On the Role of Debt Maturity in a Model with Sovereign Risk and Financial Frictions*

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Abstract

We develop a model with financial frictions and sovereign default risk where the maturity of public debt is allowed to be larger than one period. When the debt portfolio has longer average maturities, public debt increases less in the event of a crisis, reducing the size of the subsequent fiscal consolidation through distorsionary taxes or public spending, with positive effects on welfare. In addition, we provide some results suggesting that optimized fiscal responses to a crisis depend on the average maturity of the debt portfolio.

Keywords: Sovereign Default Risk, Financial Crisis, Fiscal Policy.
JEL Classification: E44, E62, H63.

1 Introduction

Most DSGE models that try to analyze the effects of the 2008 financial crisis or the more recent sovereign debt crisis in Europe assume that the single support of public debt is a one-period bond. In practice however, the average maturity of public debt is much longer. Some papers have proposed simple ways of taking into account the fact that public debt has a longer maturity, including the seminal paper of Woodford (2001), or more recently Arellano and Ramanarayanan (2012), Debortoli, Nunes and Yared (2016) and Faraglia, Marcet, Oikonomou and Scott (2016).

This note investigates the importance of the average maturity of the stock of public debt in the aftermath of a financial crisis with banking frictions and sovereign risk. It is often argued that the bank-sovereign doom loop was at the heart of the double dip and 2011 debt crisis in countries of the south of the Euro Area. The initial kick however was due to the imported U.S. subprime crisis. We build a model *à la* Gertler and Karadi (2011) and incorporate sovereign default risk *à la* Corsetti, Kuester, Meier and Mueller (2014) and a distortionary tax on labor income. Finally we consider that sovereign bonds are perpetuities with a coupon decay factor ρ, as in Woodford

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(2001), where ρ is directly related to the average maturity of bonds. We run a crisis experiment as in Gertler and Karadi (2011) under very short, short, medium or long average maturities of the stock of sovereign debt.

We show that the resulting dynamics are substantially different. It has long been established that, for a given change in yields, the fluctuations in market price are greater when the term to maturity is longer (see Hopewell and Kaufman (1973)). Accordingly, in our model, longer maturities favor larger movement of bond prices that mitigate the rise in public debt and hence the size of the fiscal consolidation that follows a capital quality shock that mimics the effects of a crisis. When fiscal adjustment is achieved through the distorsionary tax rate, hours worked fall less and contribute to lower the fall in output and consumption, with positive effects on welfare. When the fiscal adjustment goes through changes in public spending, as in Cui (2016), they need to fall less for longer average maturities – they even rise under very long maturities, which contributes positively to welfare. In this sense, longer average maturities serve as a better shock absorber in the event of crises and generate welfare gains compared to shorter maturities.

2 Model

Our model of financial intermediation is an extension of the Gertler and Karadi (2011) model where banks can either grant loans to capital producers or hold risky sovereign bonds.

2.1 Banks

There is a unit continuum of banks. The balance sheet of the representative bank is

$q_t k_t$ = Private loans	d_t = domestic deposits
$q_t^b b_t$ = Sovereign bonds	n_t = net worth

where q_t is the price of capital, q_t^b the real price of the sovereign portfolio, k_t the amount of private capital and b_t the amount of sovereign bonds. The sovereign bond portfolio is defined as perpetuities with a coupon decay factor ρ, as in Woodford (2001). The parameter ρ is directly related to the average maturity of the sovereign bond portfolio, defined as $\mathcal{M} = 1/\left(1-\beta\rho\right)$ where β is the discount factor. We also introduce default risk (explained in details later) and assume that a fraction χ_t of the portfolio may be perceived by agents as being potentially defaulted on. Banks arbitrage both assets and equate their respective expected returns

$$E_t\left(r_{t+1}^k\right) = E_t\left(r_{t+1}^b\left(1-\chi_{t+1}\right)\right) \tag{1}$$

where r_{t+1}^k is the return on capital and $r_{t+1}^b = \left(1+\rho q_{t+1}^b\right)/q_t^b$ is the return on bonds. This equation shows that sovereign risk matters in the sense that agents price sovereign bonds *ex-ante* as if default was a potential event and are unaware of the *ex-post* insurance scheme. As such, sovereign risk affects the dynamics of returns both on the sovereign market but also on the capital market through the arbitrage of banks. Let a_t denote the total bank asset. The balance sheet equation is

$$a_t = q_t k_t + q_t^b b_t = d_t + n_t \tag{2}$$

As explained above, sovereign default matters *ex-ante* but not *ex-post*. Banks have access to insurance contracts and receive $T_t = \left(1 + \rho q_{t+1}^b\right) \chi_{t+1} b_t$, covering their losses in the event of sovereign default. Accordingly, net worth evolves according to

$$n_{t+1} = r_{t+1}^a a_t - r_t^d d_t + T_t \tag{3}$$

where r_t^d is the deposit rate and r_t^a the rate of return on total assets defined as

$$r_t^a = \left(r_t^b \left(1 - \chi_t\right) q_t^b b_{t-1} + r_t^k q_t \xi_t k_{t-1}\right) / a_{t-1} \tag{4}$$

where ξ_t is a capital quality shock to be defined later on. Combining both equations gives the dynamics of the bank's net worth

$$n_{t+1} = \left(r_{t+1}^a - r_t^d\right) q_t^a a_t + r_t^d n_t + T_t \tag{5}$$

The bank maximizes expected net worth given a fixed exit probability $(1 - \sigma)$, in which event net worth is rebated to the households, and discounts future outcomes at the stochastic rate $\beta_{t+1} = \beta u_{c,t+1} / u_{c,t}$. We follow Gertler and Karadi (2011), and conjecture that v_t is linear and assume

$$v_t = \varpi_t a_t + \gamma_t n_t \tag{6}$$

In addition, to prevent unlimited expansion of lending due to positive arbitrage opportunities, the representative bank may divert a fraction α of its assets. This possibility adds the following incentive constraint on bank's activities

$$v_t = \varpi_t a_t + \gamma_t n_t \geq \alpha q_t^a a_t \tag{7}$$

which will be strictly binding in equilibrium. Let $\phi_t = a_t / n_t = \left(n_t + d_t\right) / n_t$ be the leverage ratio of banks, the incentive constraint writes

$$v_t = \alpha \phi_t n_t \tag{8}$$

Banks optimization yields the following conditions for marginal values of arguments of the value function

$$\varpi_t = E_t \left(\left(1 - \sigma\right) \beta_{t+1} \left(r_{t+1}^a - r_t^d\right) + \sigma \beta_{t+1} \varpi_{t+1} \Omega_{t+1}^a \right) \tag{9}$$

$$\gamma_t = E_t \left(\left(1 - \sigma\right) + \sigma \beta_{t+1} \gamma_{t+1} \Omega_{t+1}^n \right) \tag{10}$$

