Aging, Fertility and Macroeconomic Dynamics

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Abstract

A tractable model with heterogeneous households is proposed to analyze the two-way interactions between demographic and macroeconomic variables. Total population and labor-market participation are both endogenous and affected by economic as well as demographic factors. In addition, demographic factors have direct effects on aggregate productivity through selection effects on the labor market. We show that aging and negative fertility shocks have opposite predictions in terms of their effects on GDP per capita and aggregate productivity. A quantitative exercise based on Japanese data suggests that an aging shock alone has relatively little effects and falls short in replicating the data, while considering negative fertility shocks fits the data much better. Keywords: Heterogeneous workers, Aging, Productivity, Labor markets.
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1 Introduction

Several advanced economies experience structural demographic changes such as rising life expectancy and lowered fertility. Those structural changes are often claimed to have deep economic consequences such as declining interest rates, driven by higher saving rates, lower labor-market dynamism because of shrinking active populations and stagnating productivity gains. Those trends are clearly observable in the data of many European or Asian countries and originated large amount of research to understand the economic effects of aging in developed economies or how it affects the conduct of fiscal and monetary policy. Most contributions on the subject rely on overlapping generation models (OLG) in which demographic factors are essentially exogenous, and highlight the aggregate effects of aging on savings, debt or monetary policy through the channel of pensions or retirement schemes. In addition, little (if any) studies highlight the

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potential endogenous role of demographic factors on productivity or the feedback effects that the economic environment might have on demographic factors themselves.

In this paper, we propose an alternative approach that links demographic and macroeconomic factors in a novel way to analyze the effects of aging and lowered fertility on macroeconomic dynamics, with an application to Japanese data. The model features an endogenous composition of the household sector in which household members are heterogeneous in terms of productivity, and allows for an explicit dynamics of the number of household members that results from some exogenous factors (such as life expectancy) but also from an endogenous decision of household members to invest in the creation of new members (fertility). In the model, fertility results from the confrontation of the expected lifetime sum of labor-market income and a fixed sunk cost. The dynamics of labor-market income are driven by the interaction between the distribution of individual productivity levels — drawing from a Pareto distribution — and the repeated payment of labor-market participation costs. Hence, fertility, labor-market participation, the total number of workers are all driven by both demographic *and* macroeconomic factors together. In addition, because households are heterogeneous in terms of their individual productivity and because labor-market participation and employment are endogenous, labor-market and demographic variables affect aggregate productivity endogenously through worker selection effects.

Focusing on trend shifts, these rich interactions imply that a rise in life expectancy lowers fertility, labor-market participation and total employment, as observed qualitatively in Japanese data. In addition, longer life expectancy implies a higher valuation of life for all individuals, inducing a slight rise in the creation of new household members and small improvements in aggregate productivity driven by a selection effect on the labor market that leads less productive workers to exit. As a consequence, an aging shock in the form of an exogenous rise in life expectancy is predicted to raise the aggregate level of productivity rises in the economy. However, this shock alone falls short in replicating the observed dynamics of both demographic and macroeconomic variables. Indeed, feeding the model with an empirically realistic sequence of life expectancy shocks based on Japanese data largely fails to explain the observed dynamics of fertility in Japan, and does not account for the observed fall in total population. Further, it has quantitatively limited effects on aggregate macroeconomic variables and predicts a counterfactual rise in aggregate productivity.

Hence, we consider an additional shock to match the observed dynamics of the fertility rate by means of a rise in the sunk cost of creating new household members, *i.e.* the cost of raising newborns. Combined with the exogenous dynamics of life expectancy, the model matches the observed declining trends in total population (by design), but also the declining trends in GDP, GDP per capita, employment and the slightly rising trend in labor-market participation. Last but not least, these combined demographic shocks explain up to 30% of the observed decline in aggregate productivity. These results point to the key importance of the fertility shock to generate both realistic demographic and realistic macroeconomic dynamics, and suggest that the dynamics of fertility is a potentially important factor to explain the sluggishness of aggregate productivity in Japan over the last 40 years.

Our paper contributes to the literature in various respects. First, trying to explain the lost decade in Japan, Hayashi and Prescott (2002) show the critical explanatory power of an exogenous TFP process, but do not link the latter to demographic factors. Our model provides an intuitive mapping between aging or fertility and productivity, and builds a bridge between papers trying to explain the recent productivity slowdown and papers looking at the effects of aging.

Second, most recent overlapping-generations models such as Nishiyama (2015), Kitao (2015), McGrattan and Prescott (2018) or Katagiri, Konishi, and Ueda (2020) look at the consequences of aging on the conduct of (potentially optimal) public policies, and disregard the potentially endogenous effects of aging on productivity. A few overlapping-generations (OLG) models look at the issue of how aging might affect productivity, such as Fougère and Mérette (1999) or Bouzahzah, De la Croix, and Docquier (2002), but within relatively complex frameworks relying on simulations. By contrast, our approach is highly tractable and can be understood looking at a couple of equations. In addition, the above-cited papers develop endogenous growth models with human capital to account for the endogenous productivity effects of aging. In our model, productivity is endogenously affected by the number of workers, that depends on how large total population is given both aging and endogenous fertility but also on the time-varying average productivity of workers, that results from selection effects through labor-market participation decisions.

Third, closer to our paper, Cooley and Henriksen (2018) show how aging changes the composition of the labor-force and thus alters the productivity of labor, which may account for a substantial fraction — up to quarter — of the observed slowdown in TFP growth. Relatedly, Kydland and Pretnar (2019), show how an aging population leads to structural changes in the allocation of time to care of sick old people and leads to lowered labor-market participation, which then lowers productivity and GDP per capita. Although our model works through very different channels than these two contributions, it also links demographic factors to labor-market participation and can thus be seen as an interesting complement. Its main interest, we believe, is the two-way interaction between demographic and economic factors through endogenous fertility.

Finally, our paper contributes to the rising literature using macroeconomic models with heterogeneous agents.[1] Different from these simulation based approaches, we track the distribution

[1] See Heathcote, Storesletten, and Violante (2009) for a survey including papers resorting to OLG models, and Kaplan and Violante (2018) for a more recent review, but focusing on heterogeneous-agents New Keynesian (HANK) models.

of worker productivity with summary statistics as in Ghironi and Melitz (2005) or Hamano and Zanetti (2017) among many others, the critical difference is that the heterogeneous-agent approach applies to households rather than firms. Hence, our model provides a very tractable way of introducing households heterogeneity and its labor-market, demographic and macroeconomic implications. One key difference with respect to most heterogeneous-agents models currently used is, however, that we consider full risk-sharing among household members. While it greatly simplifies the model solution, it insulates aggregate saving decisions from the income risks associated with aging and declining fertility.

The paper is structured as follows. Section 2 presents a simple model of endogenous population and labor-market participation with heterogeneous workers that links demographic and economic factors in a novel and intuitive way. Section 3 investigates the steady-state and dynamic properties of this simple model, showing the impact of various demographic and economic shocks. Section 4 extends the simple model to account for capital accumulation and various adjustment costs. It proposes a set of quantitative exercises to gauge the explanatory power of this relatively simple model. Finally, it proposes a counterfactual analysis to qualify the respective contributions of aging and fertility shocks, showing that most of the quantitative success is driven by the effects of the fertility shock.

2 Model

We propose a model of endogenous household size and labor-market participation. Households supply labor monopolistically and earn profits. Given the labor-market outcomes in terms of wages and the death rate of individuals, they can choose to increase or decrease the number of household members.

2.1 Firm and labor demand.

Firms are perfectly competitive in the market. The representative firm has the following aggregate production function

$$y_t = a_t H_t \tag{1}$$

where a_t denotes the Total Factor Productivity (TFP hereafter), and h_t is a bundle of the different types of labor:

$$H_t = \left[\int_{\omega \in \Omega} z(\omega) \, h_t(\omega)^{\frac{\theta-1}{\theta}} \, d\omega \right]^{\frac{\theta}{\theta-1}} \tag{2}$$

where the ω index stands for labor varieties, and $z(\omega)$ denotes the productivity of variety ω. The firm maximizes its profits, given that total labor expenditure are given by $\int_{\omega \in \Omega} w_t(\omega) \, h_t(\omega) \, d\omega$.

It gives the following labor demand function for variety ω:

$$h_t(\omega) = \left(\frac{w_t(\omega)}{W_t z(\omega)}\right)^{-\theta} H_t \tag{3}$$

where

$$W_t = \left[\int_{\omega\in\Omega} z(\omega)\left(\frac{w_t(\omega)}{z(\omega)}\right)^{1-\theta} d\omega\right]^{\frac{1}{1-\theta}} \tag{4}$$

We choose the price of the final good as numeraire.

2.2 Individuals and Households

In the economy, we normalize the number of households to unity. At each period t, there are two types of individuals in the households: $m_t(j)$ individuals that are already members of the household at the beginning of the period and $m_{et}(j)$ new individuals that enter the household within the period (children, immigrants). At the end of the period a fraction $\delta_t \in [0,1]$ of all existing individuals is exogenously affected by a time-varying exit shock where $1/\delta_t$ denotes life expectancy. The total number of household members thus evolves according to:

$$m_{t+1}(j) = (1-\delta_t)(m_t(j) + m_{et}(j)) \tag{5}$$

Among the $m_t(j)$ individuals in the household at the beginning of period t, only the most productive enter the labor market. Labor-market entry is subject to the repeated payment of a (potentially time-varying) cost f_{nt}, also paid in units of basket of workers. These costs can be thought to represent on-the-job training costs or various job-related types of expenditure like transport, commuting, etc...

