Online Appendix to

Taking off into the Wind: Unemployment Risk and State-Dependent Government Spending Multipliers

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A Solution method

Our solution method is fully non-linear and takes advantage of the continuous-time formulation of the heterogeneous-agent problem solving the Hamilton-Jacobi-Bellman and Kolmogorov forward equations. Our codes are adapted from those of Bence Bardoczy taken from the HACT project page maintained by Benjamin Moll: http://www.princeton.edu/~moll/HACTproject.htm.

A.1 Steady state

The algorithm solving for the steady state is the following.

1. Based on the calibrated separation rate and the target value for the unemployment rate, $\bar{u}$, compute $\bar{f}$, $\bar{q}$, and $\bar{\theta}$.

2. Guess initial values of the real interest rate, $\bar{r}$, and the vacancy-posting cost, ξ.

3. Compute output, $\bar{y}$. Based on the calibration targets, compute government spending, $\bar{g}$, and public debt, $\bar{b}$, and infer the tax rate, τ.

4. Compute profits and income levels for employed and unemployed households.

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5. Given income levels and the real interest rate:

   - Solve the Hamilton-Jacobi-Bellman equation.
   - Solve the marginal value of a job, $\bar{J}$.
   - Solve the steady-state Nash-bargaining problem to obtain the real wage, $\bar{w}$.
   - Adjust the value of the disutility parameter ω, such that hours per worker are normalized to one ($\bar{\ell} = 1$) in the steady state.
   - Update the value functions $\bar{W}$, $\bar{J}$, as well as $\bar{w}$.

6. Solve the Kolmogorov forward equation to recover the distributions of households over the asset grid.

7. Compute residuals from the goods-market-clearing and free-entry conditions.

8. If the residuals are larger than a tolerance level, update $\bar{r}$ and ξ. Use $\bar{r}^{new}$ and ξ^{new} as new guesses in Step 2.

9. Iterate until both residuals are smaller than the tolerance level.

A.2 Transition dynamics

The algorithm solving for the transitional dynamics is the following:

1. Guess initial sequences of the real interest rate $\{r_t\}_{t=1}^{t=T}$ and labor market tightness $\{\theta_t\}_{t=1}^{t=T}$.

2. Set all the endogenous variables to their steady-state values in all periods.

3. For $t = \{1 : T\}$, given θ_t, compute the transition probabilities f_t and q_t, as well as the transition matrix, Λ_t.

4. For $t = \{1 : T\}$, given r_t, compute the value of public debt b_t.

5. For $t = \{1 : T\}$, compute the the inflation rate, π_t, using the Taylor rule, and the associated nominal interest rate, i_t^n.

6. For $t = \{T : 1\}$, compute backward the price of intermediate goods, p_t^m.

7. For $t = \{T : 1\}$, starting from the terminal (steady-state) values of the variables:

   - Compute profits, lump-sum taxes, and income levels for employed and unemployed households.
   - Solve the Hamilton-Jacobi-Bellman equation and compute the value function W_t.
   - Solve the marginal value of a job, J_t.
   - Compute the real wage, w_t, according to the wage rule.
   - Compute hours per worker, ℓ_t.

8. For $t = \{1 : T\}$, solve the Kolmogorov forward equation to obtain the distributions of households over the asset grid.

9. From those distributions, compute the paths of the unemployment rate $\{u_t\}_{t=1}^{t=T}$ and vacancies $\{v_t\}_{t=1}^{t=T}$ (using $\{v_t\}_{t=1}^{t=T} = \{\theta_t\}_{t=1}^{t=T} \times \{u_t\}_{t=1}^{t=T}$).

10. Compute the sequence of residuals in the asset-market-clearing and free entry conditions, denoted, respectively, by $\{\zeta_{asset,t}\}_{t=1}^{t=T}$ and $\{\zeta_{fe,t}\}_{t=1}^{t=T}$.

11. If the residuals are larger than a tolerance level, update $\{r_t\}_{t=1}^{t=T}$ and $\{\theta_t\}_{t=1}^{t=T}$. The updating process for the real interest rate is: $\{r_t^{new}\} = \{r_t\} - d_r \{\zeta_{asset,t}\}$ where d_r is a small number: whenever households hold assets in excess of asset supply, lower the real interest rate. Similarly, we impose $\{\theta_t^{new}\} = \{\theta_t\} + d_\theta \{\zeta_{fe,t}\}$ where d_θ is a small number: whenever the labor market is not tight enough, raise tightness $\{\theta_t\}$. Use $\{r_t^{new}\}$ and $\{\theta_t^{new}\}$ as new guesses in Step 1.

12. Iterate until the largest absolute value of each of the sequences$\{\zeta_{asset,t}\}$ and $\{\zeta_{fe,t}\}$ is less than the tolerance level.

