unemployment rate (u)	0.0760	0.0437
Job-finding probability (f)	0.1823	0.2863
Labor tax rate (τ)	0.0431	0.0130
Replacement rate (δ)	0.5468	0.2873
Annual real interest rate	0.0341	0.0101

remains consistent with past optimal commitments. Our baseline assumption is the first one, which embeds an additional degree of freedom with respect to the second. In any case the final steady-state is not affected by the assumption regarding prior commitments and this assumption only affects the transition.[12]

5.1 Steady State

We first answer the question of the optimal replacement rate in the long run. The latter is independent from key assumptions regarding price or wage stickiness, debt smoothing or commitment issues regarding period 0, which all affect dynamics.

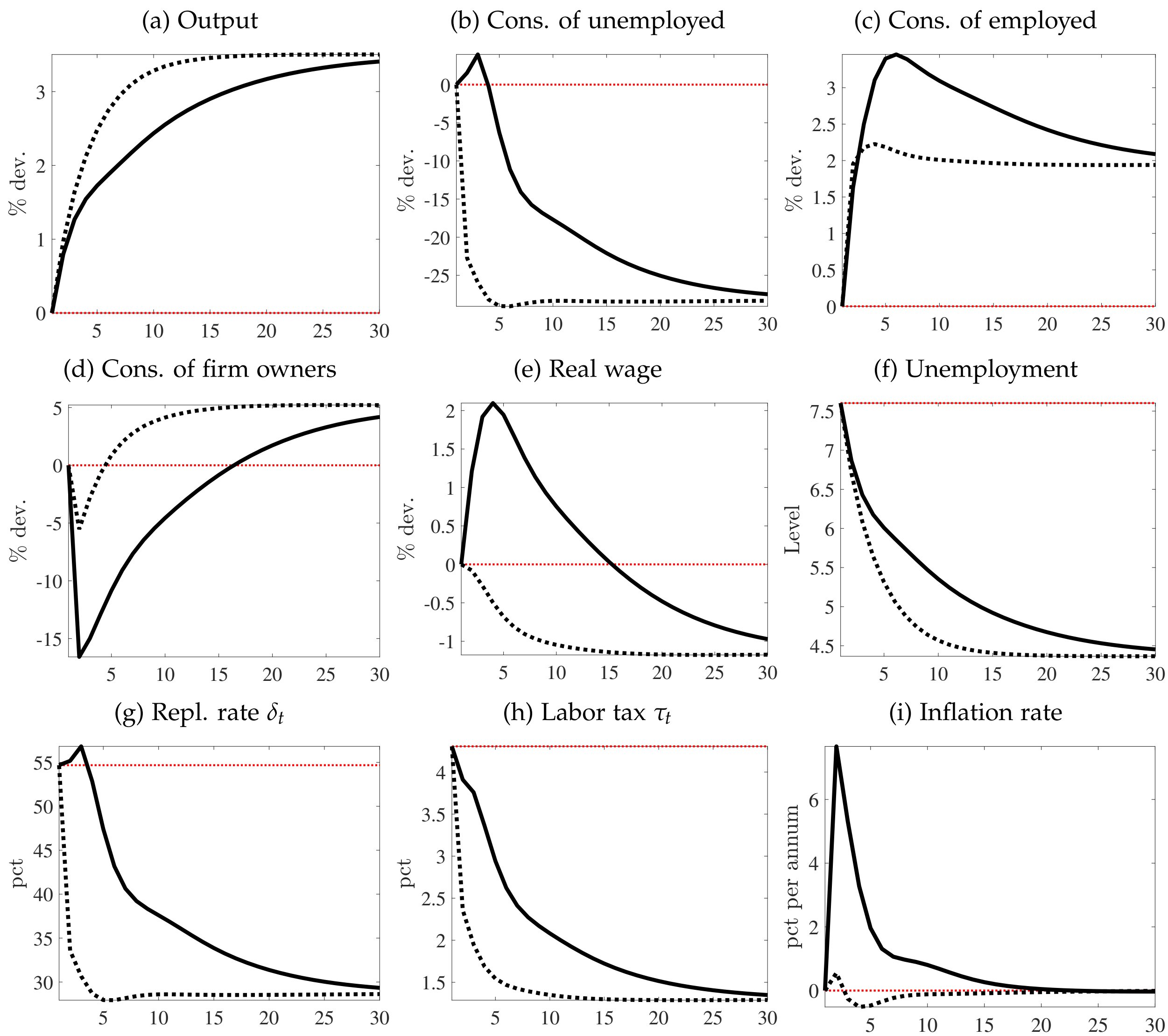
The Ramsey steady state features a lower level of the replacement rate of 28.7 percents against 54.7 percents in the initial steady state. In the long run, the vacancy creation effect clearly dominates: the lower UI benefits reduce the outside option of workers and the real wage drops. Matches become more profitable which pushes firms to post (roughly three times) more vacancies, which then lowers the unemployment rate from 7.6 to 4.4 percents. The joint fall in unemployment and UI benefits allows the government to lower the tax rate from 4.3 to 1.3 percents, which contributes to raise the individual consumption of employed workers in spite of a slightly declining pretax real wage. The aggregate consumption gains stem from three effects: *(i)* the fall in the tax rate that raise the after-tax wage of employed workers, *(ii)* the rise in firm profits that raises firm owners' consumption, and *(iii)* the composition effect by which more workers are employed and consume more than if they remained unemployed. In the mean time, the fall in UI benefits widens the consumption drop upon job loss which raises desired precautionary savings, and drives the real interest rate down.

[12] We make extensive use of the package by Lopez-Salido and Levin (2004) to derive the first-order conditions of the dual Ramsey problem, and then use Dynare to solve the resulting model files.

5.2 Optimal Transition: Time-0 Optimal Policy *vs.* Timeless Perspective

How does the Ramsey planner implement the optimal transition? Figure 3 reports the Ramsey-optimal transition path for the baseline calibration under the time-0 optimal policy and under the timeless perspective.[13]

Figure 3: Ramsey transitions: time-0 optimal *vs.* timeless perspective.



Solid black: time-0 optimal. Dotted black: Timeless perspective. Red: Initial conditions

Figure 3 shows a striking contrast between the optimal dynamics of the replacement rate δ_t depending on assumptions regarding prior commitment.

When the planner adopts the timeless perspective, the replacement rate falls monotonically

[13] Again here, the transition path is obtained solving the dual Ramsey problem under perfect foresight using the same non-linear algorithm.

