Trade Wars and the Optimal Design of Monetary Rules*

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April 22, 2024

Keywords: Protectionism, Trade Wars, Inflation Targeting

JEL Category, F30, F40, F41

Abstract

Monetary rules may have a large effect on the outcome of trade wars if central banks target the CPI inflation rate or more generally changes in the relative price of traded goods. We lay out a two-country open-economy model with sticky prices where countries engage in trade wars. In the presence of monopoly pricing markups, we show that the final level of tariffs and welfare losses from trade wars critically depend on the design of monetary policy. If central banks adopt a fixed nominal exchange rate or even better target the CPI inflation rate, trade wars are much less intense than those under PPI inflation targeting. We further show that an optimally delegated monetary rule that internalizes the formation of non-cooperative trade policy can actually completely eliminate a trade war, and even act to partly offset the welfare cost of monopoly markups.

*Devereux thanks SSHRC for research funding. Auray and Eyquem acknowledge the financial support of Projets Generique ANR 2015, Grant Number ANR-15-CE33-0001-01. Finally we acknowledge the financial support of the Europlace Institute of Finance.

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1 Introduction

The world has recently experienced a sharp breakdown in trade agreements and many countries have shifted towards non-cooperative trade policies. Can central banks do anything about it? We investigate this question in a two-country open-economy model with sticky prices and trade policy. Non-cooperative trade policies seek to exploit the so-called terms-of-trade externality (see Corsetti and Pesenti (2001), Benigno and Benigno (2003) or Ferrero (2020) and references therein for a recent survey). As in Johnson (1953)'s classic paper, each country faces a unilateral incentive to improve its terms of trade and induce welfare gains provided the other country does not respond. A trade war is then a symmetric equilibrium in which both countries raise tariffs to improve terms-of-trade – without actually succeeding to do it because of the symmetric moves – leaving the global economy worse-off compared to a free trade equilibrium.

If prices are sticky, central banks and the monetary policy they conduct interact with the above equilibrium in meaningful ways, as they endogenously shape the magnitude of the incentive to apply tariffs. Under free trade, when prices are sticky in domestic currencies (a case also known as producer currency pricing), Clarida, Gali, and Gertler (2002) and Engel (2011) among others have shown that an optimal monetary policy should target the inflation rate of the domestic goods (or the producer price index). We show that this result does not hold when there is a breakdown in cooperative trade policies. The reason is that central banks can curb the incentive to apply tariffs by targeting an inflation rate that incorporates changes in terms-of-trade, thereby partly offsetting attempts at terms-of-trade manipulation.

Our first result shows that CPI inflation strictly welfare dominates PPI targeting because it leads to lower tariffs as the outcome of a trade war. In a steady state equilibrium with either monetary rule, both PPI and CPI inflation will be stabilized around the target (which we assume is zero for convenience), but tariffs will be lower under the second rule. The logic behind this result is spelled out in a simple analytical version of our model in section 3 below. In the presence of monopoly pricing distortions, an optimal tariff has to balance the welfare gains from improving the terms of trade against the welfare costs from the worsening of the monopoly distortions. With sticky prices, the monetary rule becomes important for the impact of tariffs through the second channel, and CPI targeting leads the tariff authority to place more weight on these welfare costs, thus leading to a lower desired tariff. Since in a symmetric trade war equilibrium, each country's tariffs offset the others, welfare is higher under CPI targeting, because average tariffs are lower.

But in fact, one can do better than that. We go on to show that neither exact rule (PPI or CPI targeting) is optimal from the perspective of a cooperative authority that would design a rule which internalizes the actions of national trade authorities and the subsequent impact of the monetary rules on tariff choices. We think of this as a situation where the monetary rule is delegated to independent central banks, but the form of the rule is designed *ex ante*, taking into account the nature of trade policy and the implementation of monetary policy. In this case, we show that an optimal cooperatively designed rule will place a high weight on stabilizing a function of the tariff-adjusted terms of trade. This rule acts so as to fully offset the incentive to impose tariffs, and so in fact eliminates the trade war completely. The optimal rule actually leads to small *negative* tariffs, which have the effect of undoing part of the pre-existing monopoly distortions in production. Hence, remarkably, in the presence of endogenous tariff setting, a particular monetary policy rule can not only end the trade war, but also partly alleviate the underlying production distortion in each economy, and in so doing actually dominates a free trade outcome in welfare terms.

Finally, we show that the optimally cooperatively designed monetary policy is very close to a non-cooperative outcome where each individual country designs its monetary rule and delegates it to its own central bank. In this case also, the optimal rule leads to an endogenously elimination of the trade war, and offsets part of the monopoly distortion. Hence, we may conclude that a purely self-oriented policy of optimal monetary design and delegation can achieve major welfare gains in an environment of endogenous non-cooperative trade policy and monopoly distortions in production.

2 Literature

Our paper builds on a long tradition of macroeconomic models dealing with monetary policy in open economies. Using a two-country model with monopolistic competition, Corsetti and Pesenti (2001) show how national welfare may depend on a terms-of-trade externality. There are many subsequent papers analyzing optimal monetary policy in different open-economy frameworks, among them Benigno and Benigno (2003), Galì and Monacelli (2005), Faia and Monacelli (2008), de Paoli (2009), Bhattarai and Egorov (2016), Groll and Monacelli (2020), Fujiwara and Wang (2017), or more recently Egorov and Mukhin (2023). Most if not all of the above contributions highlight the importance of the terms-of-trade externality for the design and effects of monetary policy in open economies.

Our paper also relates to papers analyzing the interplay between trade and monetary policies. Bergin and Corsetti (2020) consider tariffs as policy instruments in addition to monetary policy, but their focus is rather on the implications of monetary policy on the building of comparative advantages. Jeanne (2021) investigates the interaction between 'currency wars' and 'trade wars' in an analytical framework of a continuum of small open economies with downward nominal wage rigidity and, in some cases, a global liquidity trap, and explores the benefits of international cooperation. Bergin and Corsetti (2023) develop a multi-country DSGE model with trade in intermediate goods and firms entry. They look at the optimal response of monetary policy to exogenous tariff shocks, which they find to be expansionary given the deflationary effects of tariff hikes.

The specificity of our paper is its focus on the design of monetary policy in an environment with non-cooperative trade policy, and shows the welfare benefits of CPI inflation targeting. Under free trade, when prices are sticky in domestic currencies (a case also known as producer currency pricing), Clarida, Gali, and Gertler (2002) and Engel (2011) among others have shown that an optimal monetary policy should target the inflation rate of the domestic goods (or the producer price index). We show that this result does not hold when there is a breakdown in cooperative trade policies. The reason is that central banks can curb the incentive to apply tariffs by targeting an inflation rate that incorporates changes in terms-of-trade, thereby partly offsetting attempts at terms-of-trade manipulation.

Another related literature concerns the design and delegation of monetary policy rules. Rogoff's Rogoff (1985) seminal paper first highlighted the implications for optimal monetary policy when policy is made in the presence of other distortions aside from price stickiness. Rogoff showed that a second-best optimal policy should place an excessive weight on inflation deviations from target relative to the socially optimal weight. A large follow up literature explored issues related to the design of optimal monetary policy rules. Walsh (1995) and Svensson (1997) placed the question of monetary policy design in the form of principal agent relationships between society and a central bank, and compared alternative forms of rules that took into account the incentives of the central bank in implementing policy. Our paper differs somewhat in that we show how an optimal monetary rule may need to take account of the incentive structure of trade policy makers.[1]

Our paper finally relates to a literature showing the potential benefits of targeting a different price index than the PPI, and of incorporating changes in the real exchange rate. This has been show to be relevant for certain values of the trade elasticity (de Paoli (2009)), for certain configurations of global value chains (Huang and Liu (2005), Wei and Xie (2020)) or when exchange rate pass-through is incomplete (Monacelli (2005)). However, in comparison to the above contributions, the motive for choosing a different inflation target is original and stems from the commitment of the central bank to offset terms-of-trade manipulations from tariff setters, implying lower tariffs and thus large steady-state welfare gains.

3 An Example Model

The complete model is described in Section 4. In this section we start with a simplified small open economy version of the model. This helps to build intuition regarding the link between monetary policy rules and the optimal tariff choice. The details of this model are set out fully in Appendix A.[2] In the small economy, trade is balanced every period, and there is an exogenous Foreign demand curve for the Home export good. The Home country government sets a tariff to maximize Home utility. Trade policy is made under discretion. Monetary policy follows a 'Taylor-type' rule, but allows for different inflation target indices. The key focus is the comparison of optimal tariffs across the different forms of the monetary rule.

[1] In this, the context is similar to Davig and Gürkaynak (2015) who show how an optimal monetary rule should be guided by the presence of other policymakers with different instruments and objectives.

[2] The model is developed more fully in Auray, Devereux, and Eyquem (2024), to which we refer readers.

3.1 Equilibrium Conditions

Households. Preferences over consumption and hours are given by:

$$\mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \left\{ u\left(C_{ht}, C_{ft}\right) - \ell\left(H_t\right) \right\}, \tag{1}$$

where, $\beta < 1$, C_{ht} ($C_{f,t}$) represents consumption of the Home (Foreign) good, u satisfies the usual conditions of differentiability and quasi concavity, $\ell(.)$ is a function of hours worked, and satisfies $\ell'(.) > 0$, and $\ell''(.) > 0$.

The Home country budget constraint is:

$$B_t + P_{ht}C_{ht} + (1+\tau_t) S_t P^*_{ft} C_{ft} = R_{t-1}B_{t-1} + W_t H_t + \Pi_t + TR_t. \tag{2}$$

Here P_{ht} (P^*_{ft}) represents the Home (Foreign) goods price in Home(Foreign) currency, and S_t is the nominal exchange rate. B_t represents holdings of Home nominal bonds and R_t is the gross nominal interest rate paid on bonds.[3] Variable W_t and Π_t are the Home nominal wage and the profit from Home firms, respectively, while τ_t is an import tariff. Finally, TR_t is a lump-sum transfer from the Home government.

Firms. A continuum of Home firms produce differentiated goods. The aggregate good is a composite of these differentiated goods, where the elasticity of substitution between goods $\epsilon > 1$. Output of firm i is: $Y_t(i) = AH_t(i)$ where A is a measure of aggregate productivity. Firm i chooses a price to maximize the present value of its expected profits subject to the demand function for individual goods $Y_t(i) = \left(P_{ht}(i)/P_{ht}\right)^{-\epsilon} Y_t$. Assuming symmetry among individual good producers, profit maximization produces the following Phillips curve:

$$\mathbb{E}_t\left\{\Omega_{t,t+1}\right\} = \mathcal{W}_t A_t^{-1} = \mathbb{E}_t \left\{ \theta + \phi\epsilon^{-1}\left(\pi_{ht}\left(\pi_{ht}-1\right) - \beta\pi_{ht+1}\left(\pi_{ht+1}-1\right)\right)\right\}, \tag{3}$$

where $\mathcal{W}_t = W_t/P_{ht}$ is the real wage and $\theta = (1+s)(\epsilon-1)/\epsilon \leq 1$ is a subsidy-adjusted measure of monopolistic distortions – the inverse of the subsidy-adjusted markup, where s is a revenue subsidy.[4] If an optimal subsidy $s = 1/(\epsilon-1)$ is in place, then $\theta = 1$ and the markup is zero. If current and future inflation is zero and the optimal subsidy is in place, then $\mathbb{E}_t\left\{\Omega_{t,t+1}\right\} = 1$ and $\mathcal{W}_t = A_t$. $\mathbb{E}_t\left\{\Omega_{t,t+1}\right\}$ measures the overall distortion bearing on the real wage. As we see below, the presence of a distorted steady state is a critical element in linking the stance of monetary policy to the choice of optimal tariffs.