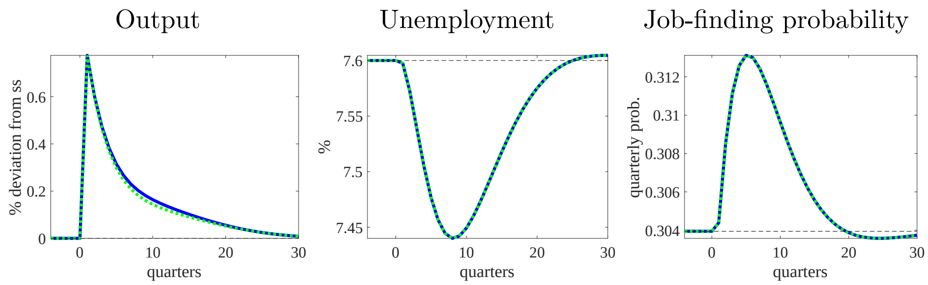
Solving for the transition dynamics takes a few minutes depending on the size and nature of the shocks and the assumptions considered.

B Unconditional Effects of Government Spending Shocks in Counterfactual Economies

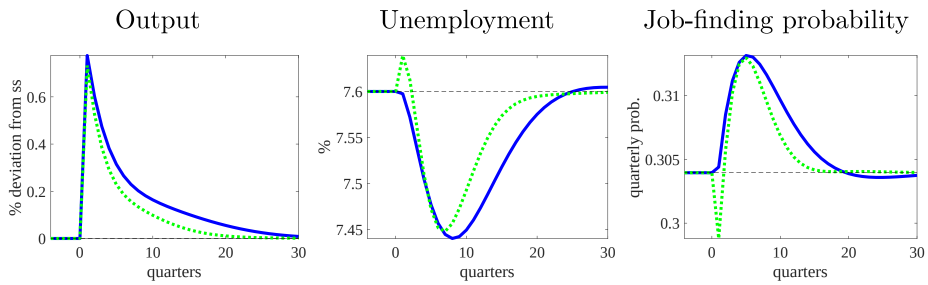
In this section, we report the dynamic responses of output, unemployment, and the job-finding probability to a 5% increase in government spending, assuming that the economy is initially in the steady state. The Figure superimposes on the responses implied by the benchmark model those obtained from each of the counterfactual economies studies in Section 3.2 of the paper.

Figure 1: Impulse responses to a 5% government spending shock: Benchmark vs. counterfactual.

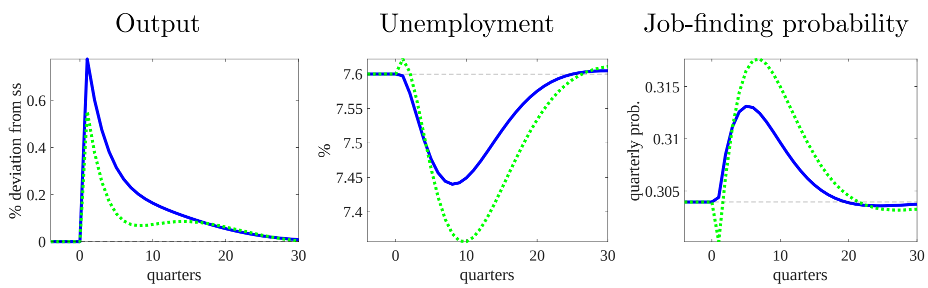
(a) Benchmark vs. no composition effect



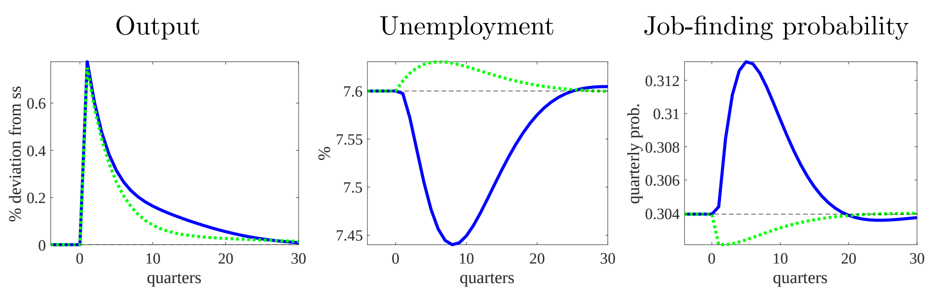
(b) Benchmark vs. complete markets



(c) Benchmark vs. no intensive margin



(d) Benchmark vs. flexible prices



Blue solid: Benchmark. Light green: Counterfactual.

C Sensitivity Analysis

In this section, we study the sensitivity of our results along two dimensions. First, we consider alternative values of the following parameters, one at a time: the replacement rate, h; the matching-curvature parameter, α; the inverse of the Frisch elasticity of labor supply, ψ; the tax-feedback parameter, d_T, and the value of steady-state inflation, $\bar{\pi}$. In each case, we evaluate the present-value multiplier for aggregate output, unemployment, aggregate consumption, and the per capita consumption of employed and unemployed households conditional on different sizes of the productivity shock. The results are depicted in Figure 2. Second, we calibrate the model to represent the U.S. labor market.