where $\Omega_t^n = n_t/n_{t-1}$ is the growth rate of net worth and $\Omega_t^a = a_t/a_{t-1}$ the growth rate of intermediated assets, respectively evolving according to $\Omega_t^n = (r_t^a - r_{t-1}^d)\phi_{t-1} + r_{t-1}^d$ and $\Omega_t^a = (\phi_t/\phi_{t-1})\,\Omega_t^n$. Using the expression of the value function finally allows to reformulate the binding incentive constraint as

$$\phi_t = \frac{\gamma_t}{\alpha - \varpi_t} \tag{11}$$

2.2 Intermediate and capital goods producers

Intermediate goods producers use capital k_{t-1} in the production process. They also hire labor in quantity ℓ_t, that they combine to build the intermediate good, with the following production function

$$y_t^m = (\xi_t k_{t-1})^\iota \, \ell_t^{1-\iota} \tag{12}$$

and sell intermediate goods at real relative price p_t^m. In this production function, the total stock of physical capital can be affected by an AR(1) quality shock ξ_t. The optimizing condition with respect to labor is

$$p_t^m \,(1-\iota)\, y_t^m/\ell_t = w_t \tag{13}$$

where w_t is the real wage and the zero-profit condition implies that intermediate goods producers pay the *ex-post* return on capital to the capital goods producers, *i.e.*

$$r_{t+1}^k = \left(p_{t+1}^m \left(\iota y_{t+1}^m/k_t\right) + q_{t+1}\xi_t\,(1-\delta)\right)/q_t \tag{14}$$

Capital goods producers buy the depreciated capital of intermediate goods producers and choose investment to accrue the total amount of available capital based on the evolution of its real price q_t.[1] Their profit writes

$$E_t \sum_{s=0}^{\infty} \beta_{t+s} \left(q_{t+s} i_{t+s} \left(1 - \left(\varphi^i/2\right)\left(i_{t+s}/i_{t+s-1} - 1\right)^2\right) - i_{t+s}\right) \tag{17}$$

and optimization yields

$$q_t - 1 = q_t \varphi^i \left(x_t\,(1+x_t) + x_t^2/2\right) - E_t \left(\beta_{t+1} q_{t+1} \varphi^i x_{t+1}\,(1+x_{t+1})^2\right) \tag{18}$$

where $x_t = i_t/i_{t-1} - 1$. Given this optimizing condition for investment, the law of capital

[1] More formally, they maximize

$$E_t \sum_{s=0}^{\infty} \beta_{t+s,t+s+1} \left(q_{t+s}\left(k_{t+s} - (1-\delta)\,k_{t+s-1}\right) - i_{t+s}\right) \tag{15}$$

subject to the law of motion of capital accumulation

$$k_t - (1-\delta)\,\xi_t k_{t-1} = i_t \left(1 - \left(\varphi^i/2\right)\left(i_t/i_{t-1} - 1\right)^2\right). \tag{16}$$

accumulation gives the dynamics of the capital stock

$$k_t - (1-\delta)\,\xi_t k_{t-1} = i_t\left(1 - \left(\varphi^i/2\right)x_t^2\right) \tag{19}$$

2.3 Final goods producers

Final goods producers j differentiate the intermediate good y_t^m in imperfectly substitutable varieties. The aggregate bundle of the final good and the corresponding aggregate price level are

$$y_t = \left[\int_0^1 y_t\left(j\right)^{\frac{\theta-1}{\theta}}\,dj\right]^{\frac{\theta}{\theta-1}},\ p_t = \left[\int_0^1 p_t\left(j\right)^{1-\theta}\,dj\right]^{\frac{1}{1-\theta}} \tag{20}$$

Final goods producers take into account households demands $y_t\left(j\right) = \left(p_t\left(j\right)/p_t\right)^{-\theta} y_t$ when optimally setting prices subject to Calvo price contracts of average length $1/\left(1-\gamma\right)$ with indexation parameter γ^p. [2]

2.4 Households

Households face a simple optimization problem as they choose consumption, labor supply and deposits maximizing lifetime welfare

$$E_t \sum_{s=0}^{\infty} \beta^s u\left(c_{t+s}, g_{t+s}, \ell_{t+s}\right) \tag{21}$$

where $u_{\ell,t} \leq 0$, $u_{c,t} \geq 0$ and $u_{g,t} \geq 0$ are the first-order partial derivatives with respect to hours worked, consumption and public spending.[3] Households optimize subject to the budget constraint

$$d_t + c_t = r_{t-1}^d d_{t-1} + \left(1-\tau_t\right) w_t \ell_t + \Pi_t \tag{22}$$

where d_t denote deposits to banks returning r_t^d between t and $t+1$, c_t is consumption, w_t denotes the real wage, τ_t is a distortionary tax on labor income, ℓ_t hours worked, and Π_t comprises monopolistic profits from final goods producers, plus the net worth rebated by bankrupt banks, net from the starting fund allocated to new banks. FOCs give

$$E_t\left(\beta_{t+1} r_t^d\right) = 1 \tag{23}$$

$$u_{\ell,t} + \left(1-\tau_t\right) u_{c,t} w_t = 0 \tag{24}$$

[2] The optimal pricing conditions are standard and therefore not reported.

[3] We assume that households value public spending in the utility function since public spending will be used as a potential policy instrument to stabilize the economy.

2.5 Government and Central Bank

We adopt the approach of sovereign default from Corsetti et al. (2014) but with a slight variation concerning the distribution of default probabilities. Actual *ex-post* default is neutral while the *ex-ante* probability of default is crucial for the pricing of government debt and therefore for real activity.[4] Focusing on the domestic economy, the *ex-ante* probability of default, p_t, at a certain level of sovereign indebtedness, $by_t = q_t^b b_t / (4y_t)$, will be given by:

$$p_t = \alpha_p e^{by_t - by} \tag{25}$$

where by denotes the steady-state level of debt to GDP. Default occurs with probability p_t so that

$$\chi_t = \Delta \text{ if } \mathcal{B}(p_t) = 1, \text{ and } \chi_t = 0 \text{ if } \mathcal{B}(p_t) = 0 \tag{26}$$

where $\mathcal{B}(p_t)$ is a Bernoulli. Given these assumptions, the consolidated budget constraint writes[5]

$$q_t^b b_t^g = \left(1 + \rho_t q_t^b\right) b_{t-1}^g + g_t - \tau_t w_t \ell_t \tag{29}$$

The stability of public debt is ensured either by a tax rule or by a public spending rule, as in Cui (2016)

$$\tau_t - \tau = d_\tau^b (by_{t-1} - by) \tag{30}$$

$$g_t - g = -d_g^b (by_{t-1} - by) \tag{31}$$

The central bank controls the nominal interest rate i_t^n

$$i_t^n = r_t^d E_t(\pi_{t+1}) \tag{32}$$

[4] Following Eaton and Gersovitz (1981), many have modeled default as a strategic decision of the sovereign. On the other hand, Bi (2012) considers default as the consequence of the government's inability to raise the funds necessary to honor its debt obligations. Alternatively, Juessen, Linnemann and Schabert (2016) model default as resulting from the behavior of households and their willingness to lend. Under all approaches, the probability of sovereign default is closely and non-linearly related to the level of public debt to GDP.