In the subsequent analysis, we will assume that it is infeasible for the fiscal authority to impose an optimal subsidy, so that the there exists a structural distortion in the economy due to monopoly pricing which leads output to fall below the fully efficient level of output. We see this as a completely reasonable assumption. First, we note that a large empirical literature has

[3]We introduce nominal bonds to rationalize an interest rate rule for monetary policy. In the simple model, all bonds are issued by home government and held only by domestic agents, so that the economy satisfies balanced trade.

[4]Here we simplify by assuming the firm's discount factor for the expected future inflation cost is constant at β. This makes no difference to the example model.

persuasively established the fact of markups in almost all countries.[5] More generally, we might argue that political constraints make it infeasible for the fiscal authority to subsidize monopoly firms, or alternatively that informational asymmetries between firms and the government (not modeled here) prevent the use of targeted subsidies for firms with market power.

Government and Foreign sector. The Home government earns revenue from tariffs, and in some cases may subsidize firms. It makes transfers TR_t to households and issues bonds. The government budget constraint is written as:

$$B_t + TR_t = R_{t-1}B_t + \tau_t S_t P^*_{ft} C_{ft} - sP_{ht}Y_t, \tag{4}$$

where the last expression on the right-hand side represents total subsidies paid to firms.[6]

We make the simple assumption that the small open economy faces the following Foreign demand for its exported goods:

$$C^*_{ht} = \Lambda \mathcal{S}_t^{\eta}, \tag{5}$$

where $\mathcal{S}_t = S_t P^*_{ft}/P_{ht}$ denotes the terms of trade (relative price of the Foreign good), Λ is a constant and η is the elasticity of Foreign demand.

Monetary policy. We assume that monetary policy follows a simple-type Taylor rule, although the target price index may differ across rules:

$$R_t = \beta^{-1}\left\{\pi_t^{tar}\right\}^{\mu_\pi}. \tag{6}$$

The target inflation index may vary between a producer price inflation index, in which case $\pi_t^{\text{tar}} = \pi_{h,t} = \frac{P_{h,t}}{P_{h,t-1}}$, and a consumer price inflation index, written as $\pi_t^{\text{tar}} = \pi_{\text{cpi},t} = \frac{\mathcal{P}_t}{\mathcal{P}_{t-1}}$. Define $\mathcal{P}((1+\tau_t)\mathcal{S}_t) \equiv \mathcal{P}(1,(1+\tau_t)\mathcal{S}_t)$. Note that since the CPI is homogeneous of degree one we may write:

$$\pi_{\text{cpi},t} = \pi_{h,t}\frac{\mathcal{P}((1+\tau_t)\mathcal{S}_t)}{\mathcal{P}((1+\tau_{t-1})\mathcal{S}_{t-1})} \tag{7}$$

Therefore, the goal of CPI stabilization can be thought of as amalgam of PPI stabilization, and the stabilization of a function of the change in the tariff-adjusted terms of trade.

The main goal of the paper is to show how different rules lead to substantially different outcomes for a trade war between countries. It is well known from the results of Clarida, Gali, and Gertler (2002) and Engel (2011) among others that in the basic New Keynesian model, when prices are sticky in domestic currency, an optimal monetary policy should target the inflation rate of the Home good (or the producer price index). In this paper, we show that in a trade war, due to the interaction between trade policy and monetary policy, it may be preferable to employ a rule targeting the overall consumer price index. In either case, the stance of monetary policy is measured by the reaction coefficient of the Taylor rule μ_π. We take μ_π as given, and show below

[5]See De Loecker, Eeckhout, and Unger (2020) for an empirical characterization of markups in the U.S., and De Loecker and Eeckhout (2018) for a global perspective.

[6]Without loss of generality, in equilibrium, we will assume bonds are in zero net supply.

how the choice of an inflation target affects the equilibrium degree of protection when the Home tariff is chosen optimally by the Home authority under discretion.

Equilibrium. Conditional on the following goods market clearing condition:

$$A_t H_t \Phi_t = C_{ht} + C^*_{ht},$$

where $\Phi_t = 1 - \frac{\phi}{2}(\pi_{ht} - 1)^2$, and assuming balanced trade every period, the full equilibrium reduces to:

$$\begin{aligned}
\text{Balanced trade} &: \Lambda \mathcal{S}_t^{\eta} = \mathcal{S}_t C_{ft}, && (8)\\
\text{Market clearing} &: A_t H_t \Phi_t = C_{ht} + \Lambda \mathcal{S}_t^{\eta}, && (9)\\
\text{Labor market} &: \ell'(H_t) = A_t u_{c_{ht}} \mathbb{E}_t \{\Omega_{t,t+1}\}, && (10)\\
\text{Optimal spending} &: u_{c_{ht}}(1+\tau_t)\mathcal{S}_t = u_{c_{ft}}, && (11)\\
\text{Inflation: PPI target} &: \mathbb{E}_t \left\{ \frac{\pi_{ht}^{\mu_\pi}}{\pi_{ht+1}} \frac{u_{c_{ht+1}}}{u_{c_{ht}}} \right\} = 1. && (12)\\
\text{Inflation: CPI target} &: \mathbb{E}_t \left\{ \frac{\pi_{ht}^{\mu_\pi}}{\pi_{ht+1}} \left(\frac{\mathcal{P}(1,(1+\tau_t)\mathcal{S}_t)}{\mathcal{P}(1,(1+\tau_{t-1})\mathcal{S}_{t-1})} \right)^{\mu_\pi} \frac{u_{c_{ht+1}}}{u_{c_{ht}}} \right\} = 1. && (13)
\end{aligned}$$

The last equation stems from combining the Euler equation with the monetary policy rule. In the next paragraphs, we assume that the Home government chooses tariffs to maximize the current-period argument of Equation (1) subject to Equations (8), (9), (10) and either (12) for PPI inflation targeting or (13), for CPI inflation targeting. Equation (11) is ignored since it determines the tariff rate given the equilibrium of the real economy. We assume that trade policy is made under discretion, whereby the government takes its successors decisions as given. While the economy features balanced trade, the trade authority must still take account of its choice of tariffs on the next period's problem given the future period terms in (12) and (13). We note also that the tariff adjusted terms of trade expressions in condition (13) can be replaced using (11) since:

$$(1+\tau_t)\mathcal{S}_t = \frac{u_{ch,t}}{u_{cf,t}} \qquad (14)$$

Appendix A gives the details and proofs of the following results that focus on steady-state outcomes.

But before exploring the implications of alternative monetary rules for equilibrium tariff rates, we note that under free trade, where tariffs are zero, both PPI and CPI targeting achieve the same steady state outcomes. This follows because under either monetary rule, to be consistent with a constant steady state terms of trade, both the PPI inflation and CPI inflation should be zero. Hence, there is no welfare case for either targeting rule above the other.[7] More generally, as we noted above, the literature on New Keynesian open macro models has shown that in general PPI

[7]Note that $\frac{P_t}{P_{t-1}} = \pi_{h,t}\left(\frac{\mathcal{P}_t}{\mathcal{P}_{t-1}}\right)$, so given that the second term on the right-hand side is constant and equal to one in a steady state, PPI and CPI targeting lead to the same real and welfare outcomes with zero infllation.

targeting dominates CPI targeting under producer currency pricing, since it acts to stabilize the price in which there are costs of adjustment and allows for efficient relative price change through nominal exchange rate adjustment.

3.2 Results

3.2.1 Optimal Tariff under PPI Inflation Targeting

We first derive the optimal tariff when the monetary authority follows a PPI inflation targeting monetary rule so that $\pi^{target} = \pi_{h,t}$. Hence, the optimal tariff choice must take account of (6). Since tariff policy is chosen without commitment, the policymaker chooses an optimal tariff taking as given all future variables. Then, from the monetary rule (6), tariff policy will take account of its effect on current home goods consumption and current inflation, given future consumption and inflation. But with sticky prices, this has implications for employment and output through the labor market equilibrium condition (10). We show the following result.

Result 1. *Under a PPI inflation targeting rule, ($\phi > 0$, $\pi^{target} = \pi_{h,t}$) the steady-state equilibrium inflation rate is zero, $\pi_h = 1$, and the optimal tariff is given by:*

$$1 + \tau^{ppi} = \frac{\eta}{\eta - 1} \frac{1 - \theta\Delta_1}{1 - \Delta_1} \leq \frac{\eta}{\eta - 1}, \tag{15}$$

where $\Delta_1 = \frac{A^2 u_{c_{hh}}}{\ell''(H)} \left(\theta + \frac{\phi}{\mu_\pi \epsilon}\right) < 0$.

a) When $\theta = 1$ (no monopoly distortions), the tariff rate equals $\frac{1}{\eta-1}$, the monopoly tariff formula.

b) When $\theta < 1$ (monopoly distortions), the tariff rate is lower than $\frac{1}{\eta-1}$, and is decreasing (increasing) in ϕ (μ_π).[8]

Proof. See Appendix B.

We can explain Result 1 as follows. First, when $\theta = 1$, Result 1 indicates that $\tau = \frac{1}{\eta-1}$ implying that the tariff is set according to the classic monopoly tariff formula, and is focused solely on exploiting the market power of Home firms over Foreign demand. But when $\theta < 1$, the optimal tariff is less than $\frac{1}{\eta-1}$. Intuitively, Home output is inefficiently low due to the monopoly distortion, and a tariff would exacerbate this distortion by increasing the consumption of the Home good and reducing output, due to the income effect on labor supply. Hence, the policymaker chooses to set a lower tariff than the pure monopoly rate.

When $\theta = 1$, the tariff is also independent of the degree of price stickiness. But in the general case where $\theta < 1$, Result 1 indicates that the optimal tariff rate depends on the degree of price stickiness. The purely flexible price case can be obtained from Result 1 when $\phi = 0$. But as ϕ rises, the optimal tariff falls below the flexible price case.

The logic is as follows. As before, the tariff will shift Home consumption away from Foreign imports towards Home goods. A rise in C_{ht}, given C_{ht+1}, reduces the natural interest rate

[8] This result combines Results 1 and Results 2 from Auray, Devereux, and Eyquem (2024).

$u_{c_{ht}}/u_{c_{ht+1}}$, which through the policy rule (6), pushes down inflation. When prices are sticky, a fall in inflation reduces current output through the Phillips curve (3). Given that output is already inefficiently low because $\theta < 1$, this further raises the welfare cost of the tariff and leads the policymaker to set an equilibrium tariff below the flexible price tariff. Since under discretion, the policymaker in each period behaves in the same way, taking future policy as given, it follows that with the monetary rule (6), the tariff rate under sticky prices must always fall below that with price flexibility.