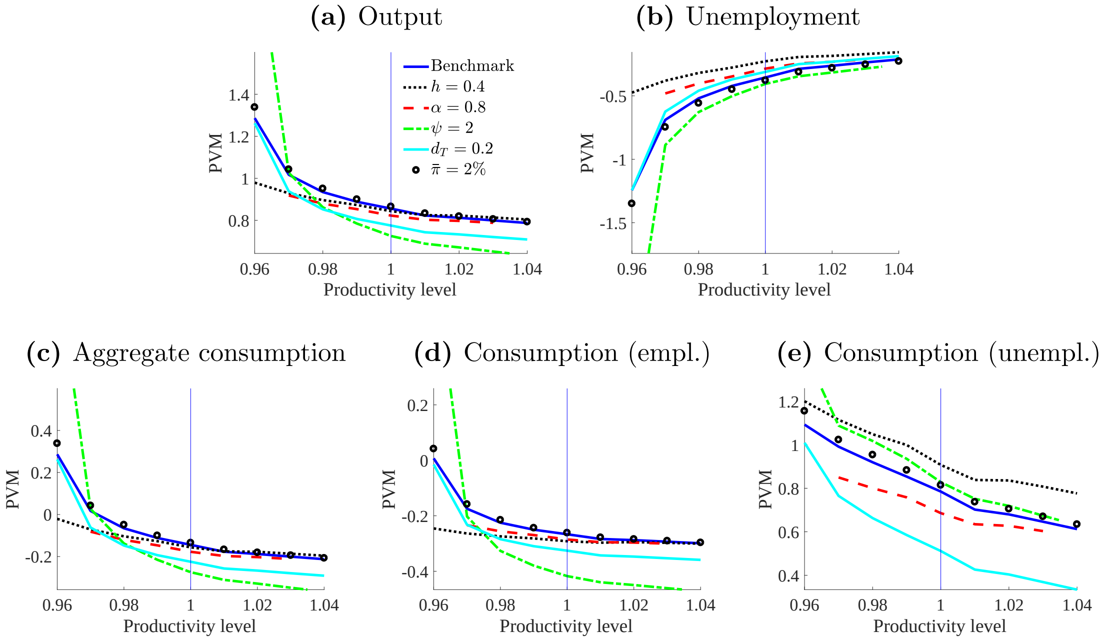
C.1 Alternative parameter values

As an alternative value for the replacement rate, we choose $h = 0.4$, a value commonly used in search and matching models calibrated to the U.S. economy, which arguably has a lower replacement rate than most of European countries. All else equal, a lower replacement rate has two effects. First, it exacerbates income losses during unemployment spells, which strengthens the precautionary motive, leading households to accumulate more assets while employed. This attenuates the decline in their consumption in response to an increase in government spending, and results in a larger spending multiplier. Second, because the replacement rate pins down the vacancy-posting cost (conditional on the values of the remaining parameters), the latter increases when h falls. Larger vacancy-posting costs imply that firms' accounting profits are larger and less sensitive (in terms of percentage changes) to shocks, mitigating firm's incentives to post vacancies,[1] and by extension, the fall in unemployment following an increase in government spending (as in Monacelli, Perotti, and Trigari (2010)). This in turn translates into a smaller output multiplier. During large expansions, the first effect dominates, such that a lower replacement rate raises the spending multiplier. For smaller expansions and recessions, the second effect dominates, reducing the multiplier when the replacement rate is lower. The reason for this result lies in the fact that lower replacement rates dampen the response of output and unemployment to productivity shocks, and that this dampening is larger the more negative is the shock. Together, these results imply that the extent of state dependence falls with the replacement rate.

Next, we lower the matching-curvature parameter, α, from its benchmark value of 1 to a value of 0.8. As shown in Figure 2, the spending multiplier falls (rises for unemployment) for any given value of the productivity shock. This is simply due to the fact that a smaller value of α implies a smaller job-finding probability for any given labor-market tightness. More importantly, the figure shows that the spending multiplier becomes less sensitive to the size of recessions/expansions. The reason is that the job-finding probability becomes less concave with respect to the labor-market tightness. In our model, the concavity of the job-finding probability plays a key role in generating asymmetric changes in unemployment, which translate into asymmetries in precautionary saving

[1] See Hagedorn and Manovskii (2008).

Figure 2: Present-value multipliers conditional on productivity shocks.



Notes: Present-value multiplier are computed according to Equation (19) in the main text, with x_t being the variable of interest. The productivity shock ranges from -4% to 4%.

and in the composition effect.[2] With a less concave job-finding probability, these asymmetries are less important.