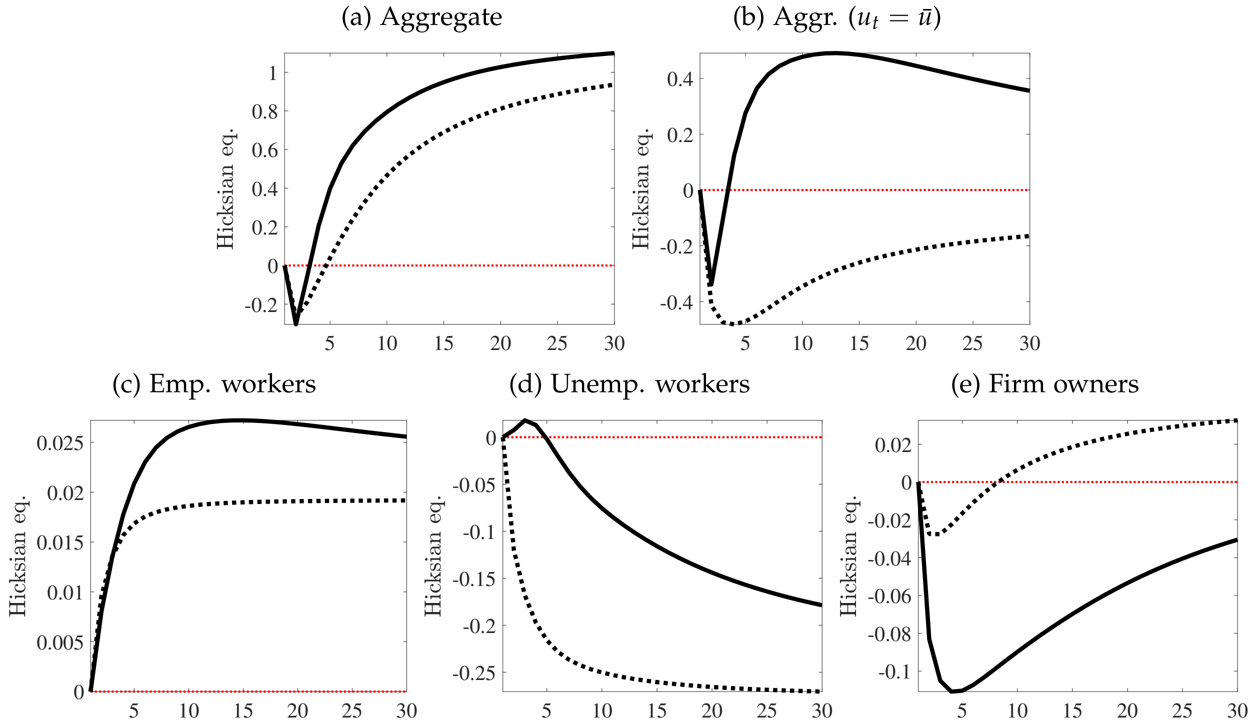
and quickly (5 quarters) to its steady-state value of 28.7%. As a result, the consumption of unemployed drops by more than 25 percents. In addition, real wages fall as the outside option of employed workers shrinks, but slowly because of wage rigidity. More vacancies are posted by firms and the rate of unemployment falls. The sharp reduction in the labor income tax induced by lower unemployment benefits more than compensates the decrease in the real wage, and the consumption of employed workers rises by 2 percents. As seen from Figure 4, the reform generates small aggregate welfare losses during the 5 first quarters, driven by the rapid fall in the consumption of unemployed workers. Aggregate welfare gains materialize after 5 quarters, and are slightly less than 1 percent of permanent consumption equivalent in the long run.

The time-0 optimal policy shows a different transition path. In this case, the planner takes advantage of the fact that its decisions in period 0 will not affect past vacancies or jobs, which are predetermined, but only future vacancies and jobs. As such, the planner chooses to *raise* the replacement rate on impact for 2 periods before *lowering* it towards its long-run value, but more *slowly* than with the timeless perspective. As in Mitman and Rabinovich (2021), commitment to lower UI in the future enables the planner to raise it today and the hiring activities of forward-looking firms are not hurt. This policy generates a temporary increase in the real wage because the outside option of employed workers rises on impact. Due to wage and price stickiness, it then takes more than 15 quarters for the real wage to fall below its initial value. These wage dynamics come at not cost in terms of unemployment, because the commitment (made in period 0) to lower the replacement rate after 2 quarters still lowers the intertemporal cost of vacancies. The latter effect leads firms to post more of them starting in period 0, and lowers the unemployment rate monotonically although at a slower pace than under the timeless policy.

The time-0 optimal wage dynamics have implications for the welfare gains of the reform, and for their distribution among households, as seen from Figure 4. First, the slower decay of the replacement rate induces a slower drop in the consumption of unemployed workers. Second, the initial rise of the real wage combines with the drop in the tax rate to produce an overshooting of the consumption of employed workers. Third, the consumption of firm owners decreases much more on impact and takes much longer to jump above its initial value. The time-0 optimal wage dynamics also has implications for inflation dynamics, as seen from Figure 3. The reform induces a very large increase in inflation because (*i*) the rising real wage increases the marginal production cost and (*ii*) the reform implies a drop – instead of an increase – in unemployment income risk, reflected in a rise of the private real interest rate.

Figure 4 shows that the time-0 optimal policy welfare dominates the timeless perspective policy reform. The latter generates average lifetime welfare gains above 1% of equivalent aggregate consumption while the former generate lifetime welfare gains below 1%. The graphical analysis of the transition already highlighted the different welfare effects: larger lifetime gains for employed workers, lower lifetime losses for unemployed, and lifetime losses for firm owners instead of lifetime gains. In addition, note that the timeless perspective reform does not bring welfare gains if the unemployment rate remains at its initial level (Figure 4, panel (b)) while the

Figure 4: Welfare effects of Ramsey transition, in percents.



Solid black: time-0 optimal. Dotted black: Timeless perspective.