Consider a continuum of individuals with heterogeneous productivity that supply differentiated types of labor within a household. The household j allows for endogenous entry in the household (fertility) and endogenous participation in the labor-market. Over the entire space of individuals, only a subset will actually work paying fixed costs in terms of consumption good. Each individual has specific random labor quality draws z from a probability density function $\mu(z)$. The specific productivity remains fixed once individuals enter the household. When she works, she supplies labor given its demand as (3). The household j and each individual member with productivity z solves

$$\max E_t\left\{\sum_{s=t}^{\infty}\beta^{s-t}\left(\frac{C_s(j)^{1-\sigma}}{1-\sigma} - \eta\frac{L_s(j)^{1+\varphi}}{1+\varphi}\right)\right\} \tag{6}$$

where $C_t(j)$ and $H_t(j)$ respectively denote total consumption in the household j and hours worked for individual with productivity z in the household j, σ and φ respectively stand for the constant degree of relative risk aversion and inverse of Frisch elasticity on labor supply,

$$L_t(j) = \left[\int_{z_{\min}}^{\infty} z\ell_t(j,z)^{\frac{\theta-1}{\theta}}\, dz\right]^{\frac{\theta}{\theta-1}}$$

and subject to the budget constraint

$$C_t(j) + x_{t+1}(j)\, v_t(j)\,(m_t(j) + m_{et}(j)) = x_t(j)\, m_t\left(v_t(j) + \widetilde{d}_t(j)\right) \tag{7}$$

In this equation, $m_t(j)$ is the number of households that belong to the household at the beginning of period t, $m_{et}(j)$ the number of new household members between the beginning and the end of period t, $v_t(j)$ is the value of human capital and $\widetilde{d}_t(j) = \int_{z_{\min}}^{\infty} w_t(j,z)\,\ell_t(j,z) - w_t f_{nt} dM(z)$ denotes the average monopolistic profits made by working household members, and $x_t(z)$ is the share of the mutual fund held by each individual with productivity. Optimization is also subject to

$$\ell_t(j,z) = \left(\frac{w_t(j,z)}{W_t(j)\,z}\right)^{-\theta} L_t(j) \tag{8}$$

The decision for total consumption and the creation of new members belongs to the household level while labor supply is made at individual level. First-order condition for workers with respect to $C_t(j)$, $x_{t+1}(j)$ and $w_t(j,z)$ give[2]

$$\begin{aligned} C_t(j)^{-\sigma} &= \lambda_t(j) && (9)\\ \beta(1-\delta_{t+1})\, E_t\left\{\frac{\lambda_{t+1}(j)}{\lambda_t(j)}\frac{v_{t+1}(j) + \widetilde{d}_{t+1}(j)}{v_t(j)}\right\} &= 1 && (10)\\ \chi L_t(j)^{\varphi} &= W_t(j)\,\lambda_t(j) && (11) \end{aligned}$$

where $\chi = \eta\theta/(\theta-1)$ and where $\lambda_t(j)$ is the marginal utility of consumption that takes into account disutility of working members. The selection in the labor market takes place. The threshold condition to work is given by

$$w_t(j, z_{nt}(j))\,\ell_t(j, z_{nt}(j)) = W_t f_{nt} \tag{12}$$

where $z_{nt}(j)$ denotes the cutoff level productivity for working within the household j and is labor basket defined as (2). This equation states that the last household member entering the labor market is productive enough for his labor income to equate the entry cost. Finally, entry incurs a once and for all (potentially time-varying) sunk costs f_{et} (education, childcare), paid in

[2]The F.O.C. with respect to $w_t(j,z)$ is given by

$$\frac{\eta\theta}{\theta-1} z\ell_t(j,z)^{-\frac{1}{\theta}} L_t(j)^{\varphi+\frac{1}{\theta}} = \lambda_t(j)\, w_t(j,z)$$

By plugging the labor demand as (8), we get $\chi L_t(j)^{\varphi} = W_t(j)\,\lambda_t(j)$.

units of basket of workers defined as (2). Thus the number of new family members is determined through the following free entry condition:

$$v_t = W_t f_{et} \tag{13}$$

2.3 Aggregation

Individual-specific labor productivity z has a Pareto distribution with lower bound $z_{\min}$ and shape parameter $\varepsilon(1+\theta\varphi) > \theta - 1$, where θ is the elasticity of substitution among the different types of labor and φ the inverse of the Frisch elasticity of labor supply. The cumulative density function is $M(z) = 1 - (z_{\min}/z)^{\varepsilon}$. Let $\widetilde{z}_t(j)$ and $\widetilde{z}_{nt}(j)$ denotes the average productivity of individuals and among workers such that , such that

$$\widetilde{z}_t(j) = \left[\int_{z_{\min}}^{\infty} z^{\frac{\theta-1}{1+\theta\varphi}} dM(z)\right]^{\frac{1+\theta\varphi}{\theta-1}}, \quad \widetilde{z}_{nt}(j) = \left[\int_{z_{nt}(j)}^{\infty} z^{\frac{\theta-1}{1+\theta\varphi}} \frac{dM(z)}{1-M(z_{nt})}\right]^{\frac{1+\theta\varphi}{\theta-1}} \tag{14}$$

The interpretation of this condition is that the average productivity of households is defined as the harmonic mean of individual productivities, weighted by the relative utility of hours worked.[3] With Pareto distribution defined previously, the average productivity of labor-market participants $\widetilde{z}_{nt}(j)$ is thus given by

$$\widetilde{z}_t(j) = \widetilde{z}(j) = \nabla z_{\min}, \quad \widetilde{z}_{nt}(j) = \nabla z_{nt}(j) \tag{16}$$

where $\nabla = \left(\frac{\varepsilon(1+\theta\varphi)}{\varepsilon(1+\theta\varphi)-(\theta-1)}\right)^{\frac{1+\theta\varphi}{\theta-1}}$. We express the variables using these average and denote that $w_t(j, \widetilde{z}_{nt}(j)) \equiv \widetilde{w}_{nt}(j)$ and $\ell_t(j, \widetilde{z}_{nt}(j)) \equiv \widetilde{\ell}_{nt}(j)$. With the average dividend of workers such that $d_t(j, \widetilde{z}_{nt}(j)) \equiv \widetilde{d}_{nt}(j) = \widetilde{w}_{nt}(j)\widetilde{\ell}_{nt}(j) - f_{nt}$, we can rewrite the cutoff condition (12) as

$$\widetilde{w}_{nt}(j)\widetilde{\ell}_{nt}(j) = \nabla^{\frac{\theta(1+\varphi)}{1+\theta\varphi}} w_t f_{nt} \tag{17}$$

Finally, once the threshold $z_{nt}(j)$ is known, the number of labor-market participants is given by $n_t(j) = (1 - M(z_{nt}(j)))\, m_t(j)$ which is rewritten with the Pareto distribution as

$$\frac{n_t(j)}{m_t(j)} = \left(\frac{\nabla}{\widetilde{z}_{nt}(j)}\right)^{\varepsilon}$$

Using the above notations, the average dividends among *all* individuals in the household j is

$$\widetilde{d}_t(j) = \frac{n_t(j)}{m_t(j)}\widetilde{d}_{nt}(j) \tag{18}$$

[3] or equivalently that:

$$\widetilde{z}_t^{-1} = \int_{z_{\min}}^{\infty} z^{-1}\left(\frac{\ell_t(z)}{\ell_t(\widetilde{z}_t)}\right)^{1+\varphi} \mu(z)\, dz \tag{15}$$

Also the labor supply for the average level of productivity is expressed as

$$\chi \widetilde{\ell}_{nt}(j)^{\varphi} = \widetilde{w}_{nt}(j)\,\lambda_t(j) \tag{19}$$

Finally, wage index and hours basket are rewritten as

$$W_t(j) = \left(n_t(j)\,\widetilde{z}_{nt}(j)\right)^{\frac{1}{1-\theta}} \widetilde{w}_{nt}(j)\,/\widetilde{z}_{nt}(j) \text{ and } L_t(j) = \left(n_t(j)\,\widetilde{z}_{nt}(j)\right)^{\frac{\theta}{\theta-1}} \widetilde{\ell}_{nt}(j) \tag{20}$$

2.4 General equilibrium

In equilibrium, households are homogeneous and we drop household index j as $C_t = C_t(j)$, $L_t = L_t(j)$, $W_t = W_t(j)$, $m_t = m_t(j)$, $n_t = n_t(j)$, $m_{et} = m_{et}(j)$, $\widetilde{z}_{nt} = \widetilde{z}_{nt}(j)$, $\widetilde{z} = \widetilde{z}(j)$, $\widetilde{d}_t = \widetilde{d}_t(j)$, $\widetilde{d}_{nt} = \widetilde{d}_{nt}(j)$, $v_t = v_t(j)$ and $\lambda_t = \lambda_t(j)$. Perfect competition in good market implies that $W_t = a_t$. The goods market clearing writes[4]

$$Y_t = C_t \tag{21}$$

The labor market clearing writes

$$L_t = \frac{Y_t}{a_t} + n_t f_{nt} + m_{et} f_{et} \tag{22}$$

The model summary and steady-state conditions are given in Appendix B and C respectively.

3 The Simple Model

We now use the simple model to investigate the macroeconomic effects of aging and lower fertility. Since the model features heterogeneous households and endogenous labor-market participation, a lengthing of life expectancy — a fall in δ — and a drop in fertility — a rise in f_e — will of course have demographic consequences but also labor-market and productivity effects through changes in the participation threshold.