We also see from Result 1 that the optimal tariff is increasing in the strength of the monetary policy rule μ_π. A tighter monetary policy rule reduces the (negative) impact of a tariff on inflation, and thus reduces the policymaker's perceived distortionary impacts of a tariff on output. In the limit, as μ_π rises arbitrarily high, the price level is fully stabilized and the tariff approaches its flexible price level.

3.2.2 Optimal Tariff under CPI Inflation Targeting

We now contrast the above results with those arising when monetary policy targets the CPI rate of inflation, $\pi_t^{\text{target}} = \pi_t$. Appendix B shows that the optimal tariff under CPI targeting is given by:

$$1 + \tau^{\text{cpi}} = \frac{\eta}{\eta - 1} \frac{1 - \theta\Delta_2}{1 - \Delta_2}(1 + \Delta_3) \leq 1 + \tau^{\text{ppi}} \leq \frac{\eta}{\eta - 1}, \tag{16}$$

where $\Delta_2 = \frac{A^2 u_{c_{hh}}}{\ell''(H)}\left(\theta + \frac{\phi}{\mu_\pi \epsilon}(1 + \mu_\pi \alpha(1-\beta))\right) < 0$, $\Delta_3 = \frac{\phi A^2 u_{c_h}}{\mathcal{S}\epsilon\ell''(H)}\alpha\frac{u_{c_{ff}}}{u_{c_f}}(1-\beta)\frac{(1-\theta)}{(1-\theta\Delta_1)} < 0$, and α is the share of Foreign goods in consumer spending.

Result 2. *The optimal tariff under CPI targeting is less than that under PPI targeting.*

Proof. Appendix B outlines the full proof. We see from (16) that if $\theta = 1$, the tariff rate is again equal to the full monopoly tariff. But for $\theta < 1$ the tariff falls below the monopoly tariff rate and also below the tariff under PPI inflation targeting (with sticky prices).

Why does CPI inflation targeting produce a lower optimal tariff rate? The intuition can be seen from the policy adjusted Euler equation (13). When the optimal tariff is determined under discretion, the policymaker takes account of a change in the tariff on current PPI inflation and the tariff-adjusted terms of trade. An increase in the tariff will tend to raise the policy-adjusted terms of trade $(1 + \tau_t)\mathcal{S}_t$. Given that (13) is always binding, this will tend to push down the PPI inflation rate $\pi_{h,t}$, further exacerbating the domestic output distortion. This leads the tariff setter to reduce the size of the optimal tariff further than in the case with only PPI targeting.

This analysis only covers the case of a small economy, taking Foreign demand (and any Foreign tariff) as given. In the extended model below, we show that when both countries choose optimal tariffs, then a policy of CPI targeting in both countries can reduce the depth of a trade war and increase welfare in all countries.

4 The Full Model

The extended model follows closely Auray, Devereux, and Eyquem (2024) and is described in details in the Appendix C. There are two countries denoted Home and Foreign. Households supply labor, consume goods from both countries with an elasticity of substitution λ and trade bonds. The world is populated with a unit mass of agents and Home has share n of these, with Foreign share $1-n$. We assume that firms supply imperfectly substitutable varieties of local goods, set prices in the currency of producers (PCP), and adjust prices constrained by Rotemberg price adjustment costs.

Using appropriate substitutions, the above equations can be reduced to a system of two Phillips Curves (Equations (C.102) and (C.103) in Appendix C), two good market clearing conditions (Equations (C.104) and (C.105)), two Euler equations (Equations (C.107)-(C.108)), and Equations (C.106), (C.109) and (C.110) that describe the external equilibrium – the terms of trade (Equation (C.109)) and two net foreign asset positions (Equation (C.106 and Equation (C.110)). Conditional on a given set of tariffs $\{\tau_t, \tau_t^*\}$ and monetary policies $\{R_t, R_t^*\}$, these equations determine $\left\{\pi_{ht}, \pi_{ft}^*, C_t, C_t^*, Y_t, Y_t^*, b_t, b_t^*, \mathcal{S}_t\right\}$.

4.1 Economic Policy

There are two separate levers of policy in the model. Trade policy may be used to levy tariffs on imports, and monetary policy can be used to stabilize inflation with a flexible exchange rate between the two countries, or to stabilize inflation in one country and the nominal exchange rate in the other country, *i.e.* a fixed exchange rate regime. As argued above, given the ubiquity of markups in the real economy, we leave aside a third possible policy lever and disregard any potential subsidy aimed at correcting markup distortions. As explained in details in Auray, Devereux, and Eyquem (2024) this has important implications for optimal tariffs set in interaction with monetary policy, as optimal country-level tariffs balance the utility gains from improved terms of trade (achieved with higher tariffs) and the implied costs from lower output while output is already inefficiently low due to markup distortions.

4.1.1 Monetary Policy

With a flexible exchange rate, the model is closed by the two following monetary policy rules:

$$R_t = \beta^{-1} \left(\pi_t^{tar}\right)^{\mu_\pi}, \tag{17}$$

$$R_t^* = \beta^{-1} \left(\pi_t^{*tar}\right)^{\mu_\pi^*}, \tag{18}$$

where π_t^{tar} and π_t^{*tar} can be either the PPI inflation rate or the CPI inflation rate.

If the Foreign country has a nominal exchange rate target, it cedes control over its domestic inflation rate, leaving the Home country to run an independent monetary policy. In this case, the

Foreign monetary policy rule is replaced by the following condition:

$$\pi_{ht} = \pi^*_{ft} \frac{\mathcal{S}_{t-1}}{\mathcal{S}_t}. \tag{19}$$

Because the nominal exchange rate is fixed, the terms of trade can change only due to changes in nominal price levels, implying that the terms of trade follows the dynamics of relative inflation rates.

4.1.2 Trade Policy

With a flexible exchange rate, a discretionary Nash equilibrium of trade policy implies that the Home government solves:

$$\max_{\{C_t, C^*_t, Y_t, Y^*_t, b_t, b^*_t, \mathcal{S}_t, \pi_{ht}, \pi^*_{ft}, \tau_t\}} V(b_{t-1}) = U(C_t, H_t) + \beta \mathbb{E}_t \{V(b_t)\}, \tag{20}$$

subject to Equations (C.102)-(C.110) and monetary policy rules (17)-(18), while the Foreign government solves:

$$\max_{\{C_t, C^*_t, Y_t, Y^*_t, b_t, b^*_t, \mathcal{S}_t, \pi_{ht}, \pi^*_{ft}, \tau^*_t\}} V^*(b^*_{t-1}) = U(C^*_t, H^*_t) + \beta \mathbb{E}_t \{V^*(b^*_t)\}, \tag{21}$$

subject to the same constraints. The resulting first-order conditions determine optimal discretionary Nash tariffs and select the equilibrium.

With a fixed exchange rate, the rule (19) adds an additional state variable to the model – in addition to net foreign assets – in the form of the lagged terms of trade. Since the nominal exchange rate is pegged, the terms of trade can adjust *only* via differences in inflation rates. Under a fixed exchange rate regime, the problem can be stated as:

$$\max_{\{C_t, C^*_t, Y_t, Y^*_t, b_t, b^*_t, \mathcal{S}_t, \pi_{ht}, \pi^*_{ft}, \tau_t\}} V(\mathcal{S}_{t-1}, b_{t-1}) = U(C_t, H_t) + \beta \mathbb{E}_t \{V(\mathcal{S}_t, b_t)\}, \tag{22}$$

subject to (C.102)-(C.110) and monetary policy rules (17)-(19) for the Home policymaker and similarly:

$$\max_{\{C_t, C^*_t, Y_t, Y^*_t, b_t, b^*_t, \mathcal{S}_t, \pi_{ht}, \pi^*_{ft}, \tau^*_t\}} V^*(\mathcal{S}_{t-1}, b_{t-1}) = U(C^*_t, H^*_t) + \beta \mathbb{E}_t \{V^*(\mathcal{S}_t, b_t)\}, \tag{23}$$

for the Foreign. Assuming $\mathcal{S}_{-1} = 1$ on top of $b_{-1} = 0$ selects only symmetric equilibria in the Nash tariff game.

5 Trade Wars under Alternative Monetary Policy Regimes

Let us first discuss the numerical values assigned to key parameters before discussing our main results.

5.1 Parameter Values

The model is parametrized to an annual frequency. The discount factor of households is $\beta = 0.96$, consistent with a real interest rate of 4% *per annum*. Both countries are of equal size in the baseline calibration so that $n = 0.5$. Further, we assume a home bias parameter $x = 0.4$ which implies $\gamma = \gamma_x = (1 - \gamma^*) = (1 - \gamma_x^*) = 0.7$. Under free trade (zero tariffs), this number is associated with a 60% total trade openness ratio. We consider a baseline value of $\sigma = 1$, implying a log utility for consumption. The Frisch elasticity is $\psi^{-1} = 2.5$ following Chetty et al. (2011) and we normalize $\chi = 1$. The elasticity of substitution between varieties is $\epsilon = 6$, consistent with a 20% steady-state price-cost markup. The (annual) Rotemberg parameter is $\phi = 40$ and the baseline monetary policy rule inflation parameter is $\mu_\pi = 1.5$, in line with empirical estimates. Following Bergin and Corsetti (2023), we consider the share of intermediate goods in production to be $\alpha = 0.4$. Last, the trade elasticity is $\lambda = 5$. This is on the high end of the range estimated by Feenstra et al. (2018), but is more appropriate for the evaluation of trade policy. The bond adjustment cost parameter suggested by Ghironi and Melitz (2005) is 0.0025 in a quarterly set-up which, in our annual set-up, implies $\nu = 0.01$. Finally, the baseline results are derived under the assumption that trade is balanced in the steady state, *i.e.* $b = b^* = 0$.