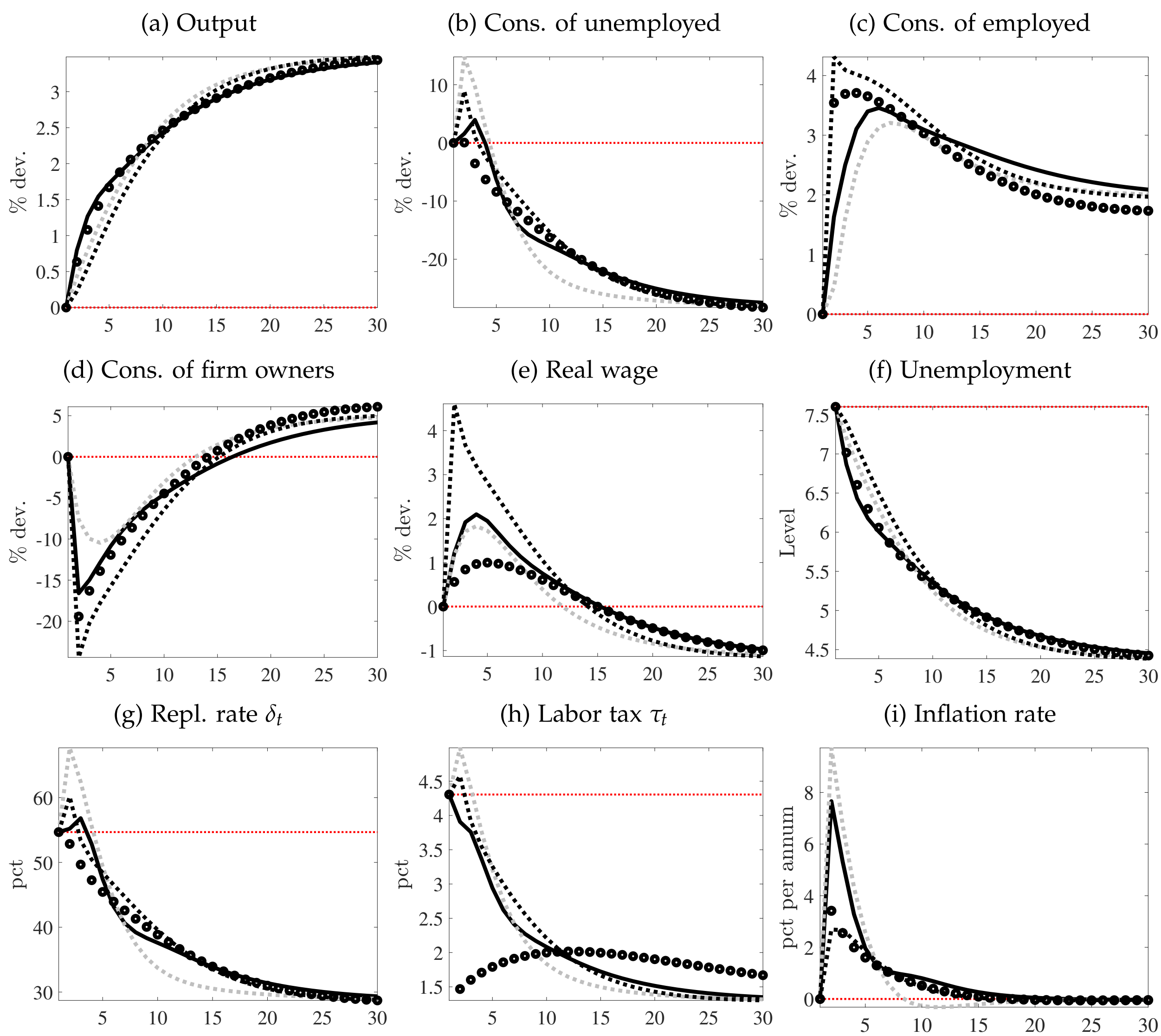
time-0 optimal reform does, by favoring a smoother transition for workers, who outnumber firm owners in our model. The remainder of the section considers time-0 optimal transition paths, and investigates the effects of other assumptions on the optimal transition.

5.3 Optimal Transition: The Role of Assumptions

We now question the role of price and wage stickiness in the optimal transition path. We also look at the role of debt financing. All these assumptions may matter because they contribute to shape the response of the real wage to changes in the replacement rate, as shown when looking at an exogenous shock on δ_t (see Section 4). If the planner targets a particular path for the real wage, its implementation through changes in the replacement rate may thus differ depending on these assumptions, as shown in Figure 5.

Figure 5 reports time-0 optimal transition dynamics and shows that alternative assumptions have qualitatively similar implications for the optimal path of the real wage: above its initial value for 10 to 15 quarters and then decreasing towards its long-run value. Quantitatively however, the short-run increase in the real wage differs depending on assumptions: more than 4% above its initial steady-state value with a flexible real wage, and less than 1% with debt financing. These qualitatively similar real wage trajectories imply very similar decreasing dynamics for the unemployment rate both qualitatively and quantitatively, from 7.6% to its long-run value of 4.4%,

Figure 5: Ramsey transitions – time-0 optimal paths.



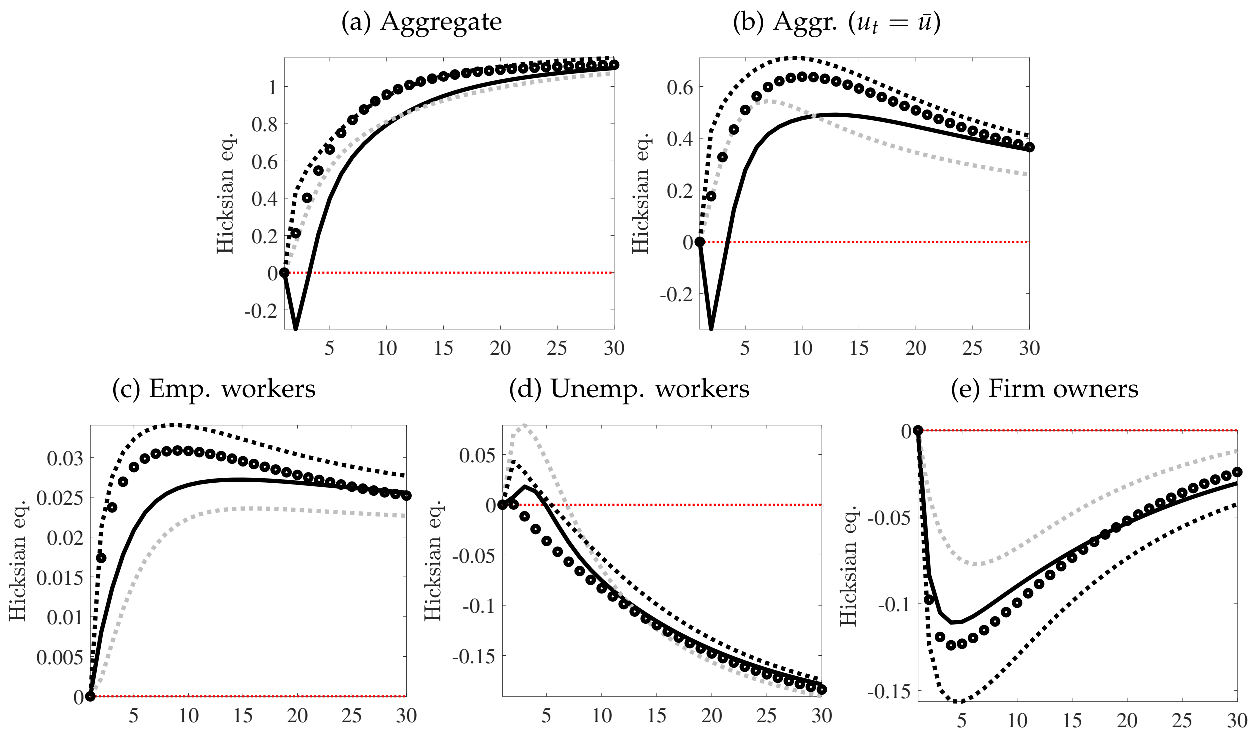
Solid black: Baseline. Dotted grey: Flexible prices. Dotted black: Flexible real wages. Circled: Debt financing.

and comparably increasing dynamics of aggregate output.