[5] The budget constraint of the government writes

$$q_t^b b_t^g = \left(1 + \rho q_t^b\right)(1 - \chi_t) b_{t-1}^g + g_t - \tau_t w_t \ell_t + T_t^b \tag{27}$$

Once again, potential losses from default are fully compensated, so that only *ex-ante* default risk matters. As a consequence

$$T_t^b = \left(1 + \rho_t q_t^b\right) \chi_t b_{t-1}^g \tag{28}$$

which yields the reported consolidated budget constraint.

where the central bank follows a Taylor-type policy rule[6]

$$\log\left(i_t^n/i^n\right)=\rho_r\log\left(i_{t-1}^n/i^n\right)+d_\pi\log\left(\pi_t/\pi\right)+d_y\log\left(y_t/\widetilde{y}_t\right) \tag{33}$$

where $\widetilde{y}_t$ is the natural level of output.[7]

2.6 Aggregation

At the end of the period, a fraction $1-\sigma$ of the total number of banks become households. Dividends are paid to households only when banks exit the banking sector. The net worth of continuing banks is simply carried to the next period, so that aggregate continuing banks' net worth evolve according to

$$n_t^e=\sigma\Omega_t^a n_{t-1} \tag{34}$$

In addition, the household provides a starting net worth to new banks, equal to a fraction $\varphi/\left(1-\sigma\right)$ of the total assets of exiting bankers, so that the net worth of new banks is

$$n_t^n=\varphi\left(q_t\xi_t k_{t-1}+q_t^b b_{t-1}\right) \tag{35}$$

Overall, aggregate net worth evolves according to

$$n_t=n_t^e+n_t^n \tag{36}$$

The clearing condition on the intermediate goods market is $y_t^m=\int_0^1 y_t\left(j\right)dj=y_t dp_t$ where $dp_t=\int_0^1\left(p_t\left(j\right)/p_t\right)^{-\theta}dj$ is the dispersion of prices. On the final goods market, the clearing condition simply writes $y_t=c_t+i_t+g_t$ and the clearing condition for government bonds is just $b_t^g=b_t$.

3 Calibration

We calibrate the model to the Euro Area but follow Gertler and Karadi (2011) in many respects. The time unit is a quarter. The functional form of preferences is

$$u\left(c_t,g_t,n_t\right)=log\left(\tilde{c}_t\right)-\omega\ell_t^{1+\psi}/1+\psi \tag{37}$$

[6]None of the scenarios investigated in this paper implies that the nominal interest rate hits the Zero Lower Bound after the shock. While such a case would be an interesting extension (obtained for instance by assuming a zero persistence in the Taylor rule), we do not consider this possibility.

[7]Variations in the mark-up will serve as a proxy for variations in the output gap.

where

$$\begin{cases} \tilde{c}_t = \left((1-\kappa)\, c_t^{\frac{\nu-1}{\nu}} + \kappa g_t^{\frac{\nu-1}{\nu}} \right)^{\frac{\nu}{\nu-1}} \text{ if } \nu \neq 1 \\ \tilde{c}_t = c_t^{1-\kappa} g_t^{\kappa} \text{ if } \nu = 1 \end{cases} \tag{38}$$

The discount factor is $\beta = 0.99$. The inverse of the Frisch elasticity on labor supply is $\psi = 3$.[8] In the utility function, $\nu > 0$ governs the extent of complementarity ($\nu < 1$), independence ($\nu = 1$) or substitutability ($\nu > 1$) between public spending and household's consumption. In the baseline case, we consider $\nu = 1$, and preferences are separable. However, alternative cases will be considered as a robustness check: one where public and private expenditure are complementary ($\nu = 0.5$, a value in line with the estimates of Bouakez and Rebei (2007)) and one where they are substitutable ($\nu = 2$). In all cases, the weight parameter κ is set so as to equate the marginal utility of private consumption and public spending in the steady state, *i.e.* $u_c = u_g$. On the production side, the share of capital is $\iota = 0.33$, the depreciation rate is $\delta = 0.018$ (7% annually). We impose a 1pp spread (in annual terms) over the risk-less rate for r^k which pins down the capital to output ratio. Investment adjustment costs are $\varphi_i = 2$, the Calvo parameters are $\gamma = 0.75$ and $\gamma^p = 0.25$, and the steady-state mark-up is 30%, implying $\theta = 4.33$. In the banking sector, we impose a steady-state leverage ratio of $\phi = 4$. The survival probability of bankers is $\sigma = 0.975$. On the monetary and fiscal policy side, we set the steady-state probability of default at $\alpha^p = 0.5\%$ and the value of $\Delta = 0.55$, implying a roughly 2pp sovereign spread (in annual terms) with respect to the risk-less deposit rate for the calibrated debt-output ratio. We assume standard Taylor rule parameters, *i.e.* $\rho_r = 0.8$, $d_\pi = 1.5$ and $d_y = 0.125$. The stability of debt to GDP is ensured either through taxes or through public spending. In the former case, Equation (30) is in place and we set $d^b_\tau = 0.33$ while at $d^b_g = 0$. In the latter case, Equation (31) is in place with $d^b_g = 0.15$ while $d^b_\tau = 0$. Based on the average maturity of sovereign debt in Euro Area countries in 2007 (7 years), the average duration of the portfolio is $\mathcal{M} = 1/\left(1 - \beta\rho\right) = 28$ implying that the steady-state value of the decay is $\rho = 0.9740$. Alternative values are considered in the paper: $\mathcal{M} = 4$ (1 year maturity), $\mathcal{M} = 8$ (2 years maturity) and $\mathcal{M} = 100$ (25 years maturity). Using OECD data for 2007, we build a measure of hours worked and set $\ell = 0.2571$. Proceeding similarly, the share of public expenditure in GDP and the level of public debt to GDP are respectively $s_g = 0.2025$ and $b^g/\left(4y\right) = 0.6959$.[9] The steady-state labor income tax rate is then adjusted to match the debt-to-GDP ratio target: $\tau = 0.4762$. This calibration implies that our model has a unique determinate equilibrium in which, in the

[8] This value is much larger than the value considered by Gertler and Karadi (2011) – they use 0.276 – but their calibration relates to the U.S. where the labor market is much more responsive than in the Euro Area.

[9] Notice that considering alternative average maturities does not affect the steady state, but only affects the price-quantity nexus of public debt. Longer maturities are associated with larger bond price and less quantities while shorter maturities are associated with low bond prices and larger quantities. The total steady-state value of the portfolio $q^b b^g$ however remains the same under all calibrations.

terminology of Leeper (1991), monetary policy is active and fiscal policy is passive.[10] Table 1 summarizes our parameter values.

Table 1: Parameter values.