5.2 Markup distortions

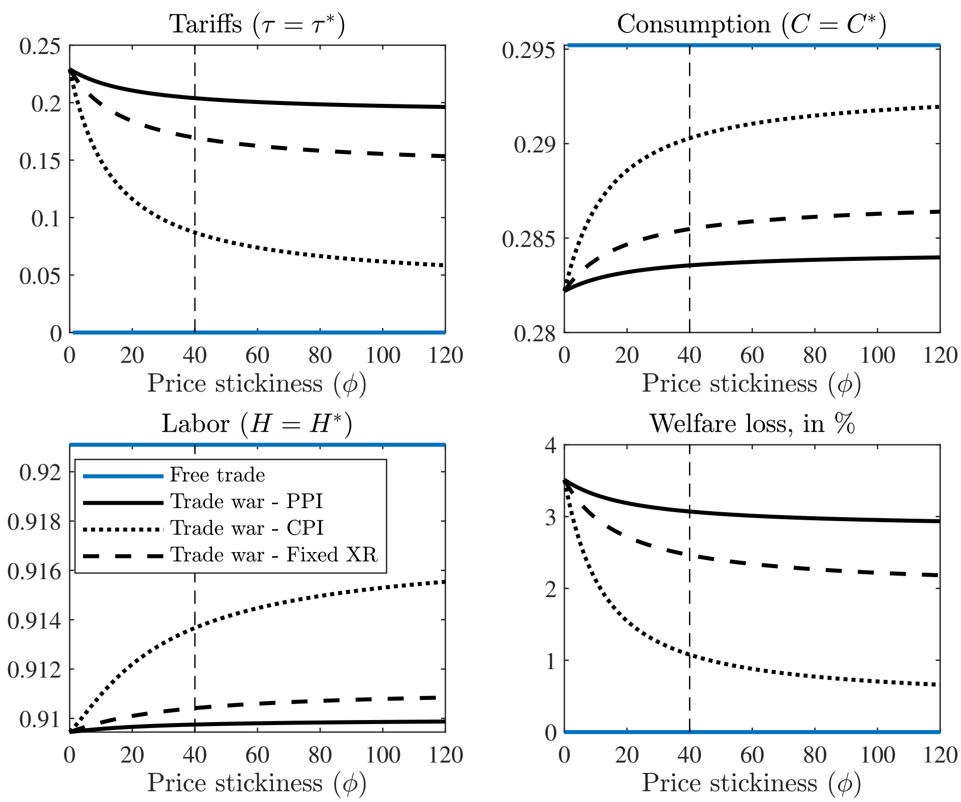
Most parameter values above are standard and, if they affect the results quantitatively they do not change the qualitative results. As we have noted above, an important maintained hypothesis is that the there are monopoly markups in each country, and markup distortions are not offset by an appropriate subsidy to firms' sales. As shown in the example model above, if steady-state markups are offset, monetary policy has no effects on the optimal tariffs resulting from the Nash game, which are simply equal to the classic monopoly formula in this case, *i.e.* $1 + \tau = \frac{\lambda}{\lambda - 1}$. When markups are not corrected, optimal tariffs under flexible prices are lower, because policy-makers do not want to lower output too much since output is already low because of markup distortions. What we show below is that sticky prices make tariffs lower than under flexible prices, but with a different magnitude depending on the type of monetary policy conducted by central banks.

5.3 Baseline Results

Figure 1 reports the steady-state levels of tariffs, consumption and labor resulting from a trade war equilibrium as a function of the Rotemberg parameter ϕ for the three monetary policy set-ups described in the model section.

First, Figure 1 confirms that when prices are flexible, monetary policy does not interact with the choice of optimal tariffs since the three policy regimes deliver similar outcomes. Optimal tariffs are 22.5 percents, slightly below the classic monopoly tariff implied by the value of the trade elasticity (25 percents).

Figure 1: Trade Wars under Alternative Monetary Policies.



Note: Welfare losses denote the Hicksian consumption equivalent loss compared to the free trade equilibrium. The vertical line indicates the baseline value of $\phi = 40$.

Second, it confirms the result of Auray, Devereux, and Eyquem (2024) according to which tariffs – and welfare losses – fall with price stickiness, but generalizes it to CPI inflation targeting. Further, it shows that the equilibrium with a fixed exchange rate yields lower tariffs than PPI inflation targeting for any given value of price stickiness. In this case, changes in the nominal exchange rate are perfectly offset by the commitment of the Foreign central bank, which is internalized by tariff setters and results is lower tariffs. Tariffs can be as low as 15 percents with a fixed exchange rate when prices are very sticky, against 20 percents with a flexible exchange rate and a PPI inflation targeting rule.

Third, Figure 1 shows that CPI inflation targeting produces an even lower level of optimal tariffs. The reason is that the CPI inflation rate incorporates changes in the real exchange rate – or equivalently, changes in terms of trade. Hence, when tariff setters seek to manipulate terms of trade, they internalize the fact that central bank with largely offset them, which then deters them from actually doing it. As a result, tariffs are much lower than under the two alternative monetary policies for any value of price stickiness, and can be as low as 6 percents.

As one would guess, with tariffs ranging from 22.5 to 6 percents, the welfare losses with respect to the free-trade equilibrium vary massively: from 3.4 percents (flexible prices) to 3 percents (with very sticky prices) under PPI inflation targeting, 2.2 percents with a fixed exchange rate regime, and 'only' 0.67 percents under CPI inflation targeting. For the baseline calibrated value of $\phi = 40$, CPI inflation targeting reduces the intensity of the trade war so as to imply welfare losses that are only a third of those arising under PPI inflation targeting.

6 Welfare-maximizing Inflation Target under Trade Wars

From the above results, we learn that central banks adopting inflation targeting rules that incorporate changes in tariff-adjusted terms of trade can attenuate the severity and welfare losses of trade wars. But can central banks completely eliminate incentives for tariff setters to improve terms of trade in a trade war? To answer this question, we now consider general monetary policy rules of the form:

$$R_t = \beta^{-1} \left(\pi_{ht} \left(\frac{\mathcal{P}_t}{\mathcal{P}_{t-1}} \right)^{d_r} \right)^{\mu_\pi}, \tag{24}$$

$$R_t^* = \beta^{-1} \left(\pi_{ft}^* \left(\frac{\mathcal{P}_t^*}{\mathcal{P}_{t-1}^*} \right)^{d_r} \right)^{\mu_\pi}. \tag{25}$$

These general rules are symmetric in that both countries target the same inflation rate, and imbed two of the three previous cases: $d_r = 0$ implies PPI inflation targeting while $d_r = 1$ implies CPI inflation targeting.[9] But the rules are more general in the sense that they allow central banks to place a larger weight on changes in $\mathcal{P}_t$, the relative price of consumption goods in terms of

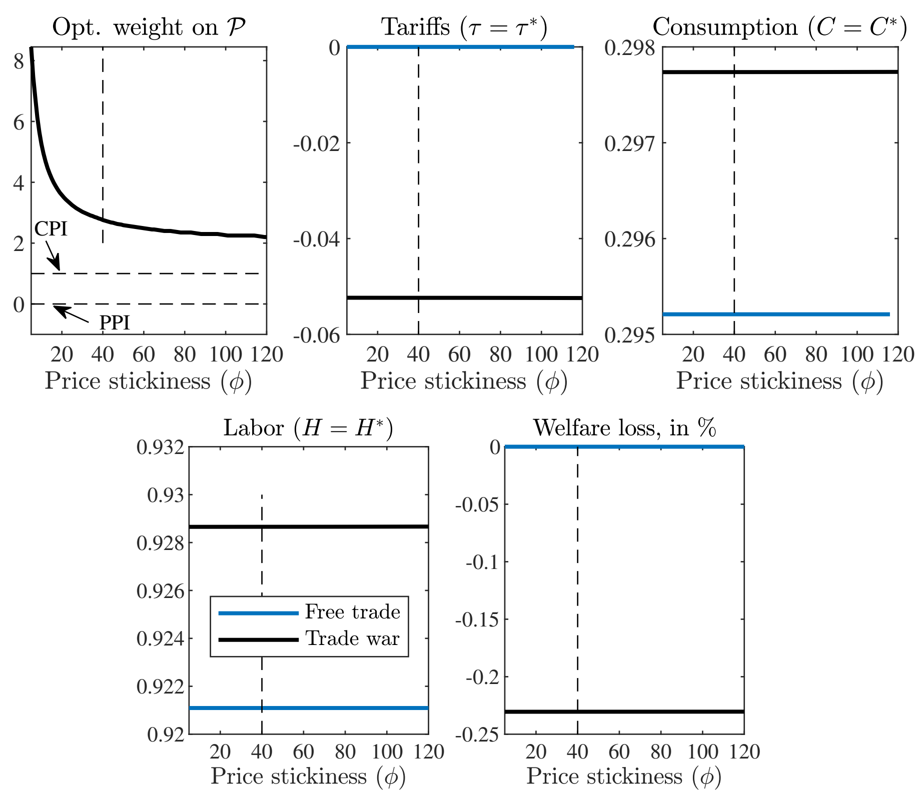
[9]Indeed, assuming $dr = 1$ and replacing the definition of $\mathcal{P}_t = \frac{P_t}{P_{ht}}$ in the Home rule yields $R_t = \beta^{-1} (\pi_t)^{\mu_\pi}$, where $\pi_t = \frac{P_t}{P_{t-1}}$ is the CPI inflation rate. Similar manipulations yield an equivalent outcome for the Foreign rule.

the domestic goods. As a matter of fact, looking at the definition of $\mathcal{P}_t$ shows that it embeds two important determinants: the domestic tariff rate and the terms of trade. A credible commitment to stabilize changes in $\mathcal{P}_t$ thus amounts to a credible commitment in stabilizing changes in tariffs and terms of trade.

How should we think about the setting of this type of monetary rule? We think of this as a case of cooperative monetary policy design, where *ex ante*, the designer chooses a weighting scheme on a monetary rule to be applied by the monetary authority, and then delegates the rule to each separate central bank, taking into account the manner in which trade policy is determined, and also how trade policy is affected by the form of the monetary rule being applied. The cooperative monetary policy design then chooses the form of the rule to maximize global *ex ante* welfare.

Can the stabilization of changes in terms of trade be large enough to fully avoid trade wars? Figure 2 below reports the welfare maximizing value of d_r for a range of price stickiness parameters ϕ and the corresponding equilibrium outcomes in terms of tariffs, consumption, labor and the welfare losses compared to the free trade equilibrium.

Figure 2: Non-cooperative trade policies under welfare-maximizing inflation targeting rules.



Note: Welfare losses denote the Hicksian consumption equivalent loss compared to the free trade equilibrium. Negative losses indicate welfare *gains*. The vertical line indicates the baseline value of $\phi = 40$. Horizontal lines in the top left panel represent the values of d_r implying CPI ($d_r = 1$) and PPI ($d_r = 0$) inflation targeting.

Figure 2 reports a stark result: inflation targeting monetary policies with an adequately chosen parameter d_r can fully prevent trade wars. As a matter of fact, they completely eliminate incentives for tariff setters to manipulate terms of trade, which leaves them with the only possible motive when setting tariffs: eliminate the (purely local) monopoly distortions.

Indeed, the implied level of tariffs is negative and corresponds to a subsidy, *i.e.* $\tau = \tau^* = -0.052$. This value in fact exactly matches the welfare-maximizing level of tariffs that tariff setters would choose cooperatively to offset monopoly distortions when sales subsidies are absent. With an optimally designed monetary rule, tariffs are thus used exclusively to offset monopoly distortions.

As such, welfare-maximizing inflation targeting rules can, in the context of non-cooperative trade policies, raise welfare by killing the terms-of-trade externality. Remarkably, as we show below, this delivers a welfare level greater than that under free trade with zero tariffs.[10] Note however that for this outcome, the weight placed on changes in the relative price of traded goods must be above the weight implied by CPI inflation targeting, and be larger (smaller) when prices are more flexible (sticky).

7 Asymmetric Inflation Targeting

Given the above results, a natural question arises regarding asymmetry in the inflation target: what happens when central banks target different inflation rates? What are the aggregate and national welfare implications and thus the national incentives to choose the inflation target? Last, what are the implications for the likely outcome?