The ways UI reforms are implementated differ though: the initial spike in the replacement rate δ_t is larger when the real wage is flexible – rising from 54.7% to 60.1% – and much larger when prices are flexible – rising from 54.7% to almost 67.7%. Under debt financing, the replacement rate falls monotonically because the sharp immediate drop in the proportional tax rate is enough to raise workers surplus and engineer the desired short-run rise in the real wage.

As a result of these different wage and replacement rate dynamics, the welfare implications of reforms under alternative assumption differ as seen in Figure 6.

Figure 6: Welfare effects of Ramsey transitions, in percents.



Solid black: Baseline. Dotted grey: Flexible prices. Dotted black: Flexible real wages. Circled: Debt financing.

Figure 6 shows that the welfare gains/losses from unemployed workers (panel (d)) exactly mirror the dynamics of the replacement rate, and that the reform under flexible prices yields the largest welfare gains in the short-run. For employed workers (panel (c)), the welfare gains depend on the size of the real wage increase and of the drop in the tax rate. The largest short-run gains stem from the case of a flexible real wage, followed by debt financing – where the large gains from lower labor taxation are distributed immediately – and the lowest welfare gains occur under flexible prices. The welfare effects experienced by firm owners (panel (e)) rank in inverse order compared to employed workers: lowest losses under flexible prices, higher losses under flexible real wages.

5.4 Summary

In our model, optimal reforms of the unemployment insurance system imply a reduction of the replacement rate in the long run. This lowers the outside option of workers, reduces labor cost and boosts vacancy creation. Our simulations suggest a potentially large employment gain: unemployment falls from 7.6% to 4.4%. In the short-run however, optimal transition paths depend on key assumptions. If the planner is tied by prior commitments, the replacement rate falls monotonically. But if the planner is allowed to take advantage of time 0, she can smooth the negative short-run effects of the reform on workers. Indeed, unemployed workers loose because the replacement rate falls, employed workers loose because the real wage falls – although the losses are more than compensated by lower labor income tax rates. The planner can alleviate these adverse short-run effects by increasing the real wage in the early stages of the reform, essentially by raising the replacement rate and committing to lowering it in the long run. The exact implementation of the time-0 optimal policy depends on key assumptions regarding real wage and price stickiness, and on whether the planner uses debt over the transition.

6 Optimal Unemployment Insurance over the Business Cycle

We now investigate the Ramsey-optimal unemployment insurance in the event of shocks hitting when the economy is at its optimal steady state.

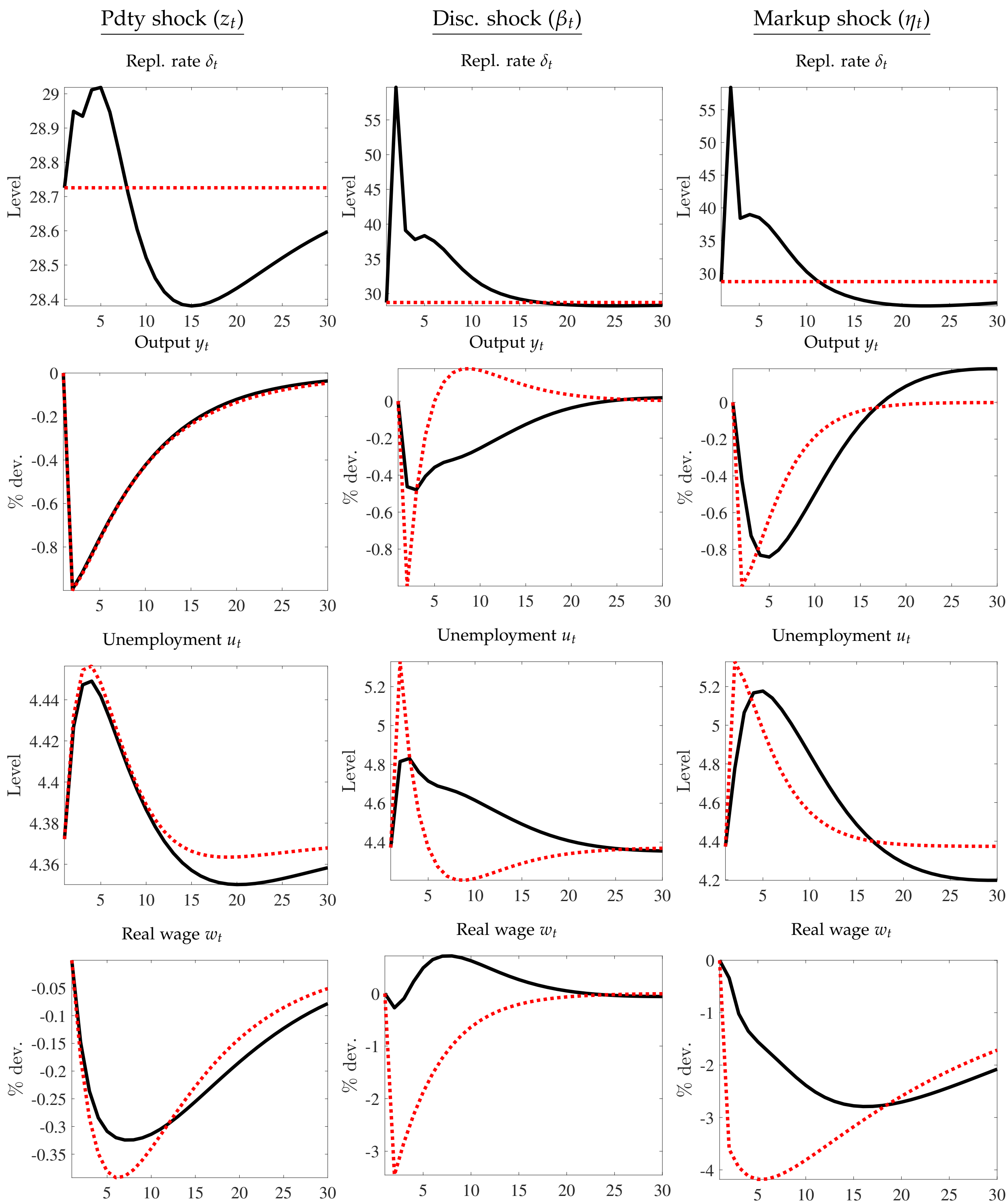
6.1 Impulse responses

We start by considering three scenarios producing an economic downturn: a negative productivity shock, a rise in the discount factor aimed at capturing contractions in aggregate demand, and an adverse markup shock corresponding to a drop in the elasticity across varieties.[14] We make use of persistence parameters estimated in Section 3, and normalize innovations to produce a 1% drop in output. Figure 7 reports the impulse response functions (IRFs) under the Ramsey-optimal UI policy for the three shocks, and compares them to the counterfactual case where the replacement rate remains constant.