Discount factor	$\beta = 0.99$
Inverse of the Frisch elasticity	$\psi = 3$
Weight on hours worked	ω adjusted to get $\ell = 0.2571$
Weight on public spending	κ adjusted to get $u_c = u_g$
Elasticity of substitution between public and private consumption	$\nu = \{1, 0.5, 2\}$
Capital share	$\iota = 0.33$
Steady-state depreciation rate	$\delta = 0.018$
Steady-state quarterly return on capital	$r^k = 1.0025/\beta$
Investment adjustment costs	$\varphi_i = 2$
Calvo probability of price adjustment	$\gamma = 0.75$
Indexation parameter	$\gamma^p = 0.25$
Firms mark-up	$1/(\theta - 1) = 0.3$
Leverage ratio	$\phi = 4$
Bankers survival probability	$\sigma = 0.975$
Steady-state probability of default	$\alpha^p = 0.005$
Size of default	$\Delta = 0.55$
Nominal interest rate persistence	$\rho_r = 0.8$
Nominal interest rate response to inflation	$d_\pi = 1.5$
Nominal interest rate response to output gap	$d_y = 0.125$
Response of labor income tax to debt	$d_\tau^b = \{0.33, 0\}$
Response of public spending to debt	$d_g^b = \{0, 0.15\}$
Average debt maturity	$\mathcal{M} = 1/(1 - \beta\rho) = \{4, 8, 28, 100\}$
Steady-state public spending to GDP	$s_g = g/y = 0.2025$
Steady-state public debt to annual GDP	$b_g/(4y) = 0.6959$
Steady-state labor income tax rate	$s_g = g/y = 0.4762$

4 Transmission of a capital quality shock

4.1 Labor income tax rule

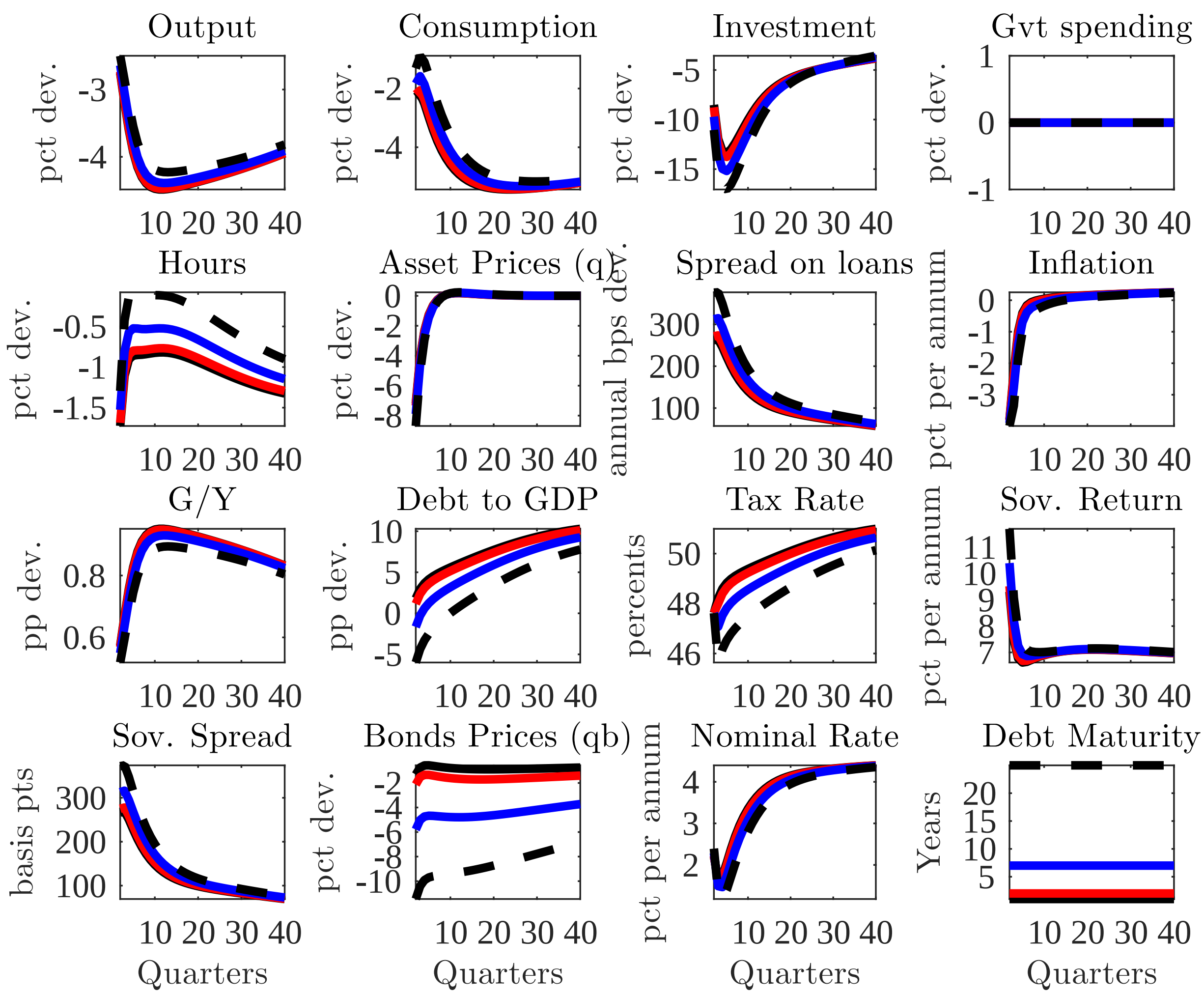
We investigate the transmission of a capital quality shock, that mimics quite accurately the effects of the 2008 financial crisis with different steady-state average sovereign debt maturities affect. The model is solved non-linearly under perfect foresight using a Newton-type algorithm.[11] We consider four alternative steady-state maturities for public debt: 1, 2, 7 and 25 years respectively.

Figure 1 plots the Impulse Response Functions (IRFs hereafter) of the economy after a 4% capital

[10] As shown in Appendix A, alternative monetary-fiscal regimes producing stable and unique equilibria exist – in particular one in which monetary policy is passive and fiscal policy is active – but their analysis is beyond the scope of the present paper. Note that the determinacy analysis is made around the steady state by computing the eigenvalues of the dynamic log-linearized system, while our simulations are conducted using a fully non-linear solution to the perfect foresight equilibrium.

[11] The algorithm is a built-in routine of Dynare. It is an application of the Newton-Raphson algorithm that takes into consideration the special structure of the Jacobian matrix in dynamic models with forward-looking variables. The details of the algorithm are explained in Juillard (1996).

Figure 1: IRFs to a capital quality shock - Labor income tax rule.



Black solid: 1 year, red: 2 years, blue: 7 years, dashed black: 25 years.

quality risk shock with persistence 0.66. As capital quality falls the price of capital q_t falls as well, deteriorating banks balance sheet and contributing to lower investment. The stock of capital shrinks, bringing output and consumption down. This dynamics tends to raise the debt to GDP ratio, which contributes to raise sovereign risk and further raises the stock of debt. Sovereign spreads rise one for one with private spreads and the price of bonds falls. The government has to change the tax rate to stabilize debt in the medium run, which results in an additional variation of hours worked, consumption, investment and output.