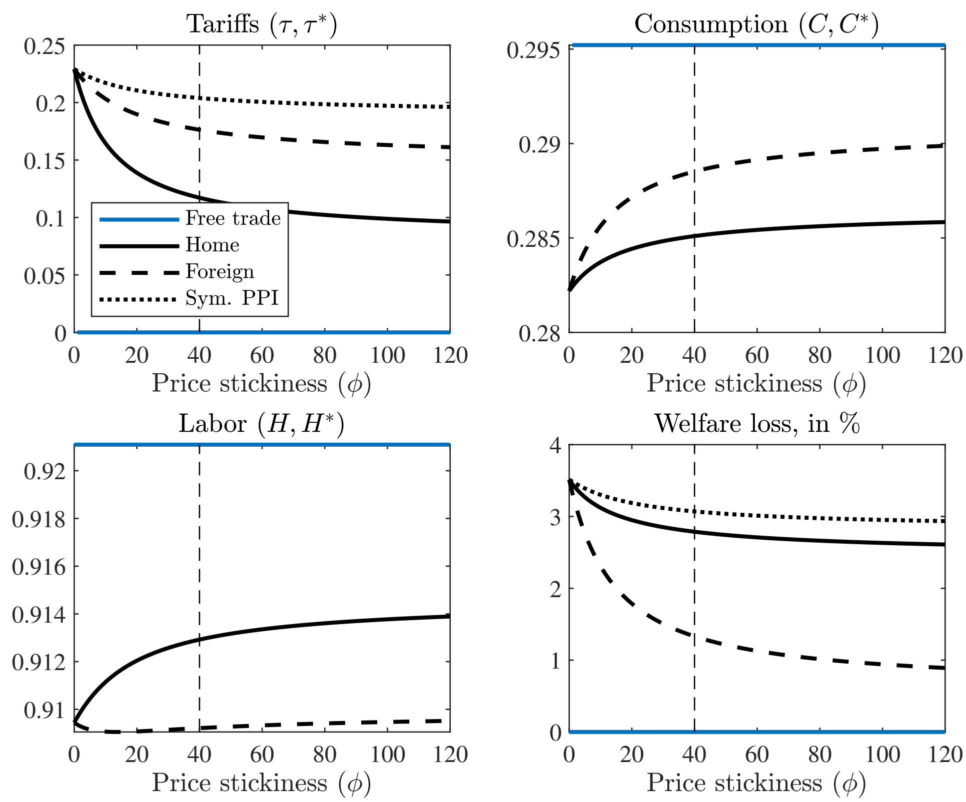
7.1 Home Targets CPI inflation and Foreign PPI inflation

We start our analysis by looking at the case where the Home central bank targets the CPI inflation rate while the Foreign central bank targets the PPI inflation rate in Figure 3.

Figure 3 shows that targeting the CPI inflation rate generates smaller tariffs for *both* Home and Foreign economies compared to symmetric PPI inflation targeting. Aggregate welfare losses are thus lower, but since CPI inflation targeting directly curbs incentives to set tariffs only in the Home economy, tariffs are much lower in the Home than the Foreign economy. However, this alleviates trade tensions for both countries, leading to lower overall tariffs and higher welfare than in the case of uniform PPI inflation targeting. This asymmetry in inflation targets and resulting tariffs also leads to an asymmetry in welfare losses: losses fall more for the Foreign economy – which sets a higher tariff than the Home economy, thus gaining a terms of trade advantage – and less for the Home economy. In any case, *given* that the Foreign country follows a PPI inflation target, targeting the CPI rate of inflation is welfare improving for the Home economy.

[10] Note however, that this does not attain the first-best outcome that would hold in the absence of markups and zero tariffs. This is because the tariff chosen in this case must distort the composition of consumption between Home and Foreign goods.

Figure 3: Trade wars with asymmetric inflation targeting.



Note: Home targets the CPI inflation rate and Foreign the PPI inflation rate. Welfare losses denote the Hicksian consumption equivalent loss compared to the free trade equilibrium. The vertical line indicates the baseline value of $\phi = 40$.

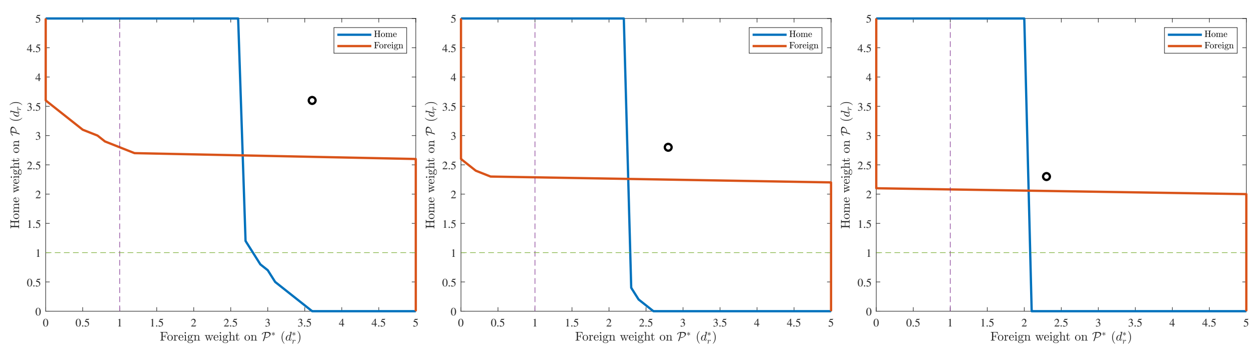
7.2 Optimal Non-cooperative Inflation Target under Trade War

The result of the last subsection describes the differential tariffs and welfare outcomes for each country when countries follow different inflation targeting strategies, but they do not directly inform us of the optimal choice of inflation target in a situation of non-cooperative strategic interaction. We thus investigate the optimal choice of inflation targets in a non-cooperative game where each country chooses its inflation-target weighting, taking the other's choice as given.

In Section 6, we considered symmetric rules and inflation targets. The implicit assumption was that authorities were cooperating in choosing their optimal inflation target. However, cooperation may be difficult to implement in practice, and we want to characterize Nash equilibria. In the policy game, both central banks commit to Taylor-type rules and choose their optimal inflation target (determined by d_r or d_r^*) given the optimal inflation target chosen by the other central bank. Note that the monetary policy set-up will be internalized by tariff setters, and that central bankers take this into account. We thus compute the reaction functions of the Home and Foreign central banks in this set-up and report them for different degrees of price stickiness in Figure 4 below.

Figure 4: Central Banks Reaction Functions.

(a) $\phi = 20$ (b) $\phi = 40$ (c) $\phi = 100$



Note: For a given d_r^* (d_r), reaction functions report the welfare-maximizing d_r (d_r^*) chosen by the Home (Foreign) central bank. The black dots represent the 'cooperative' welfare-maximizing inflation target (weight $d_r = d_r^*$) discussed in Section 6.

First, Figure 4 shows that the intuition derived from the last subsection continues to apply. Imagine the Foreign central bank targets PPI ($d_r^* = 0$). In this case we have already seen that, if the Home central bank targets the CPI, both countries as better off. So from the perspective of the Home country it is always optimal to adopt a target that stabilizes tariff-adjusted terms of trade, even if the Foreign households gain more from it. Figure 4 illustrates this and shows that the optimal weight d^r is large when the Foreign central bank adopts a low d_r^*. When the Foreign central bank increases its weight placed on $\mathcal{P}^*$ enough, the Home central bank does not need to target anything else than the PPI, and thus adopts $d_r = 0$. Hence, the Nash equilibrium stems from both countries adopting an inflation target featuring a moderate – but in any case larger-than-one – weight on tariff-adjusted terms of trade.

Second, Figure 4 shows that the optimal Nash weights are decreasing in price stickiness ϕ, which aligns perfectly with the results about the welfare-maximizing weight. When prices are more flexible, a larger weight should be placed on relative prices to stabilize changes in the tariff-adjusted relative price of traded goods.

Third, Figure 4 shows that the optimal Nash weights are always below the welfare-maximizing weight discussed in Section 6. Hence, we expect Nash equilibria to produce lower levels of welfare than cooperative equilibria. To confirm that, Table 1 reports the optimized inflation targets (characterized by d_r and d_r^*) and the steady-state allocations resulting from the Nash equilibrium for different degrees of price stickiness, and compares them to all the equilibria discussed so far, including the welfare-maximizing rules discussed in Section 6.

Table 1: Monetary Policy Design under Trade Wars.

	FB	FT	PPI	FXR	CPI	Coop.	Nash
				$\phi = 20$			
$d_r = d_r^*$	–	–	0.000	–	1.000	3.600	2.661
$\tau = \tau^*$	0.000	0.000	0.211	0.184	0.116	−0.054	−0.005
$C = C^*$	0.326	0.295	0.283	0.285	0.289	0.298	0.295
$L = L^*$	1.000	0.921	0.910	0.910	0.912	0.929	0.922
Utility	−1.407	−1.434	−1.467	−1.462	−1.450	−1.432	−1.434
Welfare loss (%)	0.000	2.659	5.763	5.306	4.165	2.435	2.620
				$\phi = 40$			
$d_r = d_r^*$	–	–	0.000	–	1.000	2.800	2.259
$\tau = \tau^*$	0.000	0.000	0.204	0.170	0.087	−0.055	−0.021
$C = C^*$	0.326	0.295	0.284	0.285	0.290	0.298	0.296
$L = L^*$	1.000	0.921	0.910	0.910	0.914	0.929	0.924
Utility	−1.407	−1.434	−1.466	−1.459	−1.445	−1.432	−1.433
Welfare loss (%)	0.000	2.659	5.648	5.058	3.709	2.436	2.520
				$\phi = 100$			
$d_r = d_r^*$	–	–	0.000	–	1.000	2.300	2.058
$\tau = \tau^*$	0.000	0.000	0.197	0.155	0.062	−0.054	−0.037
$C = C^*$	0.326	0.295	0.284	0.286	0.292	0.298	0.297
$L = L^*$	1.000	0.921	0.910	0.911	0.915	0.929	0.926
Utility	−1.407	−1.434	−1.464	−1.457	−1.441	−1.432	−1.432
Welfare loss (%)	0.000	2.659	5.533	4.818	3.346	2.435	2.455

'FB' denotes the first-best equilibrium ($\theta = 1$ and $\tau = \tau^* = 0$), 'FT' the free-trade equilibrium without a subsidy ($\theta < 1$ and $\tau = \tau^* = 0$), 'PPI' the case of PPI targeting ($d_r = d_r^* = 0$), 'FXR' the case of a fixed exchange rate, 'CPI' the case of CPI targeting ($d_r = d_r^* = 1$), 'Coop.' the case of a welfare-maximizing inflation target discussed in Section 6, and 'Nash' the non-cooperative design of targeting rules. All welfare losses are computed against the first-best equilibrium.