All three recessionary shocks produce qualitatively similar negative dynamics for output and unemployment. While shocks are calibrated to produce similar recessions with a constant replacement rate (red curves), they do not result in similar changes in unemployment: unemployment increases from 4.37% to 4.46% after a negative productivity shock while it rises to 5.3% for the two other shocks. The much larger rise in unemployment produced by these shocks is also met with a much greater fall in the real wage.

[14]Since these are again one-time 'MIT' shocks, the model is solved under perfect foresight using the Ramsey steady state with the same non-linear algorithm. Given the low steady-state value of the nominal interest rate, we impose a zero lower bound, a non-linearity that the algorithm handles easily given the nature of the exercise – although not for any size of shocks.

Figure 7: Impulse response functions to negative shocks.



Solid black: Baseline. Red: constant replacement rate.
All shocks are normalized to produce a 1% drop in output in the case of a constant replacement rate.

The optimal response of the replacement rate somehow reflects the above differences: it remains almost constant in the case of productivity shocks – it increases from 28.7% to 29% – but increases significantly for the two other shocks, from 28.7% to almost 60%. Hence, when the economy is at its Ramsey steady state, fluctuations in output and unemployment driven by productivity shocks are close to efficient and require little action in terms of unemployment insurance. On the contrary, fluctuations driven by discount factor or markup shocks are inefficient and require large and counter-cyclical changes in the replacement rate. In this case, the vacancy creation effect is weaker and the insurance motive stronger. The latter merges with what we label the 'aggregate demand stabilization motive' following Gorn and Trigari (2024). Indeed, the rise in the replacement rate does not only reduce the gap between the consumption of unemployed and employed workers, it also stimulates aggregate demand in a way that boosts output and vacancy creations. As a result, the optimal rise in UI benefits dampens the responses of output and unemployment by roughly 50% after a discount factor shock and produces a hump-shaped rise in the real wage. In the case of markup shocks, even tough the replacement rate rises to almost 60%, its stabilizing power is more limited and fluctuations in output and unemployment are less reduced.

What is the role of our various assumptions in producing these results? Figure 8 compares the response of our model economy in the baseline case to the response obtained under flexible prices, flexible real wages and debt financing.

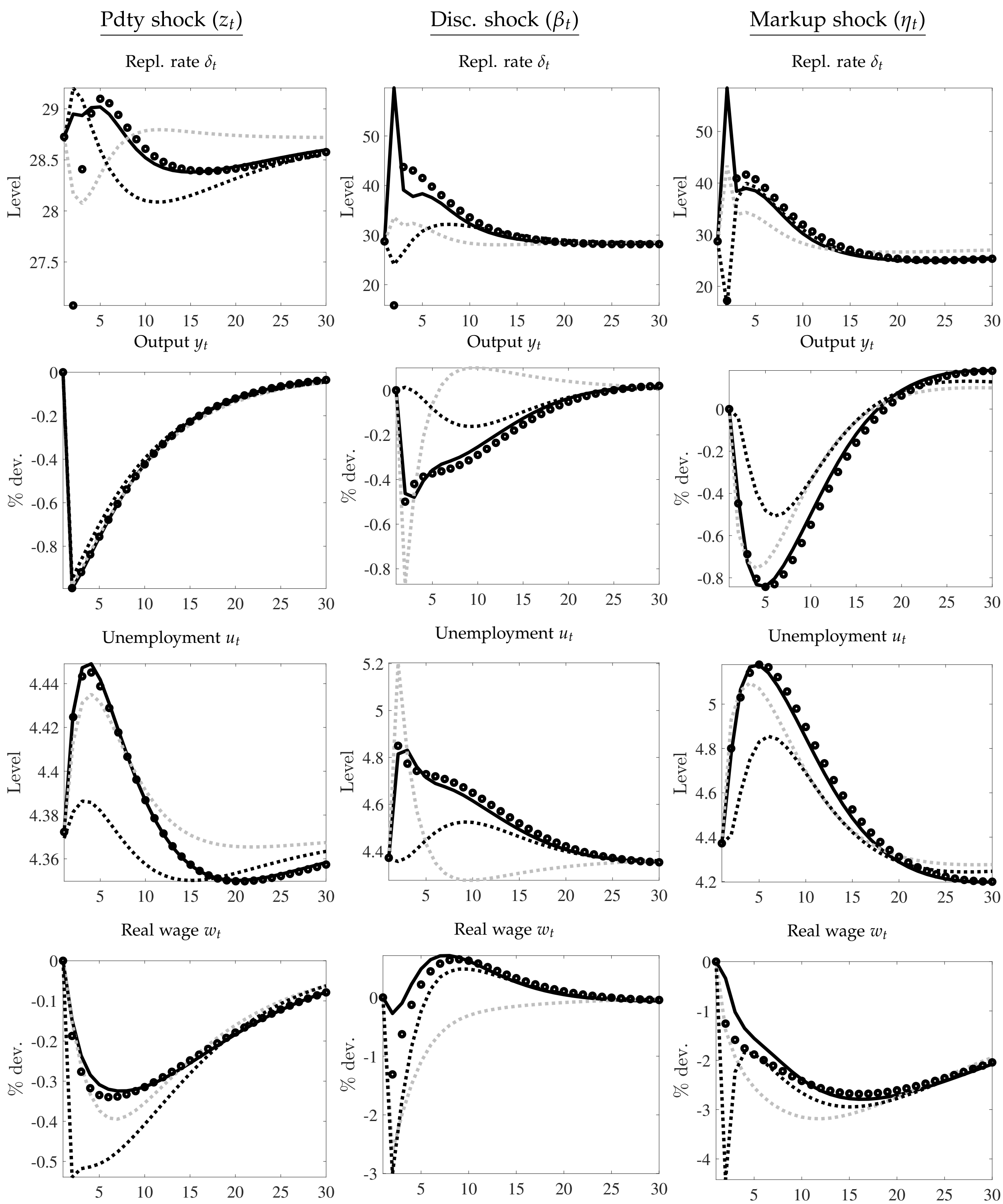
In the baseline case, we find that fluctuations are close to efficient when the economy is driven by productivity shocks, calling for little changes in the replacement rate. The same reasoning applies under alternative assumptions: optimal changes in the replacement rate remain very small and comparable to those obtained in the baseline case.