In the third experiment, we consider a Frisch elasticity of labor supply of 0.5 by increasing ψ from 1 to 2. Larger values of ψ imply that the union is less willing to increase the supply of hours after a given shock, *ceteris paribus*. As ψ tends to infinity, the model boils down to the economy without an intensive margin of labor adjustment, described in Section 3.2 of the paper. By a continuity argument, it is obvious that the unconditional output multiplier will be smaller the larger the value of ψ. This continues to be the case in expansions and in mild recessions. However, because larger values of ψ amplify the economy's response to productivity shocks, recessions will be more severe for a given fall in productivity, and this tends to increase the multiplier and thus the extent of state dependence.

In a fourth experiment, we increase the value of the tax-feedback parameter, d_T, from 0.1 to 0.2. This alternative parameter value tilts the tax schedule needed to finance the increase in government spending towards the present. Since Ricardian equivalence does not hold in our model, this further raises the real interest rate and exacerbates the fall in the consumption of employed households at short horizons. Because the present-value multiplier assigns a larger weight to changes in aggregate variables that occur in the near future, it will tend to fall (in absolute value) as d_T rises. This outcome should hold regardless of the size of productivity shocks since d_T has little effect on the economy's response to those shocks. One should therefore expect the multiplier curves to simply

[2] Conceptually, the parameters h and α affect the results through the same channel: the elasticity of unemployment in response to shocks. However, while changes in h affect this elasticity indirectly — through steady-state tightness — changes in α do so directly.

shift downward (upward for unemployment) compared to the benchmark economy, which is exactly what Figure 2 shows. Based on this discussion, we can safely conclude that the amount of state dependence implied by the model exhibits very little sensitivity to changes in the value of d_T.

In the last experiment, we consider a strictly positive inflation target of 2% per annum. Alves (2018) shows that, in a model with Calvo-type price contracts, non-zero trend inflation can help generate volatile labor-market variables. This continues to be the case under the Rotemberg-type price stickiness assumed in our model. However, raising steady-state inflation from 0 to 2% per annum generally raises the spending multipliers (in absolute value) by a negligible amount, except when the economy is a in deep recession.

C.2 Calibrating the model to the U.S. labor market

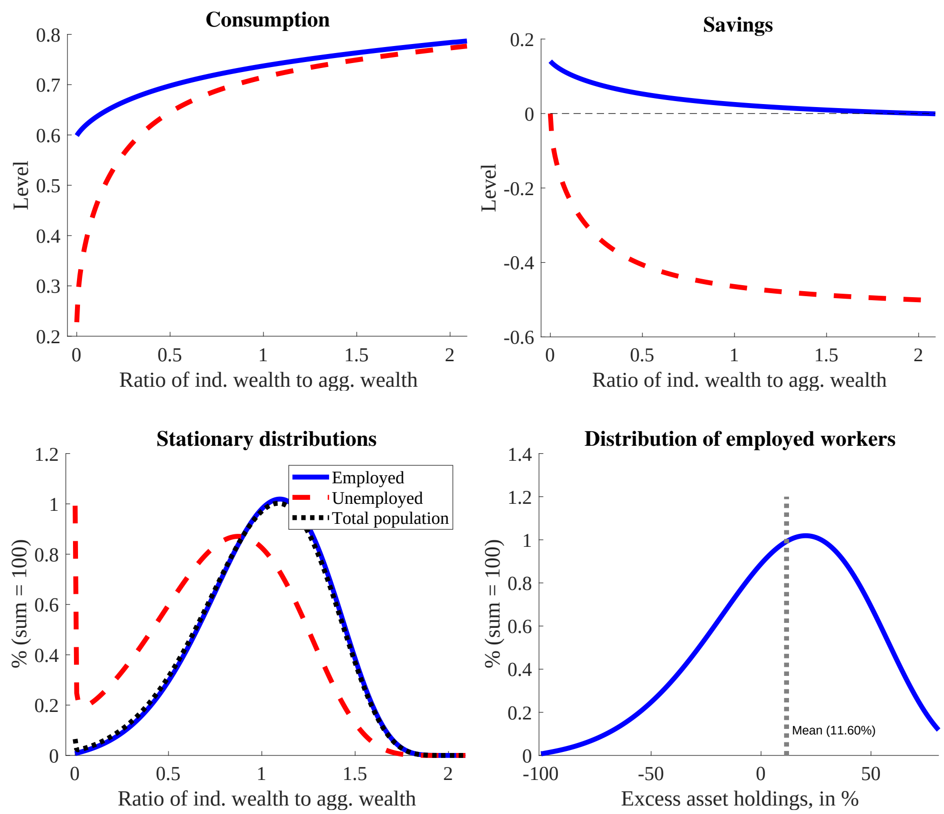
So far, we have focused on a model that captures the specificities of the labor market prevailing in major European countries, and studied the sensitivity of results by perturbing some of the structural parameters one at a time. In what follows, we evaluate the spending multiplier and its state dependence in a version of the model calibrated to the U.S. economy. The U.S. labor market is not only characterized by a lower replacement rate than our benchmark economy, it also features significantly larger separation and job-finding rates, i.e., larger labor-market turnover.[3] To capture these characteristics, we set the replacement rate to $h = 0.4$ and impose a separation rate of $s = 0.05$ — twice the benchmark value, along with a targeted unemployment rate of $u = 0.059$, which yields a job-finding probability $f \simeq 0.8$ — almost four times the benchmark value. These numbers are almost identical to those used by Challe (2020), who targets the U.S. labor market at a quarterly frequency. Finally, we impose a slightly lower bargaining power for the union, $\beta = 0.7$, in the determination of the steady-state real wage.