The first colum of Table 1 reports the first-best equilibrium in which the monopoly distortion is offset by a subsidy and tariffs are null. The second column considers the free-trade equilibrium with monopoly distortions, and shows that these generate lower output, consumption and 2.66 percent welfare loss. Columns 3-5 report the results already seen in the previous sections:

fixed exchange rate and CPI inflation targeting produce milder trade wars in comparison of PPI inflation targeting, and alleviate the corresponding welfare losses. CPI inflation targeting results in tariffs that are more than half those arising under PPI inflation targeting and cut welfare losses by 1.5-2 percentage points.

Column 6 and 7 of Table 1 compare equilibria resulting from Nash optimal weight to equilibria resulting from the cooperative choice of (welfare-maximizing) weights. It shows that optimal Nash weights stabilize tariff-adjusted terms of trade enough to bring non-cooperative tariffs below zero. They do not completely extinguish the terms of trade externality as in the case of welfare-maximizing cooperative weights, but sufficiently for the tariff-setters to focus more on correcting the monopoly distortions and less on improving their terms of trade. As a result, the allocations are quite close to the cooperative equilibrium, and generate small welfare losses from non-cooperation. Interestingly enough, Nash equilibria produce slightly negative tariffs and improve allocations compared to the free-trade equilibrium with monopoly distortions. Finally, the distance between Nash and cooperative equilibria and the welfare losses from non-cooperation shrink as prices become stickier. This happens because stickier prices make monetary policy a more powerful stabilization tool to extinguish the terms-of-trade externality.

From these results we conclude that, even without international cooperation, when the design and delegation of monetary rules is chosen independently by central banks, they may act to fully eliminate trade wars, and in fact welfare dominate a free trade equilibrium.

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A The Example Model

Here we set out the full details of the Example model discussed in Section () of the text.

Households. Preferences over consumption and hours are given by:

$$\mathbb{E}_t \sum_{j=0}^{\infty} \beta^j U\left(C_{ht+j}, C_{ft+j}, H_{t+j}\right), \tag{A.1}$$

where

$$U\left(C_{ht}, C_{ft}, H_t\right) = u\left(C_{ht}, C_{ft}\right) - \ell\left(H_t\right). \tag{A.2}$$

Here, $\beta < 1$ is the discount factor and u is continuous, twice differentiable, and satisfies $u_{c_{ii}} < 0$ and $u_{c_{ij}} \geq 0$, for $i = \{h, f\}$, and $i \neq j$. Consumption of the Home export good is C_{ht}, and consumption of the Foreign imported good is C_{ft}.[11] Function $\ell(.)$ is a continuous and twice differentiable function of hours worked, satisfying $\ell'(.) > 0$, and $\ell''(.) > 0$. The Home country budget constraint is:

$$B_t + P_{ht}C_{ht} + (1+\tau_t) S_t P^*_{ft} C_{ft} = R_{t-1}B_{t-1} + W_t H_t + \Pi_t + TR_t, \tag{A.3}$$

where P_{ht} (P^*_{ft}) is the Home (Foreign) goods price in Home (Foreign) currency, and B_t the stock of local nominal bonds. R_t is the nominal interest rate, paid on domestic one period nominal bonds maturing at time $t+1$. Variable S_t is the nominal exchange rate, τ_t is an import tariff imposed by the Home government, W_t is the Home nominal wage, Π_t represents the profits of Home firms and TR_t is a lump-sum transfer from the Home government. Optimal choices over consumption and hours lead to the following conditions:

$$\beta \mathbb{E}_t \left\{ \frac{R_t}{\pi_{ht+1}} \frac{u_{c_{ht+1}}}{u_{c_{ht}}} \right\} = 1, \tag{A.4}$$

$$u_{c_{ft}} = u_{c_{ht}} (1+\tau_t) \frac{S_t P^*_{ft}}{P_{ht}}, \tag{A.5}$$

$$\ell'(H_t) = u_{c_{ht}} \frac{W_t}{P_{ht}}, \tag{A.6}$$

where $\pi_{ht} = P_{ht}/P_{ht-1}$ is the PPI Home inflation rate.

Firms. Home firms produce differentiated goods. The aggregate good is a composite of these differentiated goods, where the elasticity of substitution between individual goods is $\epsilon > 1$. For now, assume that the firm's production depends only on labor. Output of firm i is:

$$Y_t(i) = A_t H_t(i), \tag{A.7}$$

[11] For simplicity we will assume that the cross derivative of the utility function is zero, so that $u_{c_{hf}} = 0$. This makes no difference to the results but simplifies the exposition of the example model. In the more general model of Section 4 we assume a more conventional CES preference representation.

where A_t is a measure of aggregate productivity. The profits of firm i are then:

$$\Pi_t(i) = \left[(1+s)P_{ht}(i) - \frac{W_t}{A_t} - \frac{\phi}{2}\left(\frac{P_{ht}(i)}{P_{ht-1}(i)}\right)^2 P_{ht}(i)\right] Y_t(i), \tag{A.8}$$

where $P_{ht}(i)$ is the price set by firm i and s is a sales subsidy. Firm i chooses its price to maximize the present value of its expected profits subject to the demand function for individual goods $Y_t(i) = (P_{ht}(i)/P_{ht})^{-\epsilon} Y_t$:

$$\mathbb{E}_t \sum_{j=0}^{\infty} \omega_{t+j} \Pi_{t+j}(i), \tag{A.9}$$

where ω_t is the firm's nominal stochastic discount factor, and ϕ captures the importance of price adjustment costs. Assuming symmetry among individual good producers, profit maximization produces the following Phillips curve:

$$\mathbb{E}_t \{\Omega_{t,t+1}\} = \mathcal{W}_t A_t^{-1} = \mathbb{E}_t \left\{\theta + \phi\epsilon^{-1}\left(\pi_{ht}(\pi_{ht}-1) - \beta\pi_{ht+1}(\pi_{ht+1}-1)\right)\right\}, \tag{A.10}$$

where $\mathcal{W}_t = W_t/P_{ht}$ is the real wage and $\theta = (1+s)(\epsilon-1)/\epsilon \leq 1$ is a subsidy-adjusted measure of monopolistic distortions – the inverse of the subsidy-adjusted markup.[12] If an optimal subsidy $s = 1/(\epsilon-1)$ is in place, then $\theta = 1$ and the markup is zero. Equilibrium wages are not distorted. If current and future inflation is zero and the optimal subsidy is in place, then $\mathbb{E}_t\{\Omega_{t,t+1}\} = 1$ and $\mathcal{W}_t = A_t$. In the absence of a subsidy, $\theta < 1$ which implies a positive markup distortion, and $\mathcal{W}_t < A_t$. Then $\mathbb{E}_t\{\Omega_{t,t+1}\}$ measures the overall distortion bearing on the real wage, whether stemming from nominal rigidities under sticky prices ($\phi > 0$) – in which case it depends on the inverse of the slope of the Phillips curve $\phi\epsilon^{-1}$ – and/or from monopolistic distortion through the inverse of the subsidy-adjusted markup $\theta < 1$ – in which case it depends on the elasticity of substitution between varieties ϵ.

Government and Foreign sector. As described in the text, the government budget constraint is written as:

$$B_t + TR_t = R_t B_{t-1} \tau_t S_t P^*_{ft} C_{ft} - sP_{ht}Y_t, \tag{A.11}$$

where the last expression on the right-hand side represents total subsidies paid to firms.

The Foreign demand for the Home good depends on the terms of trade $\mathcal{S}_t = S_t P^*_{ft}/P_{ht}$, and is described as:

$$C^*_{ht} = \Lambda \mathcal{S}_t^{\eta}, \tag{A.12}$$

where Λ is a constant and η the elasticity of Foreign demand.

Monetary policy.

The monetary rule is defined as:

$$R_t = \beta^{-1}\left\{\pi_t^{tar}\right\}^{\mu_\pi}. \tag{A.13}$$

[12] Here we simplify by assuming the firm's discount factor for the expected future inflation cost is constant at β. This makes little difference to the example model.

Under producer price inflation index $\pi_t^{\text{tar}} = \pi_{h,t} = \frac{P_{h,t}}{P_{h,t-1}}$. Under a a consumer price inflation index, $\pi_t^{\text{tar}} = \pi_{\text{cpi},t} = \frac{\mathcal{P}_t}{\mathcal{P}_{t-1}}$. This can also be written as:

$$\pi_{\text{cpi},t} = \pi_{h,t} \frac{\mathcal{P}((1+\tau_t)\mathcal{S}_t)}{\mathcal{P}((1+\tau_{t-1})\mathcal{S}_{t-1})}$$

Equilibrium. Conditional on the following goods market clearing condition:

$$A_t H_t \Phi_t = C_{ht} + C_{ht}^*,$$

where $\Phi_t = 1 - \frac{\phi}{2}(\pi_{ht} - 1)^2$, and assuming balanced trade every period, the full equilibrium reduces to:

$$\begin{aligned}
\text{Balanced trade} \quad &: \quad \Lambda \mathcal{S}_t^{\eta} = \mathcal{S}_t C_{ft}, && \text{(A.14)}\\
\text{Market clearing} \quad &: \quad A_t H_t \Phi_t = C_{ht} + \Lambda \mathcal{S}_t^{\eta}, && \text{(A.15)}\\
\text{Labor market} \quad &: \quad \ell'(H_t) = A_t u_{c_{ht}} \mathbb{E}_t \{\Omega_{t,t+1}\}, && \text{(A.16)}\\
\text{Optimal spending} \quad &: \quad u_{c_{ht}} (1+\tau_t) \mathcal{S}_t = u_{c_{ft}}, && \text{(A.17)}\\
\text{Inflation: PPI target} \quad &: \quad \mathbb{E}_t \left\{ \frac{\pi_{ht}^{\mu_\pi}}{\pi_{ht+1}} \frac{u_{c_{ht+1}}}{u_{c_{ht}}} \right\} = 1. && \text{(A.18)}\\
\text{Inflation: CPI target} \quad &: \quad \mathbb{E}_t \left\{ \frac{\pi_{ht}^{\mu_\pi}}{\pi_{ht+1}} \left(\frac{\mathcal{P}(1,(1+\tau_t)\mathcal{S}_t)}{\mathcal{P}(1,(1+\tau_{t-1})\mathcal{S}_{t-1})} \right)^{\mu_\pi} \frac{u_{c_{ht+1}}}{u_{c_{ht}}} \right\} = 1. && \text{(A.19)}
\end{aligned}$$

The last two equations stem from combining the Euler equation with the monetary policy rule.

We assume that trade policy is made under discretion, whereby the government takes the actions of its successors as given. Since this simplified economy features balanced trade, the government essentially faces a static problem in each period.