The major difference with the various maturities lies in the size of the response of bond prices: the longer the average maturity, the larger the fall in bond prices induced by the shock. Since the shock implies a surge in sovereign rates – partly through the financial accelerator and partly through sovereign risk – bond prices fall. The sensitive of prices to the rise in the yields is much longer for longer maturities. Hence, for a given rise of sovereign rates, longer maturities contribute to lower the rise in public debt, and so the rise in distorsionary taxes. The stabilizing movement of sovereign bonds price is so large for very long maturities that it reverses the initial response of the value of the stock of debt, leading to a temporary fall in distortionary taxes, an increase in hours worked, consumption and output. This "positive" outcome comes at the cost of larger sovereign and private spreads, and hence at the cost of a larger fall in investment. Summarizing, longer maturities reduce the fall in output, consumption and hours worked, and magnify the fall in investment.

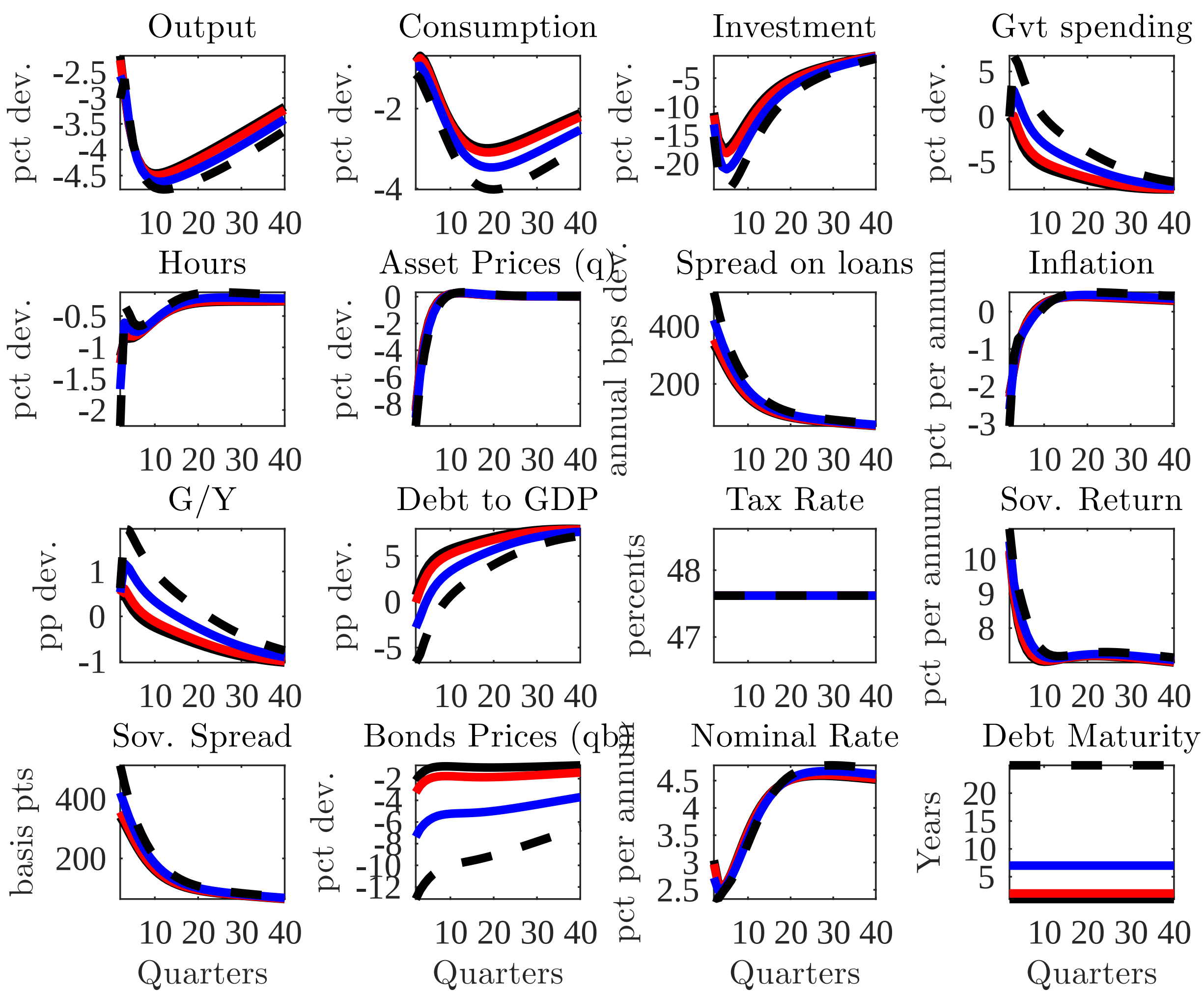
4.2 Public spending rule

Figure 2 plots the IRFs to the very same shock but when the government stabilizes the stock of public debt using public spending instead of the labor income tax rate.

The story shares a lot with the previous case: output, consumption, hours worked and investment fall, private and sovereign spread rise, and the debt-output ratio increases. In addition, as in the previous case, the adjustment of sovereign bond prices is much larger for longer maturities than for short maturities. Again, this stabilizing effect is large enough to overturn the response of the value of debt in the first quarters after the shock from positive to negative for very long maturities. However, the fiscal adjustment is different in that public spending is the instrument to stabilize debt, and the labor income tax remains constant. Hence, the additional adverse effects from higher taxes are absent in this case: output and consumption fall less, hours worked fall more. The dynamics of consumption is affected by public spending through the crowding-out effect by which any fiscal consolidation using public spending will make private consumption rise, which contributes positively to the dynamics of output.[12] As in the previous case, the positive spillovers from long average maturities of the sovereign debt portfolio favor debt sustainability and lower the size of the required fiscal adjustment.

[12] We report in Appendix B the IRFs obtained with a public spending rule when public and private goods are complementary or substitutable, as a robustness check.

Figure 2: IRFs to a capital quality shock - Public spending rule.



Black solid: 1 year, red: 2 years, blue: 7 years, dashed black: 25 years.

5 Welfare

What are the implications of these results for the welfare losses generated by the crisis? Table 2 below reports the welfare losses under different average maturities, under alternative fiscal rules and at different horizons.[13]

Table 2: Welfare losses from a negative capital quality shock, in percents.