When fluctuations are driven by other shocks, the optimal response of UI benefits changes more drastically depending on our key assumptions, and in a very similar way for discount factor and markup shocks.

Under flexible prices, the replacement rate remains counter-cyclical but the size of changes is dramatically dampened: it rises to 33% after a discount factor shock and to 44% after a markup shock. These number contrast with the 60% replacement rate obtained with the same shocks with sticky prices. When prices are flexible, fluctuations in output and unemployment driven by discount factor and markup shocks are much less inefficient, calling for a less active policy in terms of UI benefits because the aggregate demand stabilization motive is almost completely muted.

Under flexible real wages, the replacement rate becomes pro-cyclical on impact and remains counter-cyclical over the medium run, as it overshoots its long-run value in period 2 before converging to the steady state from above. These changes produce a persistent drop in the real wage that significantly reduces the size of fluctuations in output and unemployment. Finally, debt smoothing has little qualitative effects except on impact: compared to the balanced-budget

Figure 8: Impulse response functions - sensitivity.



Solid black: Baseline. Dotted grey: Flexible prices. Dotted black: Flexible real wages. Circled: Debt financing. All shocks are normalized to produce a 1% drop in output in the case of a constant replacement rate.

case, it simply allows for a smaller impact rise in the replacement rate, followed by a slower decrease. As a result, changes in output and unemployment are almost not affected.

6.2 Business Cycle and Welfare Gains

How large are the potential gains from optimal UI policy over the business cycle? Given the above results, the answer should depend on the relative importance of shocks and on price stickiness. If productivity shocks are the main driving force of business cycles or if prices are flexible, the potential welfare gains should be small. If price are sticky and if other shocks play a key role in driving the business cycle, welfare gains should be large. It is then a matter of relative importance of shocks in driving the business cycle, which justifies our estimation of shocks from the data in Section 3.

We simulate the model around the Ramsey steady state with our estimated parameters for shocks.[15] We then compare the business cycle properties of the model under alternative assumptions regarding the replacement rate – constant vs. Ramsey-optimal. We also proceed to a welfare evaluation looking at the difference between conditional and unconditional welfare means and convert them in percents of consumption equivalents. We do it at the aggregate level and for each type of household. The results are reported in Table 3.

Table 3: Optimal UI policy - business cycle and welfare

Calibration →	Baseline				Flexible prices				Flexible real wage			
UI policy →	$\delta_t = \delta$		Optimal		$\delta_t = \delta$		Optimal		$\delta_t = \delta$		Optimal	
↓ Variable (x)	$\sigma(x)$	$\rho(x,y)$	$\sigma(x)$	$\rho(x,y)$	$\sigma(x)$	$\rho(x,y)$	$\sigma(x)$	$\rho(x,y)$	$\sigma(x)$	$\rho(x,y)$	$\sigma(x)$	$\rho(x,y)$
Output (y_t)	1.53	–	1.55	–	1.48	–	1.46	–	1.36	–	1.38	–
Unemp. rate (u_t)	6.56	−0.72	7.61	−0.71	4.77	−0.73	5.06	−0.65	1.11	−0.58	3.78	−0.40
Aggregate cons. (c_t)	1.41	0.98	1.41	0.98	1.38	0.99	1.38	0.99	1.34	1.00	1.35	1.00
Cons. unemp. (c_t^u)	0.59	0.61	14.39	−0.24	0.48	0.55	2.85	0.60	0.98	0.63	7.78	−0.16
Cons. employed (c_t^e)	1.32	0.61	1.00	0.67	1.06	0.55	1.05	0.58	2.18	0.63	1.84	0.67
Cons. firm own. (c_t^f)	5.84	0.53	5.11	0.79	5.68	0.73	5.34	0.72	7.34	0.15	6.49	0.29
Real wage (w_t)	1.25	0.59	0.84	0.58	1.01	0.53	1.00	0.61	2.17	0.63	1.84	0.62
Replacement rate (δ_t)	–	–	33.24	−0.25	–	–	6.61	0.50	–	–	17.51	−0.24
↓ Welf. effect of BC												
Aggregate	−1.82		−1.64		−1.49		−1.48		−1.58		−1.49	
Unemployed	−0.97		−2.37		−0.89		−2.74		−0.87		−1.61	
Employed	−2.13		−2.05		−1.96		−1.91		−1.91		−1.77	
Firm owners	+11.74		+12.81		+13.16		+13.41		+12.55		+12.35	

Notes: Models are solved using a second-order approximation around the Ramsey steady state. Variables are then taken in log-deviation from their ergodic mean and HP-filtered before computing the business cycle moments. $\sigma(x)$ denotes the standard deviation of variable x, expressed in percent. $\rho(x,y)$ means contemporaneous correlation with output. Welfare numbers reports the difference between conditional and unconditional level of welfare, expressed in percents of steady-state consumption. A '-' denotes welfare losses from fluctuations while a '+' indicates welfare gains. $\delta_t = \delta$ is the case of a constant replacement rate and 'optimal' refers to the Ramsey UI policy over the business cycle.

[15] We solve using a second-order approximation around the Ramsey steady state using perturbation methods.

Let us first focus on the business cycle properties of our economy when the replacement rate is constant ($\delta_t = \delta$). Table 3 shows that our model produces reasonable business cycle features. With a standard deviation around 1.5%, output is as volatile as in the data. Unemployment is clearly counter-cyclical and between 3 times and 4.5 times more volatile than output.[16] Aggregate consumption and the real wage are less volatile than output and pro-cyclical.

In the baseline case, the effects of an active UI policy are the following. Changes in the replacement rate δ_t are relatively large, as shown by the 33.2% reported volatility, and counter-cyclical. They mainly result in a stabilization of the real wage, reducing its volatility from 1.25% to 0.8%, which then implies less volatility for the consumption of employed workers and firm owners, and more volatility in the unemployment rate, from 6.56% to 7.61%. Large changes in the replacement rate also massively raise the volatility of consumption of unemployed workers, from 0.59% to 14.39%.