3.1 Steady-state analysis: Permanent rise in $1/\delta$, f_e or f_n

Our primary interests are the long-term effects of aging. In the model, aging is easily captured by a rise in life expectancy $1/\delta$. Appendix C shows that the steady-state fertility rate writes

$$\frac{m_e}{m} = \frac{\delta}{1-\delta} \tag{23}$$

which immediately shows that a rise in life expectancy will lower the fertility rate in the long run. Further, the fall in δ implies that the subjective discount factor of households rises, lowering the real interest rate and raising the value of an individual life. However, since $v = W f_e = a f_e$ in

[4] The same condition is derived by aggregating the budget constraint (7) across individuals.

equilibrium, the average profits from working in the household $(n/m)\,\widetilde{d}_n$ have to fall, which is achieved by a rise in the labor-market participation cut-off z_n and hence $\widetilde{z}_n$

$$\widetilde{z}_n = \left[\left(\frac{\beta\left(1-\delta\right)}{1-\beta\left(1-\delta\right)}\right)\frac{f_n}{f_e}\left(\nabla^{\frac{\theta(1+\varphi)}{1+\theta\varphi}}-1\right)\right]^{\frac{1}{\varepsilon}}\nabla \tag{24}$$

Consequently, the average dividends of working members $\widetilde{d}_n$ increases while the participation rate (n/m) decreases, which more than offsets the rise in $\widetilde{d}_n$. Hence, an aging shock increases the number of non-working household members. Looking at shocks on f_e and f_n, Equation (23) shows that m_e/m is unaffected in the long run. Further, Equation (24) shows that a rise in f_e decreases $\widetilde{z}_n$ with an elasticity of $1/\varepsilon$ while a rise in f_n increases $\widetilde{z}_n$ with the same elasticity. These effects imply that labor-market participation n/m rises after an increase in f_e and falls after an increase in f_n.

Now focusing on consumption, in the steady state, the model implies

$$C = \left\{\frac{a^{1+\frac{1}{\varphi}}}{\chi^{\frac{1}{\varphi}}}\left[1-\frac{1+\frac{m_e}{n}\frac{f_e}{f_n}}{\nabla^{\frac{\theta(1+\varphi)}{1+\theta\varphi}}}\right]\right\}^{\frac{1}{1+\frac{\sigma}{\varphi}}} \tag{25}$$

Whether the consumption increases or not in case of aging shocks depends on whether the proportion of newborns with respect to workers m_e/n increase or not.[5] Specifically, it is shown that

$$\frac{m_e}{n} = \left(\frac{\beta\delta}{1-\beta\left(1-\delta\right)}\right)\frac{f_n}{f_e}\left(\nabla^{\frac{\theta(1+\varphi)}{1+\theta\varphi}}-1\right) \tag{26}$$

An aging shock — a larger $1/\delta$ — decreases m_e/n. As a result, consumption increases in the new steady state. Intuitively, when the number of newborns — which do not produce and hence are dependent of the working members of the household — drops, investment in the creation of new household members falls which frees up resources to increase aggregate consumption. A rise in f_e (f_n), decreases (increases) m_e/n proportionally so that $(m_e/n)\,(f_e/f_n)$ and thus consumption are constant in the long run. As a result, the steady-state level of consumption is independent from these changes or, put differently, these shocks induce only temporary changes in consumption. It also follows that $L = \left(\frac{C^{-\sigma}a}{\chi}\right)^{\frac{1}{\varphi}}$ inherits the long-run properties of consumption. Finally, total population m is given by[6]

$$m = \left(\frac{\beta\left(1-\delta\right)}{1-\beta\left(1-\delta\right)}\right)\frac{a^{\frac{1}{\varphi}-\left(1+\frac{1}{\varphi}\right)\frac{\sigma}{1+\frac{\sigma}{\varphi}}}}{\chi^{\left(\frac{1}{\varphi}+1\right)\frac{1}{\varphi}\frac{\frac{\sigma}{\varphi}}{1+\frac{\sigma}{\varphi}}}f_e}\left(1-\frac{1}{\nabla^{\frac{\theta(1+\varphi)}{1+\theta\varphi}}}\right)^{\frac{1}{1+\frac{\sigma}{\varphi}}}\left[1+\frac{\delta}{1-\delta}\left(\frac{\beta\left(1-\delta\right)}{1-\beta\left(1-\delta\right)}\right)\right]^{\frac{-\frac{\sigma}{\varphi}}{1+\frac{\sigma}{\varphi}}} \tag{27}$$

[5]Note that the economy has a balanced-growth path with respect to technology a for W, and for C and Y with a constant L if $\sigma = 1$.

[6]When $\sigma = \varphi = 1$, m is stationary with respect to the technology shock.

In case of an aging shock — a rise in δ — the direction of changes of population depends on $\frac{\beta(1-\delta)}{1-\beta(1-\delta)}$, which clearly points to an increase in total population. A negative fertility shock — a rise in f_e — induces a proportionally lower value of m. In addition, since

$$m_e = \frac{\delta}{1-\delta} m, \quad n = \frac{L}{\nabla^{\frac{\theta(1+\varphi)}{1+\theta\varphi}} f_n} \tag{28}$$

the same shock decreases m_e proportionally but does not affect n in the long run. Different from the fertility shock, the labor-market regulation shock f_n does not impact the long-run value of m but decreases the number of working members n in the new steady state.

In summary, a permanent aging shock — an exogenous rise in life expectancy — lowers the fertility rate, labor-market participation and total labor but increases consumption permanently. A permanent fall in the cost parameter f_e lowers the fertility rate and raises labor-market participation but only temporarily. All macroeconomic variables return to their steady-state levels and population drops permanently. Last, a permanent rise in f_n reduces labor-market participation permanently but keeps most other variables unchanged — even in the short-run.

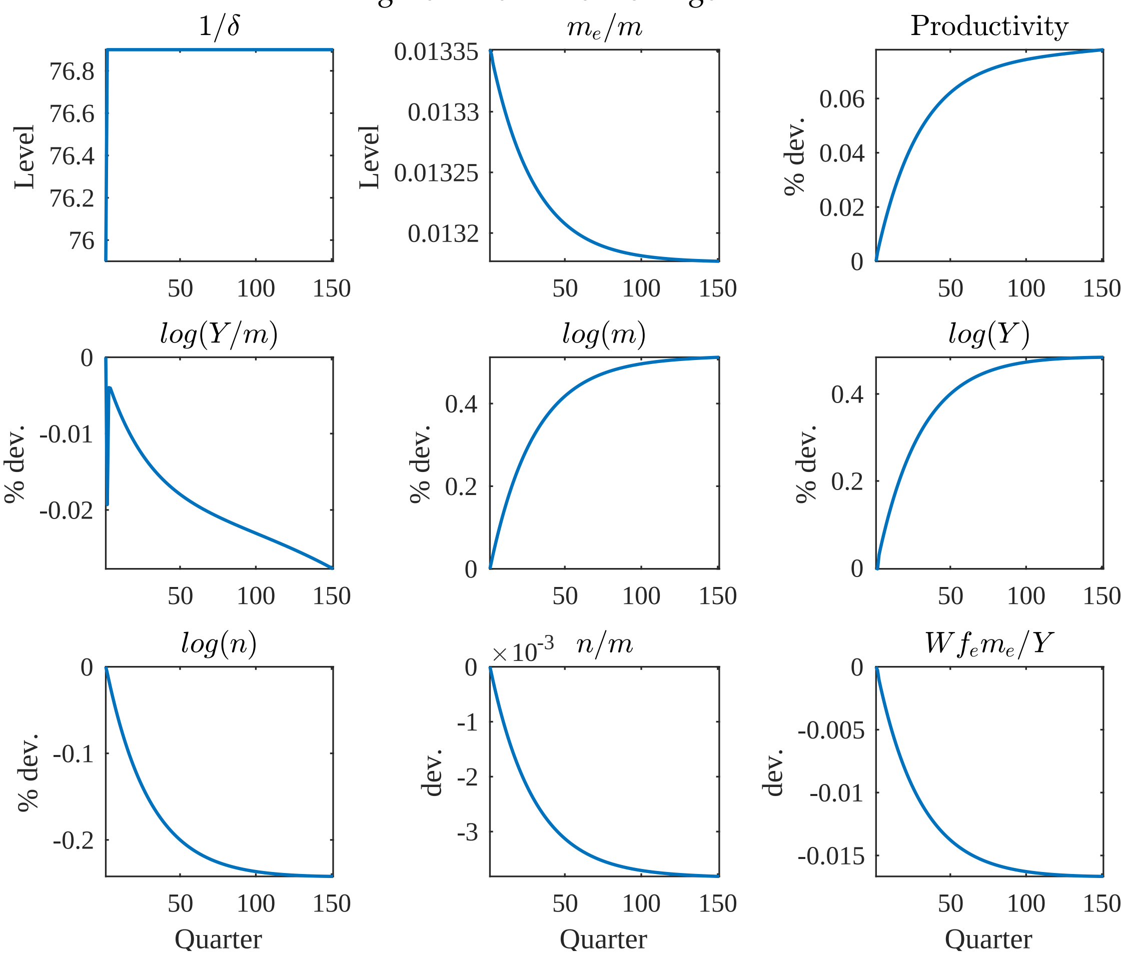
3.2 Impulse responses to permanent shocks

We now illustrate the above effects using deterministic simulations. We restrict our attention to a permanent fall in δ and to a permanent rise in f_e, given that shocks on f_n have little (if any) macroeconomic and demographic effects. We fix the following parameters values, trying to replicate key features of the Japanese economy. The time unit is a quarter so that $\beta = 0.99$ implies an annual real interest rate of 4%. We restrict the utility function so that $\sigma = 1$ and assume a $\varphi^{-1} = 0.5$ Frisch elasticity. The initial value of δ is set to imply a 75.5 life expectancy — the observed value in Japan in 1976 — implying $\delta = 0.01325$. The wage mark-up is fixed to 10%, which implies $\theta = 11$ and we set the wage dispersion parameter Pareto to $\varepsilon = 2.4085$ following Atkinson, Piketty, and Saez (2011) based on 2005 data for Japan (Table 6).[7] Last, we set $f_e = a = 1$ without loss of generality, and $f_n = 0.05$ to replicate the observed employment to total population in 1976, *i.e.* $n/m = 0.505$.