B Optimal Tariff with an Inflation Targeting Monetary Policy Rule

The optimal policy problem for the choice of the Home tariff, given the monetary policy rule writes:

$$V(\mathcal{Z}_t) = \text{Max}_{\{C_{ht}, C_{ft}, H_t, \mathcal{S}_t, \pi_{ht}, \tau_t\}} \; u\left(C_{ht}, C_{ft}\right) - \ell(H_t) + \beta \mathbb{E}_t \{V(\mathcal{Z}_{t+1})\}, \tag{B.20}$$

subject to equations (B.21)-(B.25):

$$
\begin{aligned}
\text{Balanced trade} &: \quad \Lambda \mathcal{S}_t^{\eta} = \mathcal{S}_t C_{ft}, && \text{(B.21)}\\
\text{Market clearing} &: \quad A_t H_t \Phi_t = C_{ht} + \Lambda \mathcal{S}_t^{\eta}, && \text{(B.22)}\\
\text{Labor market} &: \quad \ell'(H_t) = A_t u_{c_{ht}} \mathbb{E}_t \{\Omega_{t,t+1}\}, && \text{(B.23)}\\
\text{Optimal spending} &: \quad u_{c_{ht}} (1+\tau_t) \mathcal{S}_t = u_{c_{ft}}, && \text{(B.24)}\\
\text{Inflation: PPI target} &: \quad \mathbb{E}_t \left\{ \frac{\pi_{ht}^{\mu_\pi}}{\pi_{ht+1}} \frac{u_{c_{ht+1}}}{u_{c_{ht}}} \right\} = 1. && \text{(B.25)}\\
\text{Inflation: CPI target} &: \quad \mathbb{E}_t \left\{ \frac{\pi_{ht}^{\mu_\pi}}{\pi_{ht+1}} \left(\frac{\mathcal{P}((1+\tau_t)\mathcal{S}_t)}{\mathcal{P}((1+\tau_{t-1})\mathcal{S}_{t-1})} \right)^{\mu_\pi} \frac{u_{c_{ht+1}}}{u_{c_{ht}}} \right\} = 1. && \text{(B.26)}
\end{aligned}
$$

Here $\mathcal{Z}_t$ represents the initial state variable constraining the planner. In the case of PPI targeting there are no endogenous state variables, so the optimal tariff problem is entirely static. Under either policy target, equation (B.24) can be omitted, since τ_t is a free variable and so this constraint will always hold. Note again that we can replace the terms $(1+\tau_t)\mathcal{S}_t$ in (B.26) using

$$
(1+\tau_t)\mathcal{S}_t = \frac{u_{ch,t}}{u_{cf,t}}
$$

B.1 Optimal Tariff under PPI targeting

Let $\xi_{1,t} - \xi_{4,t}$ be Lagrange multipliers on (B.21), (B.22), (B.23), and (B.25). The first order conditions for the optimal tariff under PPI targeting are written as[13]

$$
\begin{aligned}
C_{ht} &: \quad u_{c_{ht}} = \xi_{2t} + \xi_{3t} A_t u_{c_{hht}} \mathbb{E}_t \{\Omega_{t,t+1}\} + \xi_{4t} \mathbb{E}_t \left\{ \frac{\pi_{ht}^{\mu_\pi}}{\pi_{ht+1}} \frac{u_{c_{hht}} u_{c_{ht+1}}}{u_{c_{ht}}^2} \right\}, && \text{(B.27)}\\
C_{ft} &: \quad u_{c_{ft}} = \xi_{1t} \mathcal{S}_t, && \text{(B.28)}\\
H_t &: \quad \ell'(H_t) = \xi_{2t} A_t \Phi_t(\pi_{ht}) + \xi_{3t} \ell''(H_t), && \text{(B.29)}\\
\mathcal{S}_t &: \quad \xi_{1t} \left(\Lambda \eta \mathcal{S}_t^{\eta-1} - C_{ft} \right) - \xi_{2t} \Lambda \eta \mathcal{S}_t^{\eta-1} = 0, && \text{(B.30)}\\
\pi_{ht} &: \quad -\xi_{2t} \phi (\pi_{ht} - 1) A_t H_t - \xi_{3t} \phi \epsilon^{-1} (2\pi_{ht} - 1) A_t u_{c_{ht}} + \xi_{4t} \mu_\pi \mathbb{E}_t \left\{ \frac{\pi_{ht}^{\mu_\pi - 1}}{\pi_{ht+1}} \frac{u_{c_{ht+1}}}{u_{c_{ht}}} \right\} = 0 && \text{(B.31)}
\end{aligned}
$$

In the absence of exogenous shocks, system Equation (B.21)-(B.25) along with (B.27)-(B.31) will have a time-invariant solution. Then, the only solution to (12) must imply $\pi_h = 1$. Then equation (B.31) implies:

$$
\xi_4 = \frac{\xi_3 \phi A u_{c_h}}{\mu_\pi \epsilon}, \tag{B.32}
$$

[13] Recall also that we assume additive separability between C_{ht} and C_{ft} to reduce the presence of cross terms in the first-order conditions.

which, plugged in Equation (B.27), gives:

$$u_{c_h} = \xi_2 + \xi_3 A u_{c_{hh}} \left(\theta + \frac{\phi}{\mu_\pi \epsilon}\right). \tag{B.33}$$

Further, under our assumption, Equation (B.29) implies:

$$\xi_3 = \frac{\ell'(H) - \xi_2 A}{\ell''(H)}. \tag{B.34}$$

Combining the two last equations and using $\ell'(H) = A u_{c_h} \theta$ when $\pi_h = 1$ implies:

$$u_{c_h}\left(1 - \frac{A^2 u_{c_{hh}}}{\ell''(H)}\theta\left(\theta + \frac{\phi}{\mu_\pi \epsilon}\right)\right) = \xi_2\left(1 - \frac{A^2 u_{c_{hh}}}{\ell''(H)}\left(\theta + \frac{\phi}{\mu_\pi \epsilon}\right)\right). \tag{B.35}$$

Finally, from Equation (B.47) and (B.28) and using $C_f = \Lambda \mathcal{S}^{\eta-1}$, in a steady state:

$$\xi_2 = \frac{\eta - 1}{\eta}\xi_1 = \frac{\eta - 1}{\eta}\frac{u_{c_f}}{\mathcal{S}}. \tag{B.36}$$

Combining we get:

$$u_{c_h}\left(1 - \frac{A^2 u_{c_{hh}}}{\ell''(H)}\theta\left(\theta + \frac{\phi}{\mu_\pi \epsilon}\right)\right) = \frac{\eta - 1}{\eta}\frac{u_{c_f}}{\mathcal{S}}\left(1 - \frac{A^2 u_{c_{hh}}}{\ell''(H)}\left(\theta + \frac{\phi}{\mu_\pi \epsilon}\right)\right), \tag{B.37}$$

and using the optimal spending condition given by Equation (11):

$$1 + \tau = \frac{u_{c_f}}{\mathcal{S} u_{c_h}} = \frac{\eta}{\eta - 1}\frac{1 - \theta \Delta_1}{1 - \Delta_1}, \tag{B.38}$$

where $\Delta_1 = \frac{A^2 u_{c_{hh}}}{\ell''(H)}\left(\theta + \frac{\phi}{\mu_\pi \epsilon}\right) < 0$ since $u_{c_{hh}} < 0$. If $\theta = 1$ (no markup distortion), then the formula implies the following tariff rate:

$$1 + \tau = \frac{\eta}{\eta - 1} \rightarrow \tau = \frac{1}{\eta - 1}. \tag{B.39}$$

Further, since $\xi_2 = u_{c_h}$ then $\xi_3 = \xi_4 = 0$ and the assumed zero inflation rate ($\pi_h = 1$) is optimal. Is the optimal tariff with markup distortions ($\theta < 1$) larger or smaller than the tariff under $\theta = 1$?

$$\frac{\eta}{\eta - 1} - (1 + \tau) = \underbrace{\frac{\eta}{\eta - 1}}_{>1} \frac{\overbrace{(\theta - 1)}^{<0 \text{ since } \theta<1} \overbrace{\Delta_1}^{<0}}{\underbrace{1 - \Delta_1}_{>1 \text{ since } \Delta_1<0}} > 0 \tag{B.40}$$

The tariff rate is smaller with monopolistic distortions. Further, the tariff with markup distortion depends on price stickiness and monetary policy, as shown below:

$$\begin{aligned}\frac{\partial \tau}{\partial \phi} &= \frac{\eta}{\eta-1}\frac{\partial \Delta_1}{\partial \phi}\frac{1-\theta}{(1-\Delta_1)^2} \\ &= \frac{\eta}{\eta-1}\underbrace{\frac{A^2 u_{c_{hh}}}{\ell''(H)\mu_\pi \epsilon}}_{<0}\underbrace{\frac{1-\theta}{(1-\Delta_1)^2}}_{>0} < 0, \qquad \text{(B.41)}\\ \frac{\partial \tau}{\partial \mu_\pi} &= \frac{\eta}{\eta-1}\frac{\partial \Delta_1}{\partial \mu_\pi}\frac{1-\theta}{(1-\Delta_1)^2} \\ &= \frac{\eta}{\eta-1}\underbrace{\frac{-A^2 u_{c_{hh}}\phi}{\ell''(H)\mu_\pi^2 \epsilon}}_{>0}\underbrace{\frac{1-\theta}{(1-\Delta_1)^2}}_{>0} > 0. \qquad \text{(B.42)}\end{aligned}$$

These equations also confirm that the absence of markup distortions ($\theta = 1$) implies $\frac{\partial \tau}{\partial \phi} = \frac{\partial \tau}{\partial \mu_\pi} = 0$, *i.e.* price stickiness and monetary policy do not affect the tariff rate.

B.2 Optimal Tariff with CPI inflation targeting

Now look at the case where the Home monetary authority targets the CPI. The optimal policy problem for the choice of the Home tariff, given the CPI monetary policy rule is described as :

$$V(\mathcal{Z}_t) = \text{Max}_{\{C_{ht}, C_{ft}, H_t, \mathcal{S}_t, \pi_{ht}, \tau_t\}} \; u\left(C_{ht}, C_{ft}\right) - \ell(H_t) + \beta \mathbb{E}_t \{V(\mathcal{Z}_{t+1})\}, \qquad \text{(B.43)}$$

subject to equations (B.21)-(B.26): Here the state variable Z_t is no longer degenerate, but depends on the initial tariff adjusted steady state, so that $Z_t = (1+\tau_{t-1})\mathcal{S}_{t-1}$. This represents a constraint on the policy problem due to the form of the monetary rule implied by (B.26).

The policy problem is to maximize (B.43) subject to (B.21), (B.22), (B.23), and (B.26). In addition, we impose the condition $(1+\tau_t)\mathcal{S}_t = \frac{u_{cf,t}}{u_{ch,t}}$ on condition (B.26). The policymaker then must take account of the impact of period t choices of $C_{f,t}$ and $C_{h,t}$ on the monetary policy constraint in the period $t+1$.

As before, let $\xi_{1t} - \xi_{4t}$ represent the Lagrange multipliers on the constraints (8), (B.22), (B.23), and (B.26) respectively.