Keeping the remaining parameters unchanged, we obtain the stationary distributions and policy functions reported in Figure 3. Compared with the results based on the European calibration, the higher transition rates produce much more similar stationary distributions of asset holdings for employed and unemployed households. These distributions indicate that a much smaller number of unemployed households hold zero assets: since unemployment spells are significantly shorter, unemployed households get to keep a larger fraction of the (precautionary) asset holdings they accumulated in the past, when they were employed. Note that this feature tends to lower the aggregate MPC of unemployed households. In addition, employed households accumulate less assets to self-insure against unemployment risk since the higher turnover tends to reduce unemployment risk, even though the income loss associated with unemployment spells is larger than under the European calibration. Figure 3 also reveals that the consumption function is steeper at low levels of asset holdings under the U.S. calibration, especially for unemployed households, implying that these households have a larger marginal utility of consumption at low levels of assets than their counterparts in the benchmark economy. Importantly, this feature tends to raise the MPC of un-

[3] From this perspective, the alternative calibration of the replacement rate performed in the sensitivity analysis above, while informative, would be insufficient to draw conclusions about the effects of government spending shocks and their state dependence in the U.S. simply because other parameters need to be simultaneously re-calibrated to match the characteristics of the U.S. labor market.

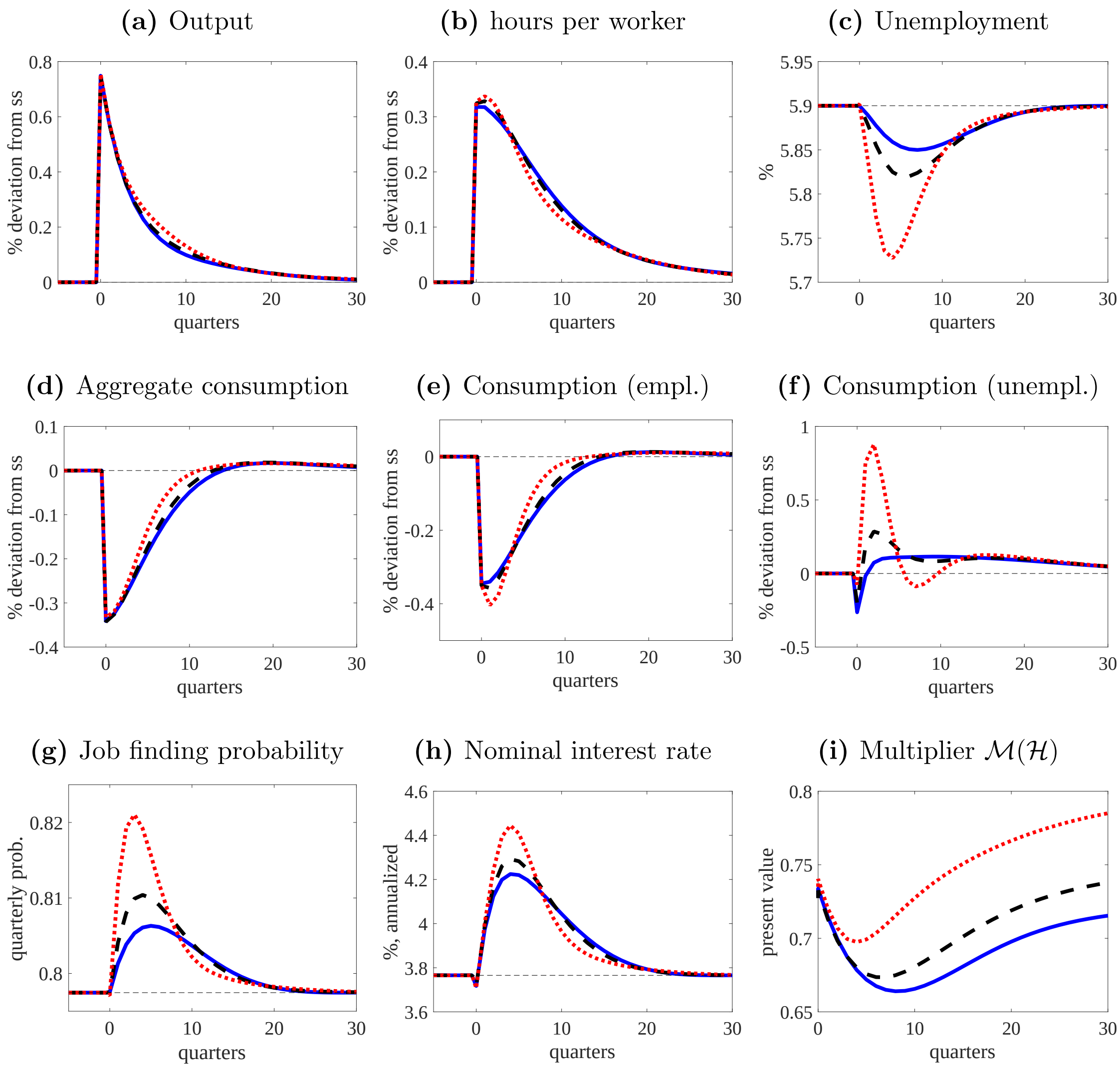
employed households holding small amounts of assets, and thus the aggregate MPC of unemployed households. Finally, the bottom right panel of Figure 3 shows that, on average, employed workers hold 11.6% more assets than in an otherwise identical economy with complete markets — where unemployment risk is fully insured. This figure is consistent with the amount of precautionary saving estimated by Hurst, Lusardi, Kennickell, and Torralba (2010) in the U.S. economy.