On the one hand, as a result of a reduced consumption volatility for employed workers and firm owners, an optimal UI policy lowers their welfare losses from fluctuations, by 0.08% for employed workers (from 2.13% to 2.05%) and by 1.07% for firm owners (from -11.74% to -12.81%). On the other hand, the magnified consumption volatility for unemployed workers results in much larger (1.4%) welfare losses from fluctuations, from 0.97% to 2.37%. So an optimal UI policy over the business cycle redistributes welfare losses from business cycles from employed workers and firm owners to unemployed workers. However, because the former represent a much larger share of the household sector compared to the latter, an optimal UI policy over the business cycle yields lower (0.18%) aggregate welfare losses from fluctuations compared to the case of a constant replacement rate, from 1.82% to 1.64%.

What if prices are flexible? First, in this case, the replacement rate becomes weakly pro-cyclical and much less volatile than under sticky prices. This is a product of productivity shocks becoming much more important in the determination of the optimal UI policy over the business cycle. Second, the welfare losses from fluctuations under a passive policy are lower than under sticky prices, and thus the scope for potential welfare improvements through an optimal UI policy is rather small: the aggregate welfare gains from an optimal UI policy become negligible (0.01%), as the optimal UI policy brings the aggregate welfare losses from fluctuations from 1.49% to 1.48%. These results are fully consistent with our shock analysis and with the aggregate demand stabilization motive being muted under flexible prices.

Last, considering a flexible real wage dramatically lowers the volatility of the unemployment rate, and raises the volatility of the real wage as well as consumption volatility for all types of households. This is clearly not surprising as real wage inertia is a well-known fix to the so-called Shimer (2005) puzzle. In this case however, as in the baseline case, an optimal UI policy calls for a counter-cyclical replacement rate to stabilize the real wage, producing moderate welfare gains

[16] Except when the real wage is flexible, a feature that is known and expected in models with search and matching models (see Shimer (2005) for a discussion).

(0.09%) – aggregate welfare losses from fluctuations drop from 1.58% to 1.49%. An optimal UI policy over the business cycle generates the same qualitative welfare pattern as in the baseline case: employed workers and firm owners gain (0.14% and 0.2% respectively) while unemployed workers lose (-0.74%). The chief reason behind these results is that part of the welfare costs from business cycles are still present because prices are sticky. This keeps on making discount factor and markup shocks important and inefficient drivers of the business cycle, calling for an active UI policy.

6.3 Discussion

What can be learned of the above results in terms of policy? A first lesson is that the potential aggregate welfare gains from a structural reform are larger than the potential gains of an optimal policy along the business cycle. In other terms, an optimal reform implementing a lower replacement rate with an optimal transition period already allows policymakers to grasp most of the potential welfare gains, even if the replacement rate subsequently remains constant at its optimal value in the face of fluctuations.

A second important message is that, conditional on negative demand shocks (discount factor shocks in our model) and given that monetary policy shocks would have isomorphic effects, the optimal response consists in increasing the generosity of unemployment benefits. This message aligns almost perfectly with the results of Kekre (2022) and Gorn and Trigari (2024), who document strong stabilizing power of unemployment benefit extensions when the economy is hit by negative demand shocks.

A third lesson is that, once the optimal replacement rate is in place, a policy keeping the replacement constant would be optimal if other policy instruments such as monetary or fiscal policy perfectly stabilized demand shocks. The results of our baseline model for a monetary policy achieving full price stability ($d_\pi \to \infty$) are exactly isomorphic to those arising under flexible prices (and therefore not reported). Of course, in case other policy instruments are constrained, either by the institutional set-up – monetary policy is suboptimal and/or decided by a different policy institution than the institution in charge of the design of UI – or by the zero lower bound on the nominal interest rate, a counter-cyclical UI policy during large negative demand shocks would help smoothing the shocks and bringing welfare gains.

In our analysis, we left some assumptions aside for the sake of tractability. Two assumptions strike us as potentially relevant to the problem of an optimal UI policy: endogenous search effort of unemployed workers, and endogenous separations. Endogenous search effort is usually introduced as a non-convex cost in the utility or value function of unemployed workers, and affects positively the job-finding probability, as in Moyen, Stähler, and Winkler (2019). If effort depends positively on the job surplus, then it is very likely to be pro-cyclical and to amplify unemployment fluctuations. As such it would magnify rather than dampen our effects. Regarding endogenous separations, Fujita and Ramey (2009) find that separation rates are counter-cyclical in the data, and as such contribute to amplify fluctuations of the unemployment rate. Adding endogenous separations to our model would thus likely reinforce our results.

7 Conclusion

We propose a THANK model with endogenous unemployment risk and sticky prices to analyze the macroeconomic and distributional effects of an optimal UI policy. After calibrating the model to represent the average economy of the Euro Area, we characterize the optimal policy both in the transition, and over the business cycle, *i.e.* after the transition is achieved.

In the long run, the optimal reform implies a reduction of the replacement rate, which boosts vacancy creations and lowers unemployment. In our baseline experiment, unemployment fall from 7.6% to 4.4%. In the short-run however, if the planner is not tied by past commitments, she raises the replacement rate on impact while it commits to lower it in the long run. This generates a smoother path for the real wage and produces larger welfare gains from the reform than an abrupt drop in the replacement rate. The exact timing and size of changes in the replacement rate however both depend on key assumptions regarding real wage inertia and price stickiness.

Around the Ramsey-optimal steady state, we find that the optimal UI policy in response to shocks depends (*i*) on the nature of shocks and (*ii*) on the presence of price stickiness and wage inertia. Productivity shocks generate quasi-efficient fluctuations requiring little changes in the replacement rate. Discount factor or markup shocks generate inefficient fluctuations calling for large counter-cyclical changes in the replacement rate to help stabilize aggregate demand in response to these shocks. If prices are flexible, the role and potential welfare gains from an active UI policy becomes negligible provided the replacement rate is at its optimal level.