Figure 1 reports the effects of a permanent drop in δ that raises life expectancy by exactly one year. In addition to the variables already mentioned in the model, we also report a variety-corrected measure of total productivity that takes into account the endogenous composition of the working population, $tfp_t = a_t \left(n_t \widetilde{z}_{nt}\right)^{\frac{\theta}{\theta-1}}$. This definition shows that total productivity depends on two opposite forces. On the one hand, productivity is positively affected by the average level of labor productivity $\widetilde{z}_{nt}$ through a selection effect. The latter depends on the fraction of

[7]Note that 1949 data reported in the same Table point to a 2.7544 Pareto parameter, signaling a relative stability of this parameter over time.

Figure 1: Permanent change in δ



Note: the shock raises life expectancy ($1/\delta$) by 1 year.

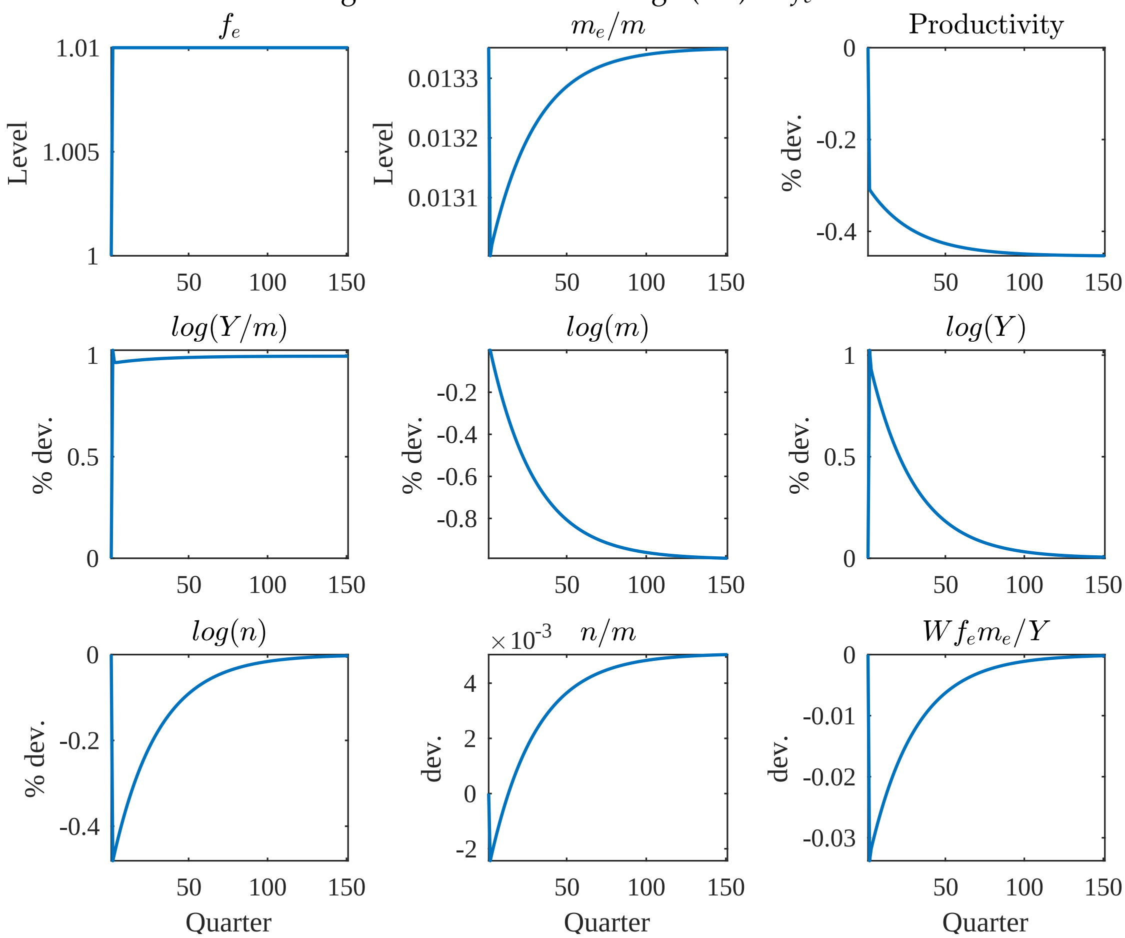
workers in total population. A lesser proportion of workers derive from a higher cut-off productivity and results in a higher $\tilde{z}_{nt}$. On the other hand, productivity depends positively on the total number of workers through a usual variety effect.

As already explained, a permanent rise in life expectancy lowers the fertility rate (m_e/m) and labor-market participation (n/m). The selection effect thus pushes aggregate productivity to rise. However, the fall in the total number of workers makes aggregate productivity fall. As the selection effect dominates, the net effect of the aging shock is a rise in productivity. While the aging shock raises aggregate output and consumption, it increases total population more than aggregate output, which then lowers — although by a negligible amount — output per capita.

Figure 2 contrasts the effects of a 1% permanent rise in f_e.

As already explained, a rise in f_e has only temporary — although potentially very persistent — effects on all macroeconomic and demographic variables except on total population, that falls

Figure 2: Permanent change (1%) in f_e



permanently. Nevertheless, the shock lowers the fertility rate for 150 quarters and raises labor-market participation. Indeed, the rise in f_e raises the value of individual life and requires average dividends and thus the average level of productivity to fall, which is the case if labor-market participation rises. In this case the selection effect — a larger proportion of workers in total population — and the variety effect — lower total number of workers — go hand in hand to lower aggregate productivity. Notice that the variety effect is only temporary as n reverts slowly to its steady-state value, while the selection effect is permanent. As a result, the productivity cut-off is permanently lowered and aggregate productivity falls permanently.

4 Model with capital

4.1 Assumptions

We now extend the simple model to introduce capital accumulation. The details are given in Appendix E and sketched below. Most aspects of the model are kept unchanged, especially on the labor market and household sector. Let K_t denote the stock of capital that now enters the production function

$$Y_t = a_t K_{t-1}^{\alpha} H_t^{1-\alpha} \tag{29}$$

While the real wage was simply equal to the exogenous productivity factor before, it is now determined by

$$W_t = (1-\alpha)\, a_t^{\frac{1}{1-\alpha}} \left(K_{t-1}/Y_t\right)^{\frac{\alpha}{1-\alpha}} \tag{30}$$

Further, the inception of capital accumulation subject to investment costs gives rise to the following Euler equation on capital:

$$\beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left(q_{t+1} \left(1-\delta_k\right) + r_{kt+1} \right) \right\} = q_t \tag{31}$$

where Tobin's Q is given by

$$q_t \left(1 - \phi g_{i,t}^2/2 - \phi g_{i,t} \left(1+g_{i,t}\right)\right) + \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left(q_{t+1} \phi g_{i,t+1} \left(1+g_{i,t+1}\right)^2 \right) \right\} = 1 \tag{32}$$

and where $g_{i,t} = I_t/I_{t-1} - 1$ denotes the growth rate of investment. The dynamics of capital accumulation is finally given by

$$K_t = \left(1-\delta_k\right) K_{t-1} + I_t \left(1 - \phi g_{i,t}^2/2\right) \tag{33}$$

while the equilibrium condition on goods market writes

$$Y_t = C_t + I_t \tag{34}$$

A final change compared to the simple model is the introduction of sluggishness in the dynamics of f_{et} and f_{nt} and assume

$$f_{et} = f_e \left(\frac{m_t}{m_{t-1}}\right)^{\phi_e} \text{ and } f_{nt} = f_n \left(\frac{n_t}{n_{t-1}}\right)^{\phi_n} \tag{35}$$

4.2 Quantitative exercise

We now use our model with capital accumulation to perform simulations. Our goal is to replicate the structural evolution of key demographic and macroeconomic variables from the Japanese economy from 1977 to 2017.