Figure 3: Policy functions and stationary distributions under the U.S. calibration.



How does the U.S. calibration affect the degree of state dependence of the spending multipliers? To answer this question, we again evaluate the net effects of a government spending shock conditional on positive and negative 3% productivity shocks. The responses obtained in each state are reported in Figure 4. These responses resemble those obtained under the European calibration: An increase in government spending raises aggregate demand and lowers unemployment. The job-finding probability increases and unemployment risk drops, which eventually crowds-in the consumption of unemployed households, thus fueling the rise in aggregate demand and further lowering unemployment. These effects are larger conditional on a recession than on an expansion. Quantitatively, the output multiplier is 0.73 in expansion and 0.80 in recession, a difference of 9.7%. This amount of state dependence, albeit smaller than under the European calibration is still non negligible.

Figure 4: Impulse responses to a 5% government spending shock. Net effect in recession and expansion. U.S. calibration.



Solid blue: conditional on an expansion. Dashed black: around steady state. Dotted red: conditional on a recession.

D An Extended Model

We propose an extension of the model that comes closer to replicating the distribution of assets and MPCs observed in the data. We introduce two additional sources of heterogeneity. First, we assume that there are patient households, with a discount factor $\rho_p < \rho$, and impatient households, with a discount factor $\rho_i > \rho$. Patient and impatient households represent roughly equal shares of the total population. In line with Krusell and Smith (1998), households can switch type with a very small probability, which helps preserve the equal split of the population into patient and impatient agents. This assumption generates a large density of households close to the zero-asset limit, many of which are impatient employed households. Second, instead of assuming that aggregate profits are distributed to employed households as in the baseline model, we posit that they are distributed to a third type of households: entrepreneurs. We introduce a very small probability p_{e^+} of becoming an entrepreneur and a small probability $p_{e^-} > p_{e^+}$ of losing this status. This implies a relatively low stationary share of entrepreneurs in the economy. Combined with the fact that they receive all the profits from retailers and intermediate-good producers, entrepreneurs are very rich in terms of per capita income compared with the other households. They are also large savers because of the small probability of losing the status, and the extremely small probability of ever becoming an entrepreneur again in the future, once this status is lost. This additional assumption stretches the distribution of asset holdings to the right — a small fraction of the population becomes asset-rich — and generates a fat right tail in the distribution of asset holdings. Formally, the household's budget constraint in the extended model is

$$a_t^i + c_t^i = (1 + r_{t-1})\, a_{t-1}^i + \left(1 - \mathbb{1}_{ent}^i\right) \left[(1 - \tau_t) \left(\mathbb{1}_e^i w_t \ell_t + \left(1 - \mathbb{1}_e^i\right) h\overline{w}\right) - \mathbb{1}_e^i T_t^i\right] + \mathbb{1}_{ent}^i \Pi_t^i,$$