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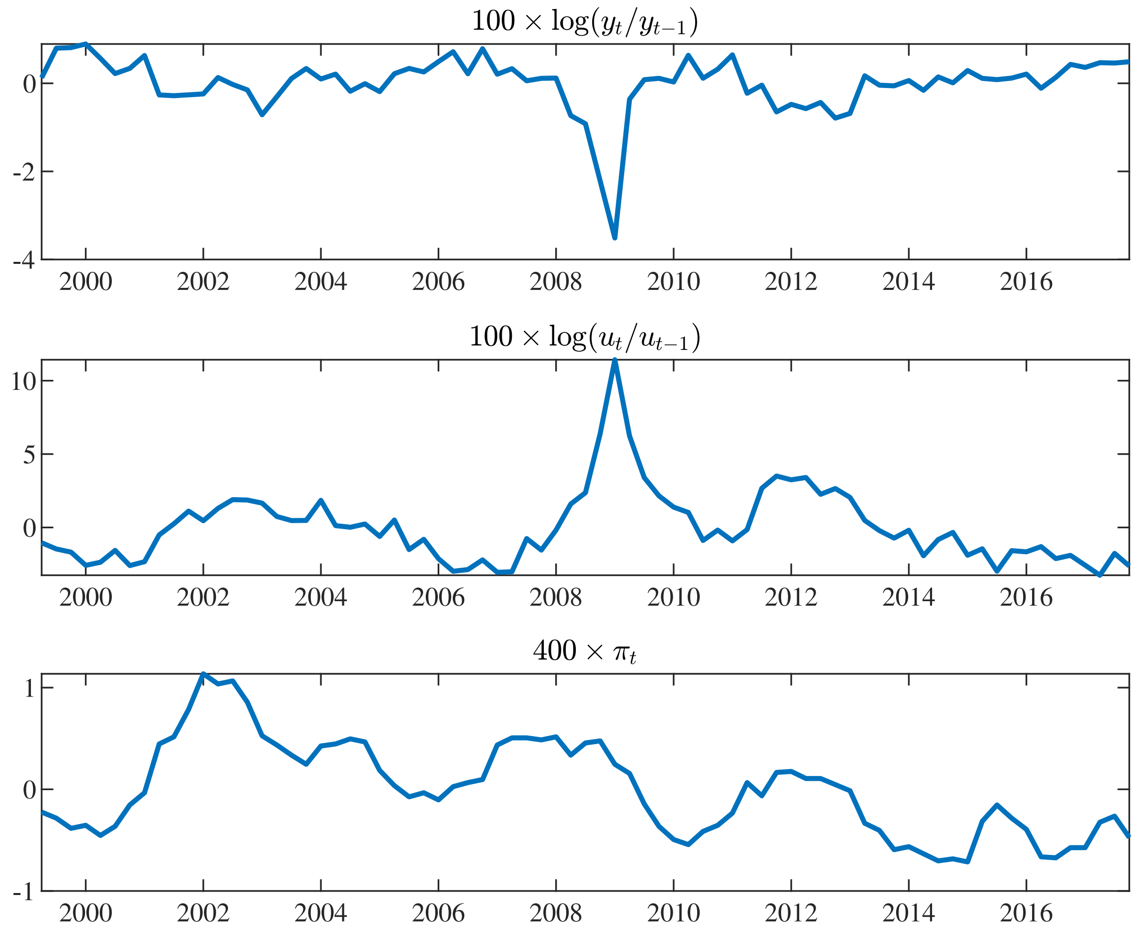
A Summary of equilibrium conditions

$$
\begin{aligned}
1 &= \mathbb{E}_t \left\{ \beta_t \left(1 + r_t\right) \left(\left(1 - s\left(1 - f_{t+1}\right)\right) \frac{c_t^e}{c_{t+1}^e} + \sigma_{t+1} \frac{c_t^e}{c_{t+1}^u} \right) \right\} \\
1 &= \mathbb{E}_t \left\{ \beta_t \left(1 + r_t^d\right) \left(c_t^f / c_{t+1}^f \right)^{\rho} \right\} \\
c_t^e &= \left(1 - \tau_t\right) w_t \\
c_t^u &= \left(1 - \tau_t\right) \delta_t w_t + \mu \\
y_t &= \chi n_t z_t \\
J_t &= \varphi_t z_t - w_t + \mathbb{E}_t \left\{ \beta_t \left(c_t^f / c_{t+1}^f \right)^{\rho} \left(1 - s\right) J_{t+1} \right\} \\
\kappa &= q_t J_t \\
\eta_t - 1 &= \eta_t \varphi_t - \phi \left(\pi_t (1 + \pi_t) - \mathbb{E}_t \left\{ \beta_t \left(c_t^f / c_{t+1}^f \right)^{\rho} \pi_{t+1} (1 + \pi_{t+1}) y_{t+1} / y_t \right\} \right) \\
n_t &= \left(1 - s\left(1 - f_t\right)\right) n_{t-1} + f_t \left(1 - n_{t-1}\right) \\
q_t &= \psi \left(\frac{u_{t-1} + s n_{t-1}}{v_t} \right)^{\gamma} \\
f_t &= \psi \left(\frac{v_t}{u_{t-1} + s n_{t-1}} \right)^{1-\gamma} \\
u_t &= 1 - n_t \\
S_t &= \log\left(c_t^e\right) - \log\left(c_t^u\right) + \beta_t \mathbb{E}_t \left\{ \left(1 - s\left(1 - f_{t+1}\right) - f_{t+1}\right) S_{t+1} \right\} \\
w_{nt} &= \frac{\theta \kappa / q_t}{\left(1 - \theta\right) S_t} \\
w_t &= w_{t-1}^{\alpha} w_{nt}^{1-\alpha} \\
c_t &= \chi \left(n_t c_t^e + u_t \left(c_t^u - \mu \right) \right) + \left(1 - \chi\right) c_t^f \\
y_t &= c_t + \kappa_t v_t + \phi \pi_t^2 y_t / 2 \\
d_t &= (1 + r_{t-1}^d) d_{t-1} + \chi u_t b_t - \tau_t \chi \left(n_t + \delta_t u_t \right) w_t, \; d_\infty = 0 \\
\tau_t &= \tau + \phi_\tau \left(d_t - d_\infty \right) \\
\log\left(\frac{1 + i_t}{1 + i} \right) &= d_\pi \log\left(1 + \pi_t\right) \\
1 + r_t &= \mathbb{E}_t \left\{ \frac{1 + i_t}{1 + \pi_{t+1}} \right\} \\
\log z_t &= \rho_z \log z_{t-1} + \sigma_z \varepsilon_{zt} \\
\log\left(\beta_t / \beta\right) &= \rho_\beta \log\left(\beta_{t-1} / \beta\right) + \sigma_\beta \varepsilon_{\beta t} \\
\log\left(\eta_t / \eta\right) &= \rho_\eta \log\left(\eta_{t-1} / \eta\right) + \sigma_\eta \varepsilon_{\eta t}
\end{aligned}
$$

B Data and posterior distributions

Figure 9: Data used for estimation.



Real GDP y_t, the real wage w_t, the unemployment rate u_t and the HICP inflation rate are taken from the AWM database. All variables range from 1999Q2 to 2017Q4. Graphs represent the demeaned log-difference of output y_t, the unemployment rate u_t, the real wage w_t and the demeaned inflation rate of the HICP.

Figure 10: Prior and posterior distributions.