Regarding demographic variables, over this period of time, the Japanese economy is characterized by a strong rise in life expectancy from $1/\delta = 75.5$ to $1/\delta = 84.1$ years. This feature is matched through a permanent change in δ. More precisely, we assume

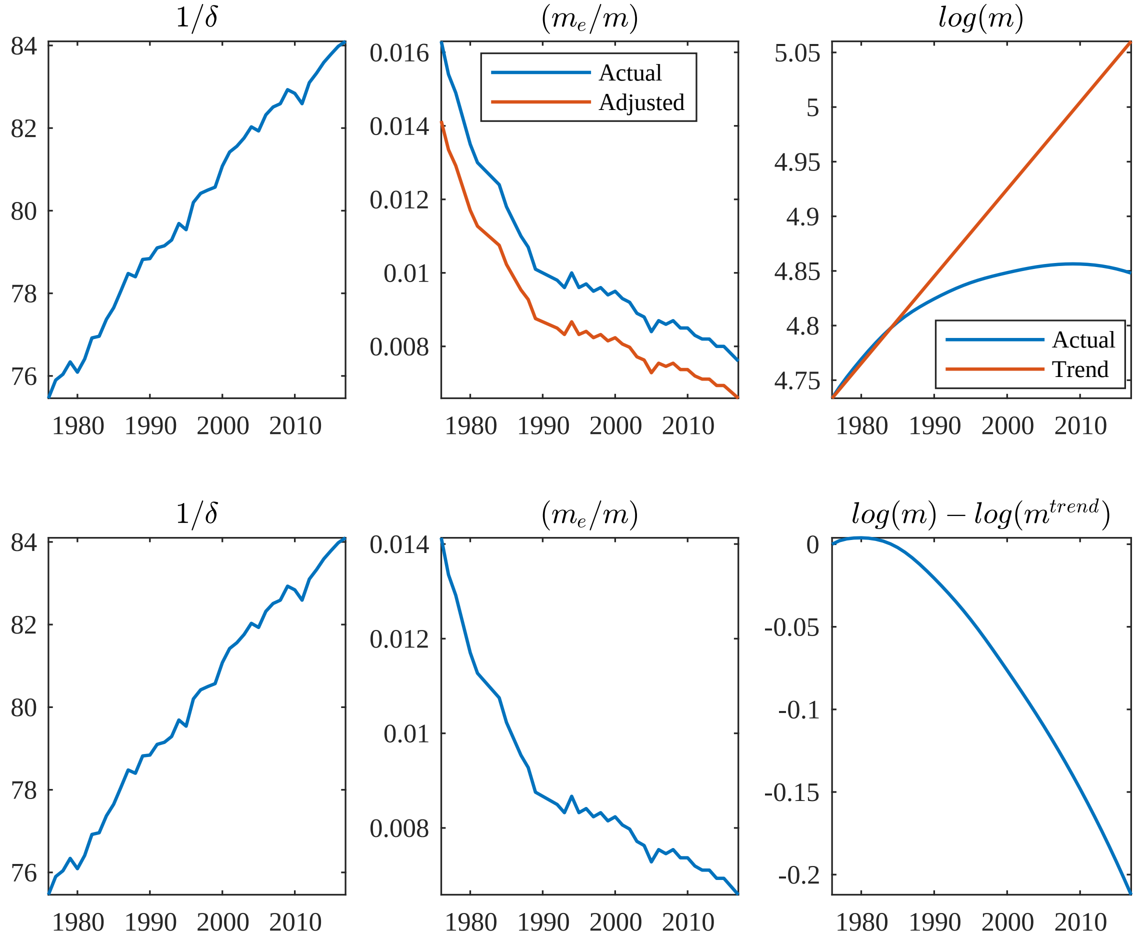
$$\delta_t = (1 - \rho_\delta)\overline{\delta} + \rho_\delta \delta_{t-1} \tag{36}$$

and consider a shock on $\overline{\delta}$ with a value of ρ_δ that match the time profile of life expectancy. In addition, the raw fertility rate (m_e/m in our model) from the data falls from 1.6% to 0.76%. However, population is not constant in the data while our model, in the steady state, predicts a constant level of population. More precisely, our model implies $fert = me/m = \delta/(1-\delta)$ in the steady sttae, where $fert$ is the fertility rate. Since the data imply $fert^{data} > \delta^{data}/(1 - \delta^{data})$, we normalize fertility rate so that this relation is verified in 1976, that is $fert^{norm} = fert^{data}\left(\delta^{1976}/(1-\delta^{1976})/fert^{1976}\right)$. With this lower the fertility rate, $fert^{norm} = me/m = \delta/(1-\delta)$ in 1976. Further, in our model, the fertility rate m_e/m is partly endogenous and partly driven by f_{et}. Since changes in life expectancy $1/\delta$ fall short in replicating the dynamics of the fertility rate m_e/m, as made clear later on, we augment our simulations with a permanent shock on f_{et} and assume

$$f_{et} = (1 - \rho_{f_e})\overline{f}_e + \rho_{f_e} f_{et-1} \tag{37}$$

The size of the shock on $\overline{f}_e$ and the value of ρ_{f_e} are adjusted to match the time profile of the adjusted fertility rate taken from the data. The quality of the match of demographic variables is assessed by comparing the log-level of total population predicted by the model from to its data counterpart. Since the model predicts a constant log-level of population in the steady state while the latter is growing in the data, we must detrend the observed log-level of population. Given that our simulations start in 1977, we detrend the log-level of population using the average

Figure 3: Demographic variables: actual (top) vs. transformed (bottom).

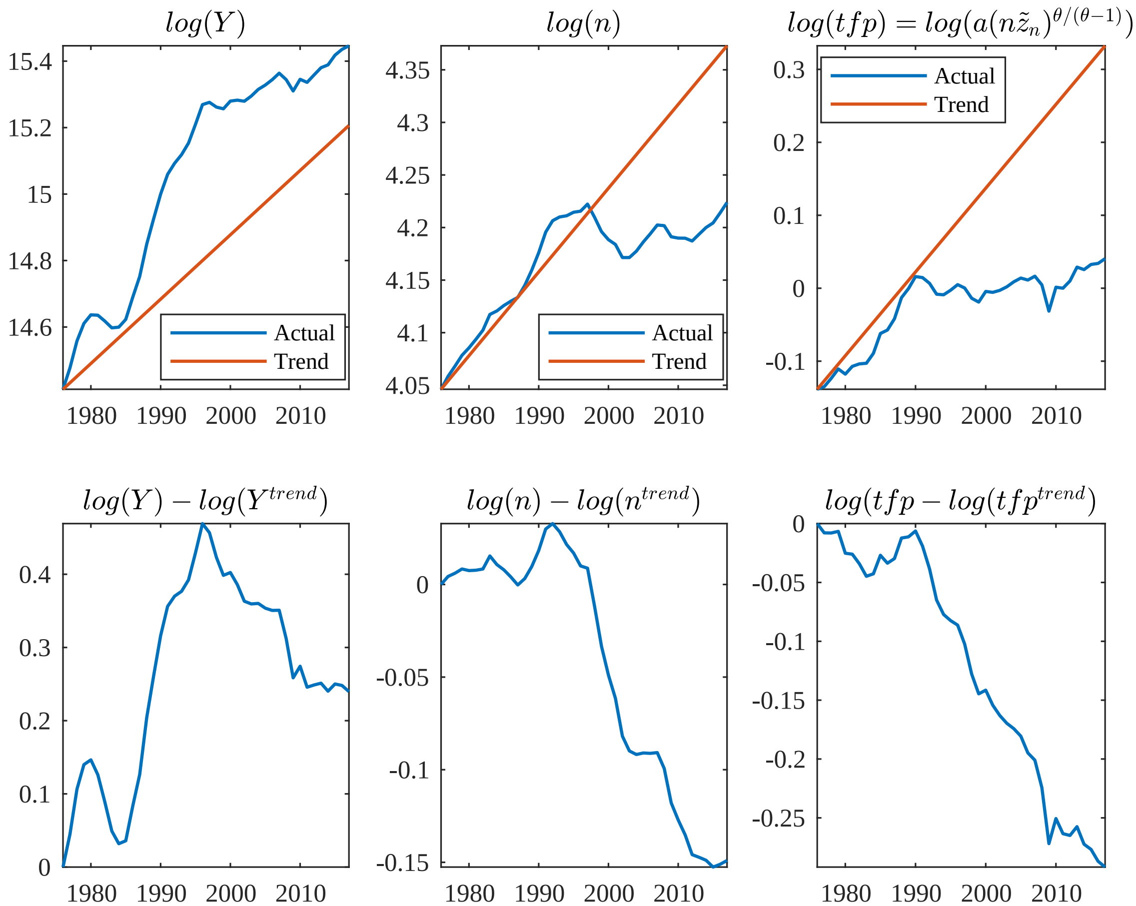


observed growth rate of population before 1976, *i.e.* from 1951 to 1976. As such, the reported population variable represents the level of population in log-deviation from what it would have been if population had been growing at the average rate observed from the data between 1951 and 1976. The resulting time series are reported in Figure 3.

Now regarding macroeconomic variables, we want to know whether our model is able to replicate the dynamics of TFP over time in Japan. Empirically, we consider the TFP measure provided by the Bergeaud, Cette, and Lecat (2016) database, and compute the log-deviation of TFP from a linear trend from 1951 to 2017. We then compute GDP as the log-deviation from a linear trend where the slope is the sum of the trend population growth rate and the trend TFP growth rate. Finally, we look at the log-deviation of total employment from a linear trend computed using the growth rate of total population. The resulting time series are reported in Figure 4.

Overall, our quantitative exercise amounts to match as closely as possible the observed dynamics of the fertility rate, the log-deviation of total population, the log-deviation of GDP and

Figure 4: Macroeconomic variables: Actual (top) vs. transformed (bottom).



the log-deviation of employment feeding the model with two permanent shocks respectively on $\overline{\delta}$ and $\overline{f}_e$. Most parameters are similar to those used for the simple model. New parameters are set as follows. The capital elasticity is set to $\alpha = 0.4$ to match the observed (40%) investment to GDP ratio in 1976. Capital depreciation is 8% per year which implies $\delta_k = 0.02$. The persistence and size of the shock on $\overline{\delta}$ are respectively $1/15$ — a 15 years rise in life expectancy in the very long run — and $\rho_d = 0.98$. These numbers track very closely the observed dynamics of life expectancy in Japan. The persistence of the fertility shock is $\rho_{f_e} = 0.99$ and the size of the shock on $\overline{f}_e$, the investment adjustment cost parameter ϕ, the labor-market participation cost parameter ϕ_n and entry cost parameter ϕ_e are jointly fixed to minimize the distance between the observed time series and the model's prediction between 1977 and 2017, given that we consider the economy to be in the steady state in 1975.[8] The targeted time series are the fertility rate, the log-deviation of total population, of GDP and total employment. We obtain $\Delta\overline{f}_e = 1.4636$, $\phi = 32.5254$, $\phi_n = 9.6278$ and $\phi_e = 0$. The resulting model-based time series are reported in Figure 5 and confronted to the observed (transformed) time series.