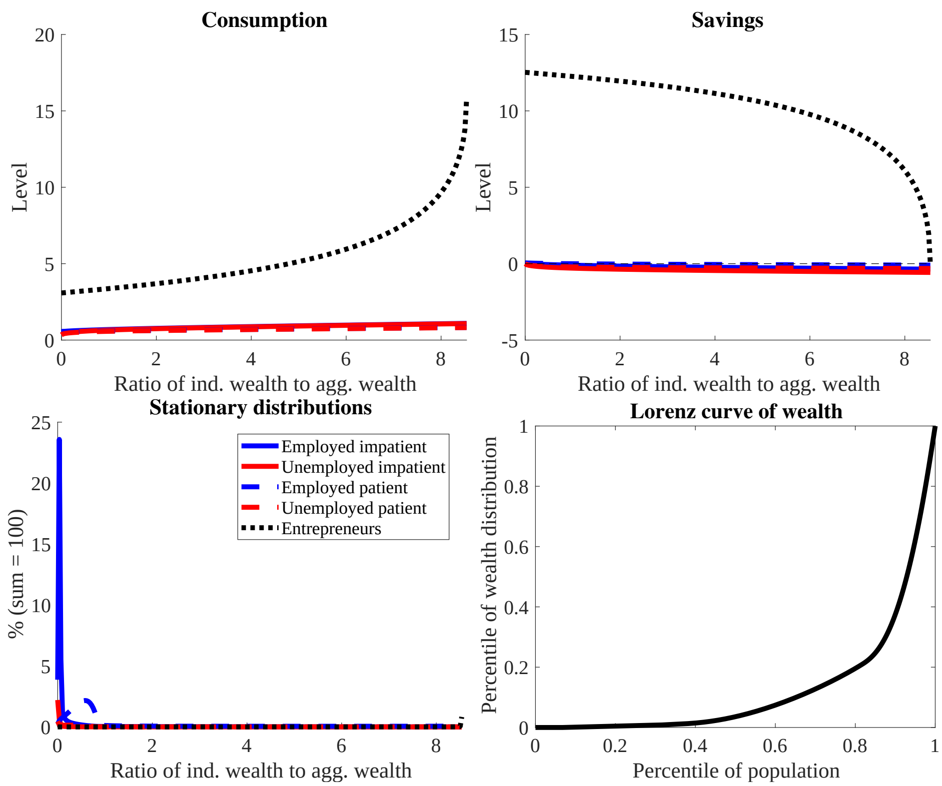
where $\mathbb{1}_e^i$ is an indicator function that takes the value of 1 if household i is employed and 0 otherwise, and $\mathbb{1}_{ent}^i$ an indicator function that takes the value 1 if household i is an entrepreneur and 0 otherwise. In the extended model, the transition matrix expands because we consider a total of 5 states: employed impatient, unemployed impatient, employed patient, unemployed patient and entrepreneur. Hence,

$$\Lambda_t = \begin{bmatrix} (1-p_{e^+})\, p_{oi}\,(1-s) & (1-p_{e^+})\, p_{oi} s & (1-p_{e^+})\,(1-p_{oi})\,(1-s) & (1-p_{e^+})\,(1-p_{oi})\, s & p_{e^+} \\ (1-p_{e^+})\, p_{oi} f_t & (1-p_{e^+})\, p_{oi}\,(1-f_t) & (1-p_{e^+})\,(1-p_{oi})\, f_t & (1-p_{e^+})\,(1-p_{oi})\, 1-f_t & p_{e^+} \\ (1-p_{e^+})\, p_{ni}\,(1-s) & (1-p_{e^+})\, p_{ni} s & (1-p_{e^+})\,(1-p_{ni})\,(1-s) & (1-p_{e^+})\,(1-p_{ni})\, s & p_{e^+} \\ (1-p_{e^+})\, p_{ni} f_t & (1-p_{e^+})\, p_{ni}\,(1-f_t) & (1-p_{e^+})\,(1-p_{ni})\, f_t & (1-p_{e^+})\,(1-p_{ni})\, 1-f_t & p_{e^+} \\ p_{e^-} p_{ni} & 0 & p_{e^-}\,(1-p_{ni}) & 0 & 1-p_{e^-} \end{bmatrix},$$

where p_{oi} is the (large) probability of staying impatient while p_{ni} is the (small) probability of becoming impatient. We assume that entrepreneurs become workers when losing their status. In addition, their discount factor is ρ, in-between the discount factor of patient and impatient households.

The rest of the model remains unchanged and the calibration is adapted when needed to deliver similar targets to those in the benchmark model. We add a couple of targets: a Gini coefficient on wealth of 0.75, the upper bound of the estimates available for European economies (see Carroll, Slacalek, and Tokuoka (2014)), and a share of entrepreneurs of 1%. The latter target is achieved by assuming $p_{e^+} = 0.001$ and $p_{e^-} = 0.05$, while we obtain the former by imposing $\rho_p = 0.0075$, $\rho_i = 0.015$, along with $p_{oi} = 0.995$ and $p_{ni} = 0.005$. The corresponding policy functions and stationary distributions are reported in Figure 5.