	$\mathcal{M}=4$	$\mathcal{M}=8$	$\mathcal{M}=28$	$\mathcal{M}=100$
	Tax rule - $\left(d_\tau^b, d_g^b\right)=(0.33,0)$			
$T=4$	1.3688	1.2976	1.0210	0.6304
$T=8$	2.2873	2.2007	1.8690	1.4057
$T=32$	4.1371	4.0984	3.9366	3.6961
$T=\infty$	3.0216	3.0115	2.9607	2.8710
	Spending rule - $\left(d_\tau^b, d_g^b\right)=(0,0.15)$			
$T=4$	0.5049	0.4126	0.0956	-0.2806
$T=8$	1.4624	1.3295	0.8883	0.3585
$T=32$	3.8507	3.8026	3.6556	3.4801
$T=\infty$	2.4568	2.4679	2.5039	2.5522

First, Table 2 shows that, when the government follows a tax rule, longer maturities unambiguously help stabilizing debt, taxes and thus consumption, which generates positive effects on welfare. Welfare losses are clearly decreasing functions of maturities, at all horizons, and the reduction in welfare losses can be quite substantial. Second, Table 2 also shows that, when the government follows a spending rule, longer maturities generate smaller welfare losses at short horizon against a negligible larger lifetime welfare loss. Finally, our results point to the much smaller welfare losses from the shock under public spending adjustments than under distorsionary tax adjustments, at all horizons. Even though the analysis of optimal policies is clearly beyond the scope of this note, our results suggest that adjusting public spending rather than distortionary taxes is the way to go for governments experiencing large negative shocks, at least for short to moderately long maturities. Does this result hold when considering that public and private consumption expenditure are not separable in the utility function?

Table 3 reports the same calculations when consumption and public spending are not separable in the utility function for the two cases ($\nu = 0.5$ and $\nu = 2$), under a public spending rule. It shows that complementarity reduces the welfare losses (or increases the gains) from the shock in the short run against larger losses in the medium and long run. The opposite pattern (larger losses in the short run against smaller losses in the long run) is observed under substitutability. In both cases, varying ν does not overturn our main results: short-run welfare losses are decreasing

[13] Welfare losses are computed as the Hicksian consumption equivalent that makes households indifferent between experiencing the crisis and remaining at the initial steady state. The calculation is made for different dates T after the shock. When T is small, the calculation captures the short-run gains or losses, when T is larger, is gets closer to the lifetime welfare gains or losses. The "true" welfare effect corresponds to $T=\infty$ but considering shorter horizons also sheds light on the short-run effects of the shock under alternative policies and for different average maturities.

Table 3: Welfare losses with complementarity or substitutability, in percents.

	$\mathcal{M}=4$	$\mathcal{M}=8$	$\mathcal{M}=28$	$\mathcal{M}=100$
Spending rule with $\nu=0.5$				
$T=4$	0.1174	0.0000	−0.3727	−0.7873
$T=8$	1.1746	0.9835	0.3983	−0.2664
$T=32$	3.9304	3.8441	3.6011	3.3178
$T=\infty$	2.6217	2.6330	2.6671	2.7155
Spending rule with $\nu=2$				
$T=4$	0.6931	0.6125	0.3320	−0.0148
$T=8$	1.6079	1.4997	1.1301	0.6773
$T=32$	3.8285	3.7962	3.6947	3.5737
$T=\infty$	2.3777	2.3893	2.4280	2.4786

with the average maturity of the public debt portfolio and public spending adjustments generate smaller welfare losses at all horizons compared to tax adjustments, even when public and private consumption expenditure are strongly complementary.

We finally push the analysis a bit further and compute optimized fiscal rules by picking the parameters of the fiscal rules d^b_τ and d^b_g – allowing both rules to be in play simultaneously – that minimize the lifetime welfare losses generated by a negative shock on capital quality. The optimized parameters and corresponding welfare losses are reported in Table 4 below.[14]

Table 4: Welfare losses under optimized rules, in percents.

	$\mathcal{M}=4$	$\mathcal{M}=8$	$\mathcal{M}=28$	$\mathcal{M}=100$
Independence ($\nu=1$)				
d^b_τ	0.0000	0.0000	0.0000	0.000
d^b_g	0.5441	0.5504	0.4857	0.3822
$T=4$	0.8069	0.5355	−0.2282	−0.8836
$T=8$	2.5932	2.2240	1.0436	−0.0792
$T=32$	4.2146	4.1759	4.0743	3.9182
$T=\infty$	2.3089	2.3203	2.3716	2.4564
Complementarity ($\nu=0.5$)				
d^b_τ	0.0000	0.0000	0.2013	0.6974
d^b_g	0.3808	0.3798	0.3456	0.2802
$T=4$	0.2418	−0.0842	−0.5197	−0.8452
$T=8$	2.3222	1.7215	0.7734	0.1824
$T=32$	4.4530	4.3718	4.1651	3.9764
$T=\infty$	2.3958	2.4123	2.4706	2.5297
Substitutability ($\nu=2$)				
d^b_τ	0.0000	0.0000	0.0000	0.0000
d^b_g	0.7373	0.7994	0.6985	0.5102
$T=4$	1.1086	0.9051	0.1794	−0.4664
$T=8$	2.5680	2.3767	1.4553	0.4380
$T=32$	4.1137	4.0905	4.0263	3.9380
$T=\infty$	2.2730	2.2827	2.3261	2.3988

[14]The IRFs under optimized rules are reported in Appendix C for the baseline case with separable preferences.

Table 4 confirms that, for short to moderately long maturities and in most cases, the best instrument to stabilize debt remains public spending.[15] The response of the tax rule is muted for almost all maturities, except for the long maturities when public and private goods are strong complements. In addition, the optimized response of public spending to deviations of the debt-output ratio are be substantially stronger than the value imposed in the baseline calibration. The welfare gains of optimized rules are observed in the last lines of Table 4 by looking at the lifetime welfare losses and comparing them to the corresponding losses with simple rules, in particular with the spending rule (last lines of Table 2 and Table 3). They can be as large as 0.25% of permanent consumption, especially for short maturities. For longer maturities, the potential lifetime welfare gains are smaller while still non-negligible, around 0.1% of permanent consumption. At shorter horizons, optimized rules tend to increase the welfare losses for short average maturities, while they yield smaller losses (even gains) under longer maturities.

Overall, our results point to the crucial importance of taking into account the structure of the sovereign debt portfolio, its average maturity and its effects on the banking system when analyzing the transmission of shocks and the effects of monetary and fiscal policies. These features make a large difference in terms of the dynamics of public debt, sovereign risk and aggregate macroeconomic dynamics.

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[15] Viegas and Ribeiro (2016) report similar results. However, our point is far from general and remains contextualized to this model and the associated assumptions, in particular regarding the restricted form the fiscal rules.