We present the simulation results following structural permanent demographic shocks, *i.e.* a permanent increase in life expectancy ($1/\delta_t$) and a rise in sunk costs for newborns (f_{et}) in Figure 5. Population (m) steadily decreases while output Y steadily increases thus replicating well the increasing shift in trend of GDP per capita (Y/m). Our simulation also captures well the declining shifts in trend of birth rate (m_e/m) and employment (n) as well as the increasing trend shift of the labor market participation (n/m). In the absence of any other types of shocks such as technology shock, the simulation accounts for only partially the observed decline in TFP.

4.3 Counterfactual analysis

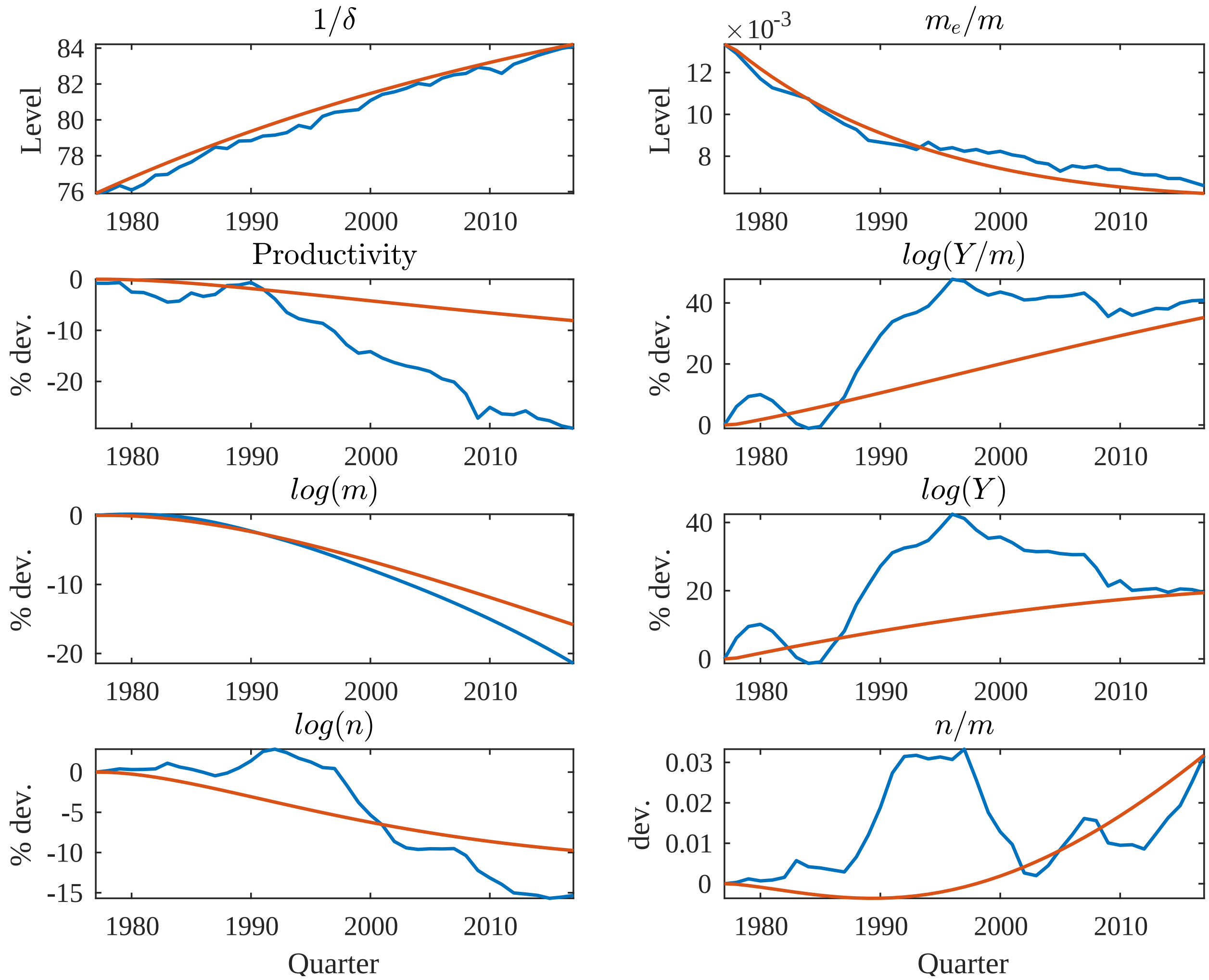
To understand better the above trend shift dynamics, Figure 6 presents the contribution of each demographic shock.

As explained in the previous section, the life expectancy shock increases the value of life and thus increase population (m) by inducing a higher number of newborns (m_e) while reducing the birth rate (m_e/m). It also increases output with a higher population. In the labor market, however, labor-market participation (n/m) decreases (creating more dependent household members) reducing the value of life in equilibrium. Not surprisingly, the impact of life expectancy shock that should end up to increase the population is found to be quantitatively small when we match the model to the data with declining population as shown with dotted lines in Figure 6.

Fertility shock thus plays a central role in explaining the trend shifts of demographic and macroeconomic variables. A rise in f_e (a higher fixed costs for the creation of newborns) de-

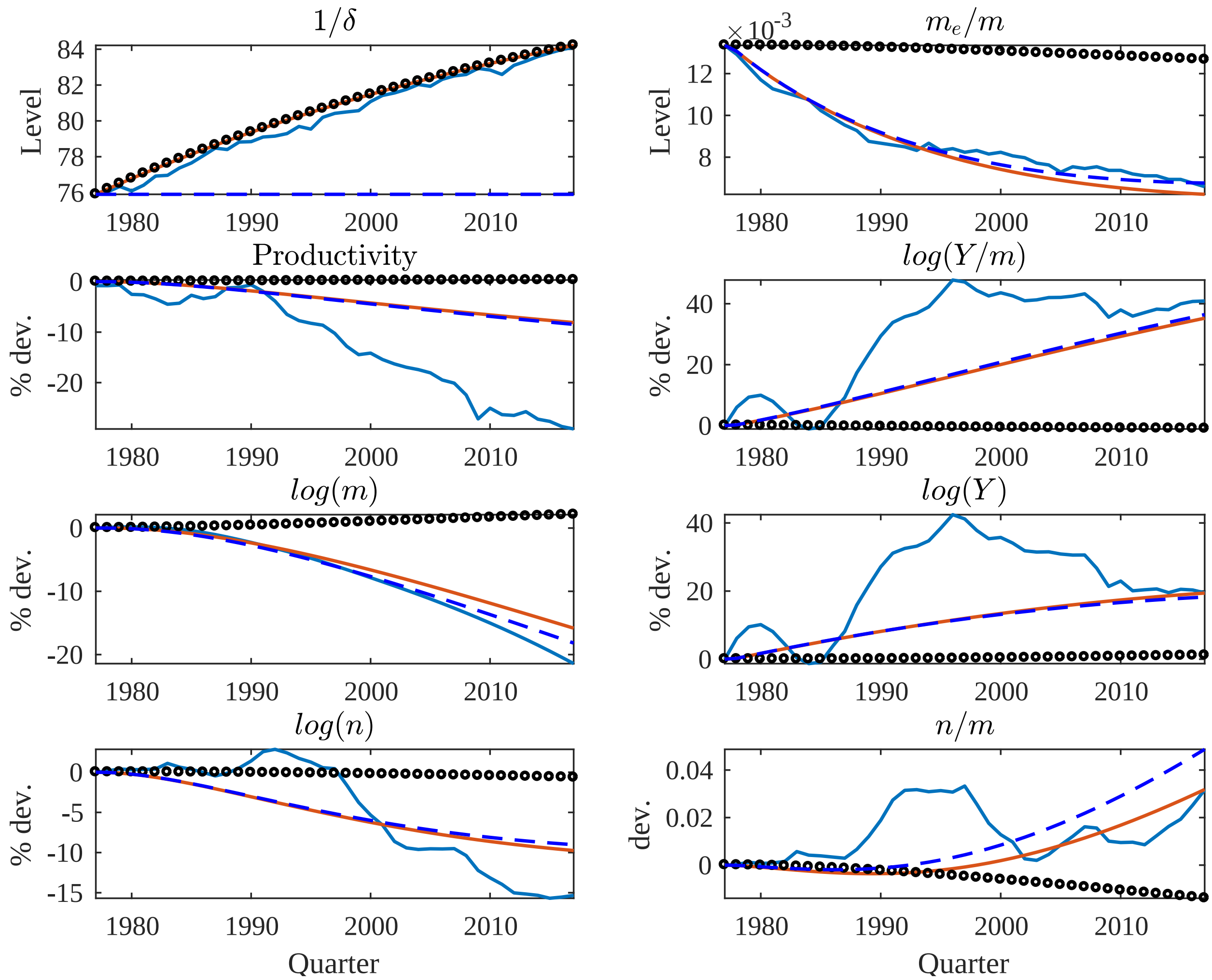
[8] We use fully non-linear deterministic simulations, that is, transition between two steady states. The algorithm used is a two-point boundary problem using a trust region method and implemented through the Dynare set-up for deterministic simulation (see Adjemian et al. (2011)).