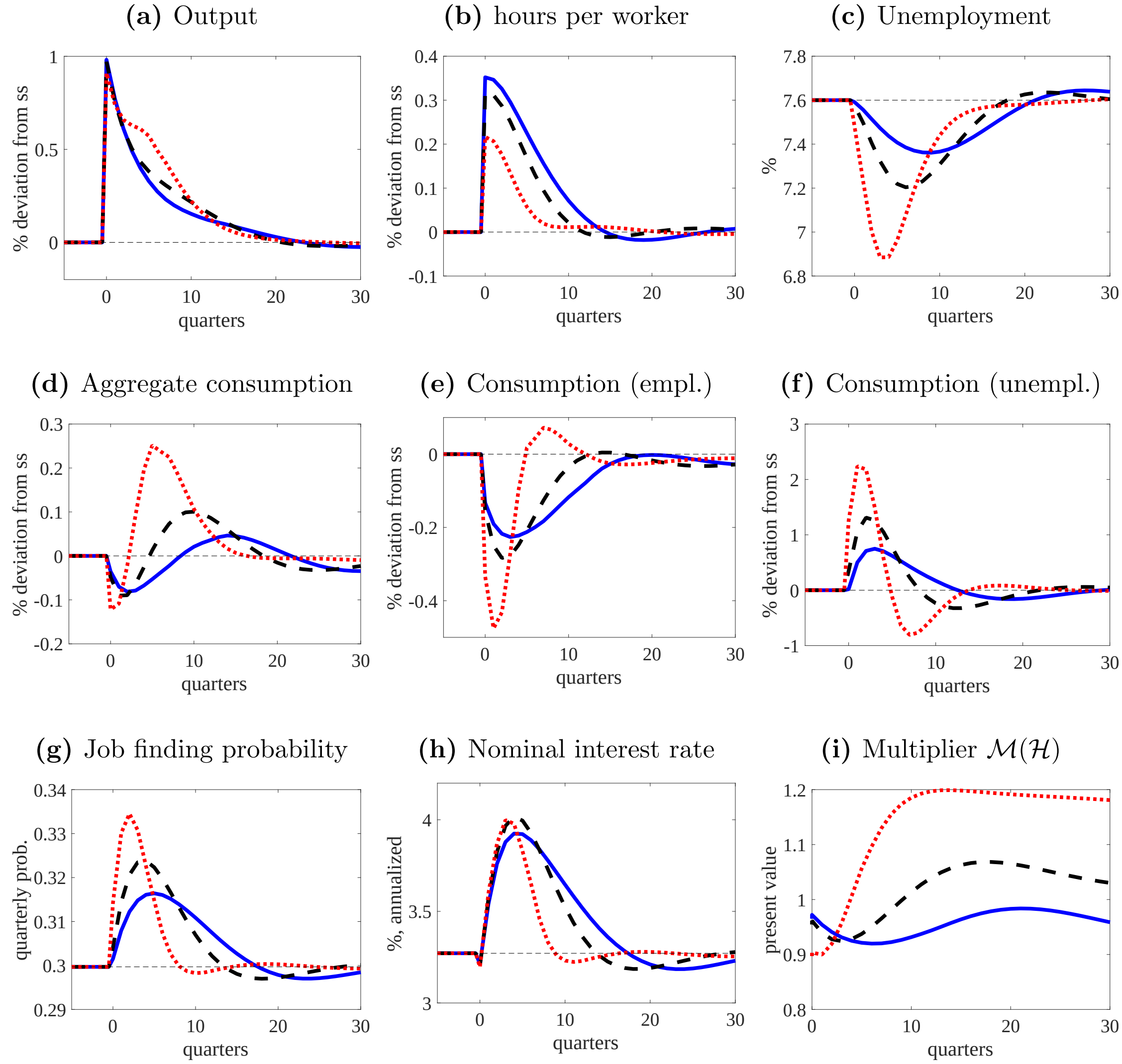
Figure 5: Steady-state distributions and policy functions in the extended model.



The model delivers a 0.7478 Gini coefficient on wealth, and the Lorenz curve for wealth reported in Figure 5. In addition, the distribution of MPCs is now more in line with empirical evidence, as the aggregate MPC is now 0.1973, the average MPCs for impatient and patient employed households are 0.2074 and 0.0556, respectively, and the average MPCs for impatient and patient unemployed households are 0.5679 and 0.4910, respectively. These numbers line-up quite well with those reported by Carroll, Slacalek, and Tokuoka (2014). Finally, we generate impulse responses based on a ±2% productivity shock — this extended model generates much larger variations of output for given values of the productivity shock — and report the net effects of a 5% government spending shock in Figure 6. For completeness, we also report the unconditional effects of the spending shock (i.e., occurring while the economy is initially is in the steady state).

Government spending shocks have very similar qualitative implications to those implied by the benchmark model, and their effects display an equally significant amount of state dependence, being larger in recession than in expansion. The output multiplier is 0.90 conditional on a 2% positive productivity shock (expansion) and 1.15 conditional on a 2% negative shock (recession), a difference of 27.1%.

Figure 6: Impulse responses to a 5% government spending shock in the extended model. Net effect in recession and expansion.



Solid blue: conditional on an expansion. Dashed black: around the steady state. Dotted red: conditional on a recession.

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