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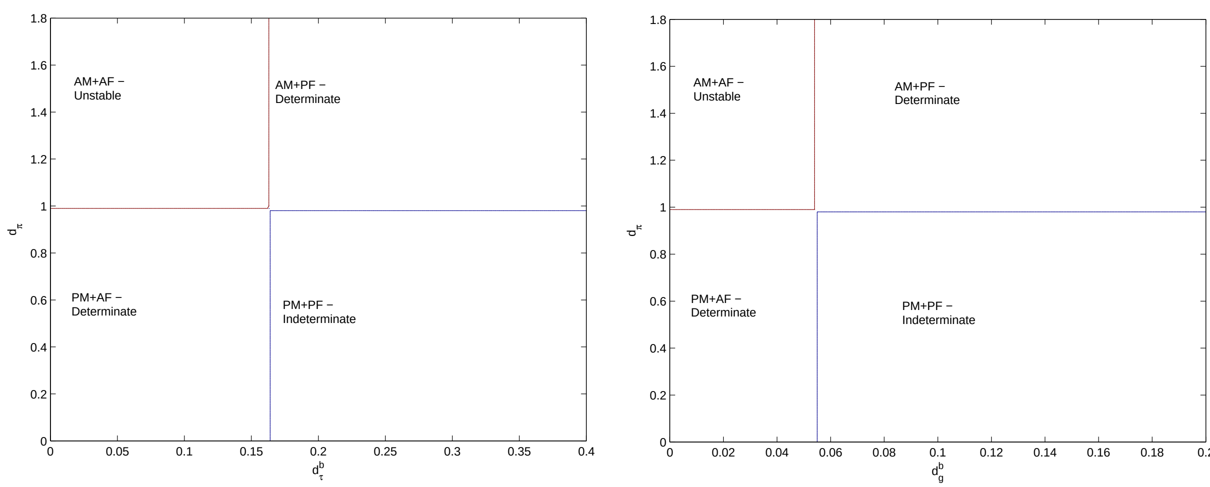
A Determinacy analysis

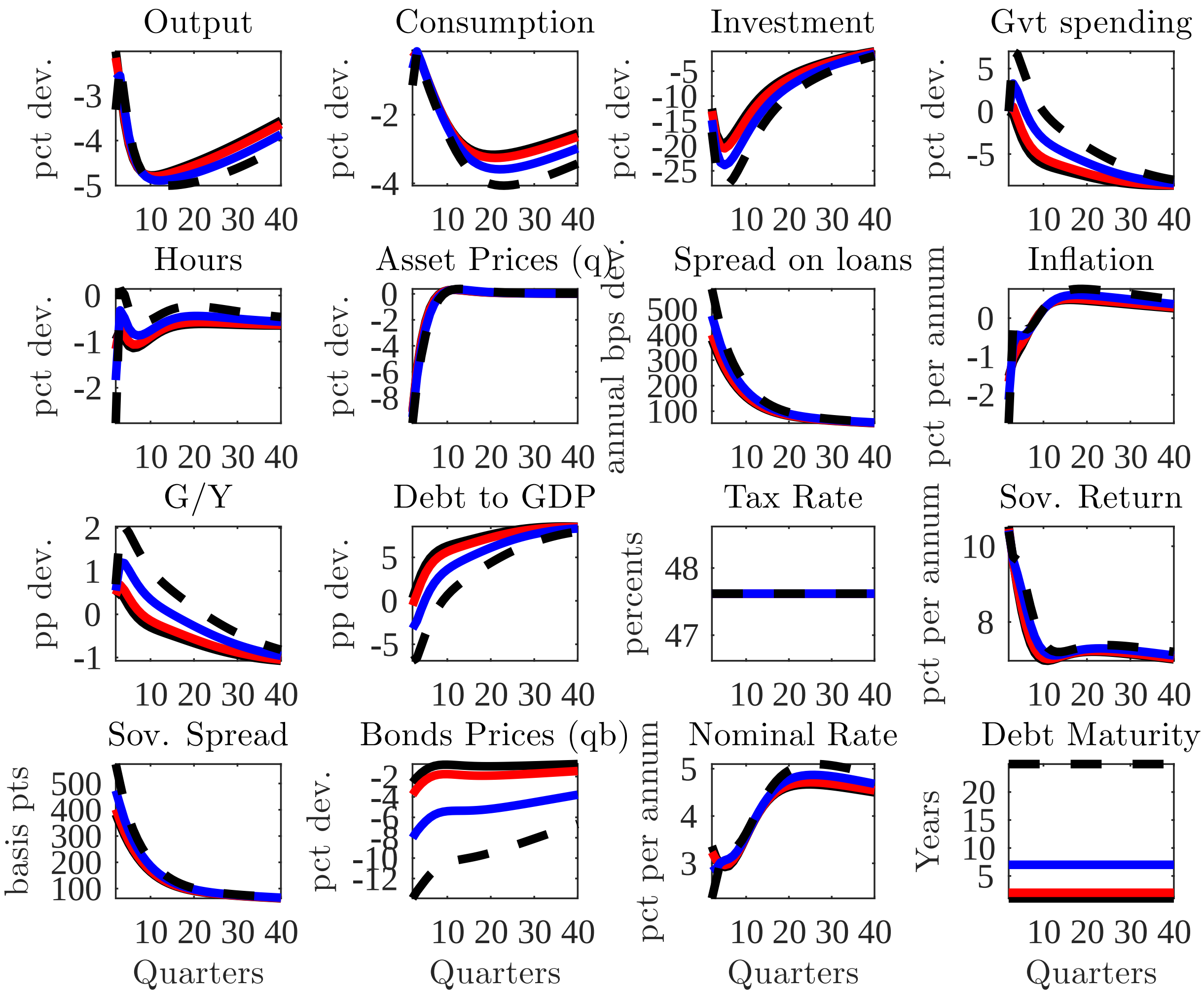
Figure 3: Determinacy analysis in the (d_π, d_τ^b) space and in the (d_π, d_g^b) space.



<u>Notes</u>: M/F means monetary or fiscal policy and A/P means active or passive policy in the sense of Leeper (1991). The unstable (resp. determinate / indeterminate) region corresponds to parameter values for which the number of explosive roots exceeds (resp. is equal to / is less than) the number of forward variables. Determinacy regions are not affected by the average maturity so these maps are valid for any steady-state value of the average maturity of the sovereign debt portfolio.