Figure 5: Dynamic simulations resulting from a permanent change in $\bar{\delta}$ and $\bar{f}_e$.



Note Blue: data / Red: model.

Figure 6: Counterfactual dynamic simulations



Note Blue: data / Red: baseline / Dots: shock on $\bar{\delta}$ only / Dashed blue: Shock on $\bar{f}_e$ only.

creases the number of newborns significantly and reducing the birth rate (m_e/m) as well as total population (m). Output (Y) increases strongly since the value of life improves simultaneously with a higher fixed costs for the creation of newborns. As a result, the labor market participation (n/m) increase (creating more independent household members) by contributing a lower level of TFP via the selection effect.

5 Conclusion

Many contributions of the macroeconomic literature trying to explain the sluggish medium-run performances of the Japanese economy have focused on aging as a critical factor. In this paper, we proposed a tractable model with heterogeneous households where total population and labor-market participation are both endogenous to analyze the effect of demographic factors on macroeconomic variables. In the model, since fertility was partly exogenous and partly tied to labor-market conditions, demographic and macroeconomic factors were intertwined. In addition, aggregate productivity was partly endogenous since also driven by the composition of the labor force.

First, we showed that aging and negative fertility shocks had opposite predictions in terms of their effects on GDP per capita and aggregate productivity. Second, a quantitative exercise based on Japanese data showed that a sequence of empirically realistic aging shocks had relatively little effects, both on total population and the economy, and thus fell short in replicating the data. Considering a sequence of negative fertility shocks that fitted the observed dynamics of the fertility rate produced dynamics for GDP per capita and productivity much more in line with the data. In particular, this sequence of shocks was shown to explain roughly 30% of the observed decline in aggregate productivity.

Our results thus point to a potentially important conclusion, that sluggish performance of the Japanese economy might be driven by the lack of fertility rather than by the rise in life expectancy. In terms of policy, an equivalently important conclusion is that stimulating fertility by appropriate family policies could result in large macroeconomic gains.

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A Proof for aggregation

Let $\widetilde{z}_t$ denote the average productivity of households, such that

$$W_t = (m_t \widetilde{z}_t)^{\frac{1}{1-\theta}} w_t(\widetilde{z}_t) / \widetilde{z}_t \tag{A.1}$$

$$L_t = (m_t \widetilde{z}_t)^{\frac{\theta}{\theta-1}} \ell_t(\widetilde{z}_t) \tag{A.2}$$

The labor supply condition in relative terms writes

$$\left(\frac{\ell_t(z)}{\ell_t(\widetilde{z}_t)} \right)^{\varphi} = \frac{w_t(z)}{w_t(\widetilde{z}_t)} \tag{A.3}$$

and the demand condition in relative terms writes

$$\frac{\ell_t(z)}{\ell_t(\widetilde{z}_t)} = \left(\frac{w_t(z)/z}{w_t(\widetilde{z}_t)/\widetilde{z}_t} \right)^{-\theta} \tag{A.4}$$

Combining gives:

$$\frac{w_t(z)}{w_t(\widetilde{z}_t)} = \left(\frac{z}{\widetilde{z}_t} \right)^{\frac{\theta\varphi}{1+\theta\varphi}} \tag{A.5}$$

and

$$\frac{\ell_t(z)}{\ell_t(\widetilde{z}_t)} = \left(\frac{z}{\widetilde{z}_t} \right)^{\frac{\theta}{1+\theta\varphi}} \tag{A.6}$$

Plugging the above condition into the wage index expressed on the space of workers z (of mass m_t) — instead of the space of labor types ω — then gives

$$W_t = \left[\int_{z_{\min}}^{\infty} z \left(\frac{w_t(z)}{z} \right)^{1-\theta} m_t \mu(z)\, dz \right]^{\frac{1}{1-\theta}} \tag{A.7}$$

$$= \underbrace{(m_t \widetilde{z}_t)^{\frac{1}{1-\theta}} w_t(\widetilde{z}_t) / \widetilde{z}_t}_{W_t} \widetilde{z}_t^{\frac{1}{1+\theta\varphi}} \left[\int_{z_{\min}}^{\infty} z^{\frac{\theta-1}{1+\theta\varphi}} \mu(z)\, dz \right]^{\frac{1}{1-\theta}} \tag{A.8}$$

which then implies

$$\widetilde{z}_t = \left[\int_{z_{\min}}^{\infty} z^{\frac{\theta-1}{1+\theta\varphi}} \mu(z)\, dz \right]^{\frac{1+\theta\varphi}{\theta-1}} \tag{A.9}$$

or equivalently:

$$\widetilde{z}_t^{-1} = \int_{z_{\min}}^{\infty} z^{-1} \left(\frac{\ell_t(z)}{\ell_t(\widetilde{z}_t)} \right)^{1+\varphi} \mu(z)\, dz \tag{A.10}$$

B Model summary and reduction

The model boils down to

$$\begin{aligned}
\text{Motion} &: \quad m_{t+1} = (1-\delta_t)(m_t + m_{et}) && \text{(B.1)}\\
\text{Labor market clearing} &: \quad L_t = Y_t/a_t + n_t f_{nt} + m_{et} f_{et} && \text{(B.2)}\\
\text{Wage} &: \quad W_t = a_t && \text{(B.3)}\\
\text{Participation} &: \quad \frac{n_t}{m_t} = \left(\frac{\nabla}{\widetilde{z}_{nt}}\right)^{\varepsilon} && \text{(B.4)}\\
\text{ZCP} &: \quad \widetilde{w}_{nt}\widetilde{\ell}_{nt} = \nabla^{\frac{\theta(1+\varphi)}{1+\theta\varphi}} W_t f_{nt} && \text{(B.5)}\\
\text{Av. dividends} &: \quad \widetilde{d}_t = \frac{n_t}{m_t}\widetilde{d}_{nt} && \text{(B.6)}\\
\text{Av. dividends of workers} &: \quad \widetilde{d}_{nt} = \widetilde{w}_{nt}\widetilde{\ell}_{nt} - W_t f_{nt} && \text{(B.7)}\\
\text{Free entry} &: \quad v_t = W_t f_{et} && \text{(B.8)}\\
\text{Euler share} &: \quad \beta(1-\delta_t) E_t\left\{\frac{\lambda_{t+1}}{\lambda_t}\frac{v_{t+1}+\widetilde{d}_{t+1}}{v_t}\right\} = 1 && \text{(B.9)}\\
\text{Goods market clearing} &: \quad C_t = Y_t && \text{(B.10)}\\
\text{Labor supply} &: \quad \chi L_t^{\varphi} = W_t \lambda_t && \text{(B.11)}\\
\text{Wage index} &: \quad W_t = (n_t\widetilde{z}_{nt})^{\frac{1}{1-\theta}}\,\widetilde{w}_{nt}/\widetilde{z}_{nt} && \text{(B.12)}\\
\text{Hours basket} &: \quad L_t = (n_t\widetilde{z}_{nt})^{\frac{\theta}{\theta-1}}\,\widetilde{\ell}_{nt} && \text{(B.13)}\\
\text{Marginal utility of C} &: \quad \lambda_t = C_t^{-\sigma} && \text{(B.14)}
\end{aligned}$$

C Aggregation

Here we show that aggregate budget constraint is equivalent to the labor market clearing. Aggregating the budget constraint across different households,

$$C_t + v_t(m_t + m_{et}) = m_t\left(v_t + \widetilde{d}_t\right) \quad \text{(C.1)}$$

Plugging the expression of $\widetilde{d}_t$,

$$C_t + v_t m_{et} = n_t \widetilde{d}_{nt} \quad \text{(C.2)}$$

Plugging the expression of $\widetilde{d}_{nt}$,

$$C_t + v_t m_{et} = n_t\left(\widetilde{w}_{nt}\widetilde{\ell}_{nt} - W_t f_{nt}\right) \quad \text{(C.3)}$$

We have $n_t\widetilde{w}_{nt}\widetilde{\ell}_{nt} = W_t L_t$, so

$$C_t + v_t m_{et} = W_t L_t - n_t W_t f_{nt} \quad \text{(C.4)}$$

With $Y_t = C_t$ and $v_t = W_t f_{et}$

$$Y_t + W_t f_{et} m_{et} = W_t L_t - n_t W_t f_{nt} \tag{C.5}$$

which, divided by W_t gives

$$\frac{Y_t}{W_t} + f_{et} m_{et} = L_t - n_t f_{nt} \tag{C.6}$$

with $W_t = a_t$ and rearranging,

$$L_t = \frac{Y_t}{a_t} + n_t f_{nt} + m_{et} f_{et} \tag{C.7}$$

D Steady state

We discuss the non-stochastic steady state. We first solve for $\widetilde{z}_n$ and next for C. Plugging the free entry and the average dividends of all household's members at the steady state, the Euler equation for share holdings becomes

$$\beta(1-\delta)\left\{1 + \frac{\frac{n}{m}\widetilde{d}_n}{W f_e}\right\} = 1 \tag{D.1}$$

Also from the zero cutoff profit condition,

$$\widetilde{w}_n \widetilde{l}_n = \nabla^{\frac{\theta(1+\varphi)}{1+\theta\varphi}} a f_n \tag{D.2}$$

and the average dividends of workers $\widetilde{d}_{nt} = \widetilde{w}_{nt}\widetilde{l}_{nt} - W_t f_{nt}$ at the steady sate, we have

$$\widetilde{d}_n = \left(\nabla^{\frac{\theta(1+\varphi)}{1+\theta\varphi}} - 1\right) W f_n$$

Noting that $\frac{n}{m} = \left(\frac{\nabla}{\widetilde{z}_n}\right)^{\varepsilon}$ and $W = a$, the Euler equation for share holdings (D.1) is expressed as

$$\beta(1-\delta)\left\{1 + \frac{\left(\frac{\nabla}{\widetilde{z}_n}\right)^{\varepsilon}\left(\nabla^{\frac{\theta(1+\varphi)}{1+\theta\varphi}} - 1\right) a f_n}{a f_e}\right\} = 1$$

which gives the solution for $\widetilde{z}_n$ as

$$\widetilde{z}_n = \left[\left(\frac{\beta(1-\delta)}{1-\beta(1-\delta)}\right)\frac{f_n}{f_e}\left(\nabla^{\frac{\theta(1+\varphi)}{1+\theta\varphi}} - 1\right)\right]^{\frac{1}{\varepsilon}} \nabla.$$

Next we solve for C, consumption at the steady state. With labor supply $\chi L^{\varphi} = W\lambda$ and the definition of marginal utility of consumption, $\lambda = C^{-\sigma}$ at the steady state, we have

$$L = \left(\frac{C^{-\sigma} a}{\chi}\right)^{\frac{1}{\varphi}} \tag{D.3}$$

Further, using the definition of wage index $n\widetilde{w}_n\widetilde{l}_n = WL$ and the zero cutoff profit condition (D.2), we have

$$n = \frac{L}{\nabla^{\frac{\theta(1+\varphi)}{1+\theta\varphi}} f_n} \tag{D.4}$$

The number of households members is given from the number of participation as

$$m = \left(\frac{\widetilde{z}_n}{\nabla}\right)^{\varepsilon} n \tag{D.5}$$

From the motion and given the solution for m, we have the solution for m_e as

$$m_e = \frac{\delta}{1-\delta} m \tag{D.6}$$

Plugging the above expressions, in the labor market clearing at the steady state $L = \frac{y}{a} + nf_n + m_e f_e$, we have

$$\left(\frac{C^{-\sigma}a}{\chi}\right)^{\frac{1}{\varphi}} = \frac{C}{a} + \left[f_n + \frac{\delta}{1-\delta}\left(\frac{\widetilde{z}_n}{\nabla}\right)^{\varepsilon} f_e\right] \frac{\left(\frac{C^{-\sigma}a}{\chi}\right)^{\frac{1}{\varphi}}}{\nabla^{\frac{\theta(1+\varphi)}{1+\theta\varphi}} f_n}$$

which gives the unique solution for C as

$$C = \left\{\frac{a^{1+\frac{1}{\varphi}}}{\chi^{\frac{1}{\varphi}}}\left[1 - \frac{1 + \frac{m_e}{n}\frac{f_e}{f_n}}{\nabla^{\frac{\theta(1+\varphi)}{1+\theta\varphi}}}\right]\right\}^{\frac{1}{1+\frac{\sigma}{\varphi}}}$$

where

$$\frac{m_e}{n} = \frac{\delta}{1-\delta}\left(\frac{\beta(1-\delta)}{1-\beta(1-\delta)}\right)\frac{f_n}{f_e}\left(\nabla^{\frac{\theta(1+\varphi)}{1+\theta\varphi}} - 1\right)$$

Given the solution of C we find the solution for L and n using (D.3) and (D.4). Also, m and m_e can be found from (D.5) and (D.6). Other variables are straightforward to compute.

E The model with capital accumulation

Let us first add the stock of capital K_t to production function of the model:

$$Y_t = a_t K_{t-1}^{\alpha} H_t^{1-\alpha} \tag{E.1}$$

Since final good producers operate under perfect competition, the factor demand functions become:

$$\alpha a_t K_{t-1}^{\alpha-1} H_t^{1-\alpha} = r_{kt} \tag{E.2}$$

$$(1-\alpha)\, a_t K_{t-1}^{\alpha} H_t^{-\alpha} = W_t \tag{E.3}$$

In addition, capital is accumulated by the households subject to investment adjustment costs, which gives the following Euler equation

$$\beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left(q_{t+1} \left(1-\delta_k\right) + r_{kt+1} \right) \right\} = q_t \tag{E.4}$$

with

$$q_t \left(1 - \phi g_{i,t}^2/2 - \phi g_{i,t} \left(1 + g_{i,t}\right)\right) + \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left(q_{t+1} \phi g_{i,t+1} \left(1 + g_{i,t+1}\right)^2 \right) \right\} = 1 \tag{E.5}$$

where ϕ captures the size of investment adjustment costs, δ_k the depreciation rate of physical capital and $g_{i,t} = I_t/I_{t-1} - 1$ is the growth rate of investment in physical capital. Finally, the clearing condition for the good market becomes

$$Y_t = C_t + I_t \tag{E.6}$$

where

$$K_t = (1-\delta_k)\, K_{t-1} + I_t \left(1 - \phi g_{i,t}^2/2\right) \tag{E.7}$$

and aggregation of households budget constraint modifies the labor market clearing condition to

$$L_t = (1-\alpha)\, Y_t / W_t + f_{et} m_{et} + n_t f_{nt} \tag{E.8}$$

We also introduce sluguishness in the dynamics of f_{et} and f_{nt} and assume

$$f_{et} = f_e \left(\frac{m_t}{m_{t-1}} \right)^{\phi_e} \text{ and } f_{nt} = f_n \left(\frac{n_t}{n_{t-1}} \right)^{\phi_n} \tag{E.9}$$

Finally, using the production function and the labor demand condition, we also find the following expression for the real wage W_t

$$W_t = (1-\alpha)\, a_t^{\frac{1}{1-\alpha}} \left(K_{t-1}/Y_t\right)^{\frac{\alpha}{1-\alpha}} \tag{E.10}$$

The model boils down to

$$
\begin{aligned}
&\text{Motion} &:&\quad m_{t+1} = (1-\delta_t)(m_t + m_{et}) &\text{(E.11)}\\
&\text{Production:} &&\quad Y_t = a_t K_{t-1}^{\alpha} H_t^{1-\alpha} &\text{(E.12)}\\
&\text{Labor market clearing} &:&\quad L_t = H_t + n_t f_{nt} + m_{et} f_{et} &\text{(E.13)}\\
&\text{Wage} &:&\quad W_t = (1-\alpha)\, a_t^{\frac{1}{1-\alpha}} (K_{t-1}/Y_t)^{\frac{\alpha}{1-\alpha}} &\text{(E.14)}\\
&\text{Participation} &:&\quad \frac{n_t}{m_t} = \left(\frac{\nabla}{\widetilde{z}_{nt}}\right)^{\varepsilon} &\text{(E.15)}\\
&\text{ZCP} &:&\quad \widetilde{w}_{nt}\widetilde{\ell}_{nt} = \nabla^{\frac{\theta(1+\varphi)}{1+\theta\varphi}} W_t f_{nt} &\text{(E.16)}\\
&\text{Av. dividends} &:&\quad \widetilde{d}_t = \frac{n_t}{m_t}\widetilde{d}_{nt} &\text{(E.17)}\\
&\text{Av. dividends of workers} &:&\quad \widetilde{d}_{nt} = \widetilde{w}_{nt}\widetilde{\ell}_{nt} - W_t f_{nt} &\text{(E.18)}\\
&\text{Free entry} &:&\quad v_t = W_t f_{et} &\text{(E.19)}\\
&\text{Euler share} &:&\quad \beta(1-\delta_t)\, E_t\left\{\frac{\lambda_{t+1}}{\lambda_t}\frac{v_{t+1}+\widetilde{d}_{t+1}}{v_t}\right\} = 1 &\text{(E.20)}\\
&\text{Euler investment} &:&\quad \beta E_t\left\{\frac{\lambda_{t+1}}{\lambda_t}\left(q_{t+1}(1-\delta_k) + r_{kt+1}\right)\right\} = q_t &\text{(E.21)}\\
&\text{Tobin's Q} &:&\quad q_t\left(1-\phi g_{i,t}^2/2 - \phi g_{i,t}(1+g_{i,t})\right) &\text{(E.22)}\\
& &:&\quad +\beta E_t\left\{\frac{\lambda_{t+1}}{\lambda_t}\left(q_{t+1}\phi g_{i,t+1}(1+g_{i,t+1})^2\right)\right\} = 1 &\text{(E.23)}\\
&\text{Investment} &:&\quad K_t = (1-\delta_k)K_{t-1} + I_t\left(1-\phi g_{i,t}^2/2\right) &\text{(E.24)}\\
&\text{Investment growth rate} &:&\quad g_{i,t} = I_t/I_{t-1} - 1 &\text{(E.25)}\\
&\text{Goods market clearing} &:&\quad Y_t = C_t + I_t &\text{(E.26)}\\
&\text{Labor supply} &:&\quad \chi L_t^{\varphi} = W_t\lambda_t &\text{(E.27)}\\
&\text{Wage index} &:&\quad W_t = (n_t\widetilde{z}_{nt})^{\frac{1}{1-\theta}}\, \widetilde{w}_{nt}/\widetilde{z}_{nt} &\text{(E.28)}\\
&\text{Hours basket} &:&\quad L_t = (n_t\widetilde{z}_{nt})^{\frac{\theta}{\theta-1}}\, \widetilde{\ell}_{nt} &\text{(E.29)}\\
&\text{Marginal utility of C} &:&\quad \lambda_t = C_t^{-\sigma} &\text{(E.30)}
\end{aligned}
$$