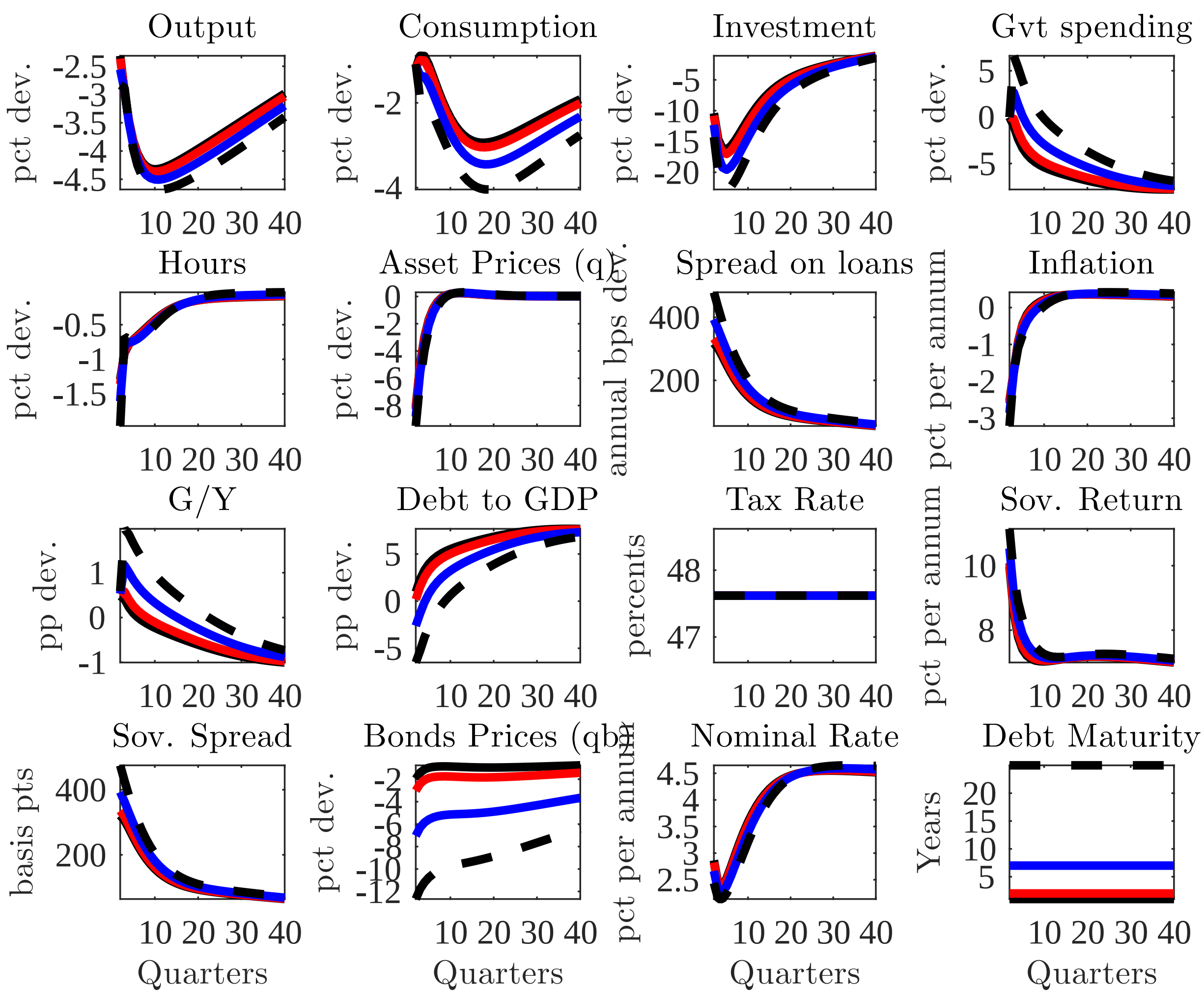
B IRFs under spending rule with complementarity or substitutability

Figure 4: IRFs with a public spending rule - $\nu = 0.5$.



Black solid: 1 year, red: 2 years, blue: 7 years, dashed black: 25 years.

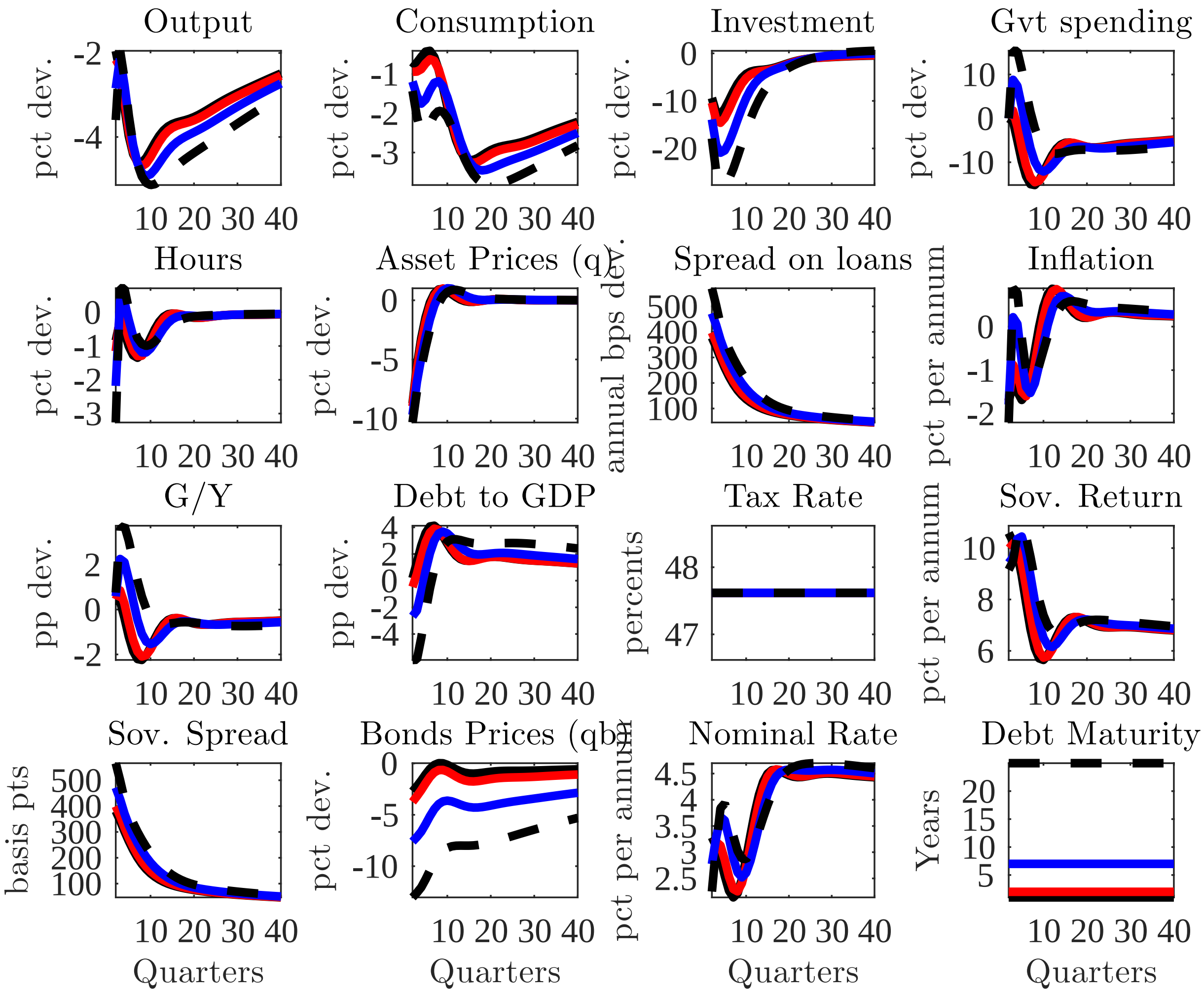
Figure 5: IRFs with a public spending rule - $\nu = 2$.



Black solid: 1 year, red: 2 years, blue: 7 years, dashed black: 25 years.

C IRFs with optimized fiscal rules

Figure 6: IRFs under optimized fiscal rules - $\nu = 1$.



Black solid: 1 year, red: 2 years, blue: 7 years, dashed black: 25 years.