The new set of first order conditions are given by

$$
\begin{aligned}
C_{ht} &: \quad u_{c_{ht}} = \xi_{2t} + \xi_{3t} A_t u_{c_{hht}} \mathbb{E}_t \left\{ \Omega_{t,t+1} \right\} + \xi_{4t} \mathbb{E}_t \left\{ \frac{\pi_t^{\mu_\pi}}{\pi_{t+1}} \frac{u_{c_{hht}} u_{c_{ht+1}}}{u_{c_{ht}}^2} \right\}, \\
&+ \quad \mathbb{E}_t \left\{ \xi_{4t} \frac{\mu_\pi \pi_t^{\mu_\pi}}{\pi_{t+1}} \frac{u_{c_{ht+1}}}{u_{c_{ht}}} \frac{\mathcal{P}_t'}{\mathcal{P}_{t-1}} \frac{u_{c_{ft}} u_{c_{hht}}}{u_{c_{ht}}^2} - \beta \xi_{4t+1} \frac{\mu_\pi \pi_{t+1}^{\mu_\pi}}{\pi_{t+2}} \frac{u_{c_{ht+2}}}{u_{c_{ht+1}}} \frac{\mathcal{P}_{t+1} \mathcal{P}_t'}{\mathcal{P}_t^2} \frac{u_{c_{ft}} u_{c_{hht}}}{u_{c_{ht}}^2} \right\} \qquad \text{(B.44)} \\
C_{ft} &: \quad u_{c_{ft}} = \xi_{1t} \mathcal{S}_t \\
&- \quad \mathbb{E}_t \left\{ \xi_{4t} \frac{\mu_\pi \pi_t^{\mu_\pi}}{\pi_{t+1}} \frac{u_{c_{ht+1}}}{u_{c_{ht}}} \frac{\mathcal{P}_t'}{\mathcal{P}_{t-1}} \frac{u_{c_{fft}}}{u_{c_{ht}}} - \beta \xi_{4t+1} \frac{\mu_\pi \pi_{t+1}^{\mu_\pi}}{\pi_{t+2}} \frac{u_{c_{ht+2}}}{u_{c_{ht+1}}} \frac{\mathcal{P}_{t+1} \mathcal{P}_t'}{\mathcal{P}_t^2} \frac{u_{c_{fft}}}{u_{c_{ht}}} \right\} \qquad \text{(B.45)} \\
H_t &: \quad \ell'(H_t) = \xi_{2t} A_t \Phi_t (\pi_{ht}) + \xi_{3t} \ell''(H_t), \qquad \text{(B.46)} \\
\mathcal{S}_t &: \quad \xi_{1t} \left(\Lambda \eta \mathcal{S}_t^{\eta-1} - C_{ft} \right) - \xi_{2t} \Lambda \eta \mathcal{S}_t^{\eta-1} = 0, \qquad \text{(B.47)} \\
\pi_{ht} &: \quad -\xi_{2t} \phi (\pi_{ht} - 1) A_t H_t - \xi_{3t} \phi \epsilon^{-1} (2\pi_{ht} - 1) A_t u_{c_{ht}} + \xi_{4t} \mu_\pi \mathbb{E}_t \left\{ \frac{\pi_t^{\mu_\pi}}{\pi_{t+1} \pi_{ht-1}} \frac{u_{c_{ht+1}}}{u_{c_{ht}}} \right\} = 0. \qquad \text{(B.48)}
\end{aligned}
$$

In a steady state, these conditions simplify greatly. In particular, the CPI inflation rate is zero, so $\pi_t = 1$. In addition, the terms $\frac{\mathcal{P}_t' \frac{u_{cf}}{u_{ch}}}{\mathcal{P}_{t+1}}$ represent the share of foreign goods in consumer expenditure, which we denote α. Imposing a steady state, the conditions become:

$$
\begin{aligned}
C_h &: \quad u_{c_h} = \xi_2 + \xi_3 A u_{c_{hh}} \theta + \xi_4 \frac{u_{c_{hh}}}{u_{c_h}} + \xi_4 \mu_\pi \alpha \frac{u_{c_{hh}}}{u_{c_h}} (1 - \beta) \qquad \text{(B.49)} \\
C_f &: \quad u_{c_f} = \xi_1 \mathcal{S} - \xi_4 \mu_\pi \alpha \frac{u_{c_{ff}}}{u_{c_f}} (1 - \beta) \qquad \text{(B.50)} \\
H &: \quad \ell'(H) = \xi A + \xi_{3t} \ell''(H), \qquad \text{(B.51)} \\
\mathcal{S} &: \quad \xi \left(\Lambda \eta \mathcal{S}^{\eta-1} - C_f \right) - \xi_2 \Lambda \eta \mathcal{S}^{\eta-1} = 0, \qquad \text{(B.52)} \\
\pi_h &: \quad -\xi_3 \phi \epsilon^{-1} A u_{c_{ht}} + \xi_4 \mu_\pi = 0. \qquad \text{(B.53)}
\end{aligned}
$$

As before (B.53) implies:

$$
\xi_4 = \frac{\xi_3 \phi A u_{c_h}}{\mu_\pi \epsilon}, \qquad \text{(B.54)}
$$

and substituting into (B.49) gives

$$
u_{c_h} = \xi_2 + \xi_3 A u_{c_{hh}} \left(\theta + \frac{\phi}{\mu_\pi \epsilon} (1 + \mu_\pi \alpha (1 - \beta)) \right). \qquad \text{(B.55)}
$$

Then equation (B.51) implies:

$$
\xi_3 = \frac{A u_{c_h} \theta - \xi_2 A}{\ell''(H)} \qquad \text{(B.56)}
$$

Following the same steps as in (B.35) gives us

$$u_{c_h}\left(1-\frac{A^2u_{c_{hh}}}{\ell''(H)}\theta\left(\theta+\frac{\phi}{\mu_\pi\epsilon}(1+\mu_\pi\alpha(1-\beta))\right)\right)=\xi_2\left(1-\frac{A^2u_{c_{hh}}}{\ell''(H)}\left(\theta+\frac{\phi}{\mu_\pi\epsilon}(1+\mu_\pi\alpha(1-\beta))\right)\right). \tag{B.57}$$

Write this as

$$u_{c_h}=\xi_2\Gamma. \tag{B.58}$$

where

$$\Gamma=\frac{\left(1-\frac{A^2u_{c_{hh}}}{\ell''(H)}\left(\theta+\frac{\phi}{\mu_\pi\epsilon}(1+\mu_\pi\alpha(1-\beta))\right)\right)}{\left(1-\frac{A^2u_{c_{hh}}}{\ell''(H)}\theta\left(\theta+\frac{\phi}{\mu_\pi\epsilon}(1+\mu_\pi\alpha(1-\beta))\right)\right)}>1$$

Then, from (B.50) and (B.54) we have

$$u_{c_f}=\xi_1\mathcal{S}-\frac{\xi_3\phi A u_{c_h}}{\mu_\pi\epsilon}\mu_\pi\alpha\frac{u_{c_{ff}}}{u_{c_f}}(1-\beta) \tag{B.59}$$

combined with (B.56) gives

$$u_{c_f}=\xi_1\mathcal{S}-\frac{\frac{Au_{c_h}\theta-\xi_2A}{\ell''(H)}\phi A u_{c_h}}{\mu_\pi\epsilon}\mu_\pi\alpha\frac{u_{c_{ff}}}{u_{c_f}}(1-\beta) \tag{B.60}$$

Then, using (B.52) and (B.58) we can get

$$u_{c_f}=\xi_1\left(\mathcal{S}-\frac{\phi A^2u_{c_h}}{\epsilon\ell''(H)}\alpha\frac{u_{c_{ff}}}{u_{c_f}}(1-\beta)(\Gamma\theta-1)\right) \tag{B.61}$$

Then using (B.52) and (B.58) we get

$$(1+\tau)=\frac{u_{c_f}}{u_{c_f}}=\frac{\eta}{\eta-1}\Gamma^{-1}\left\{\mathcal{S}-\frac{\phi A^2u_{c_h}}{\epsilon\ell''(H)}\alpha\frac{u_{c_{ff}}}{u_{c_f}}(1-\beta)(\Gamma\theta-1)\right\} \tag{B.62}$$

In the case $\alpha=0$, this is equivalent to the results under PPI inflation targeting. In that case the term Γ becomes

$$\Gamma_0=\frac{\left(1-\frac{A^2u_{c_{hh}}}{\ell''(H)}\left(\theta+\frac{\phi}{\mu_\pi\epsilon}\right)\right)}{\left(1-\frac{A^2u_{c_{hh}}}{\ell''(H)}\theta\left(\theta+\frac{\phi}{\mu_\pi\epsilon}\right)\right)}$$

Moreover, since $\Gamma>\Gamma_0$, and $\Gamma\theta<1$, so that the term inside parenthesis on the right hand side of (B.62) is less than $\mathcal{S}$, it follows that the optimal tariff under CPI targeting in (B.62) is less than that under PPI targeting given by (B.38).

C The Extended model

Households in the Home country have preferences over consumption and hours given by:

$$U\left(C_t, H_t\right) = \frac{C_t^{1-\sigma}}{1-\sigma} - \chi \frac{H_t^{1+\psi}}{1+\psi}. \tag{C.63}$$

and trade bonds across countries.

C.1 Households

The representative Home household maximizes its welfare index:

$$\mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \left(\frac{C_{t+j}^{1-\sigma}}{1-\sigma} - \chi \frac{H_{t+j}^{1+\psi}}{1+\psi} \right), \tag{C.64}$$

subject to the following budget constraint:

$$S_t B_t^* + B_t + P_{ht} C_{ht} + \left(1+\tau_t\right) S_t P_{ft}^* C_{ft} + P_t \Lambda_t = S_t B_{t-1}^* R_{t-1}^* + B_{t-1} R_{t-1} + W_t H_t + \Pi_t + TR_t, \tag{C.65}$$

where B_t^* and B_t are the amounts of Foreign-currency and Home-currency denominated bonds bought by Home households, paying returns R_t^* and R_t between t and $t+1$. Buying Foreign-currency bonds incurs the payment of a small adjustment cost $\Lambda_t = \frac{\nu}{2}\left(\frac{S_t B_t^*}{P_t} - \frac{SB^*}{P}\right)^2$, proportional to the deviation of real Foreign bonds to their steady-state value. The bundle structure of adjustment costs mimics that of final goods. The representative household in the Home economy consumes local goods in quantity C_{ht} at the price P_{ht} and foreign goods in quantity C_{ft} at the price $\left(1+\tau_t\right) S_t P_{ft}^*$. The consumption bundle is:

$$C_t = \left(\gamma^{1/\lambda} C_{ht}^{1-1/\lambda} + \left(1-\gamma\right)^{1/\lambda} C_{ft}^{1-1/\lambda} \right)^{\frac{1}{1-1/\lambda}}, \tag{C.66}$$

where $\gamma = n + x\left(1-n\right)$, and x denotes Home bias. The aggregate consumption price index is:

$$P_t = \left(\gamma P_{ht}^{1-\lambda} + \left(1-\gamma\right) \left(\left(1+\tau_t\right) S_t P_{ft}^* \right)^{1-\lambda} \right)^{\frac{1}{1-\lambda}}, \tag{C.67}$$

so that $P_{ht} C_{ht} + \left(1+\tau_t\right) S_t P_{ft}^* C_{ft} = P_t C_t$. The demand functions of Home and Foreign goods by Home households are respectively:

$$C_{ht} = \gamma \left(\frac{P_{ht}}{P_t} \right)^{-\lambda} C_t = \gamma \mathcal{P}_t^{\lambda} C_t, \tag{C.68}$$

$$C_{ft} = \left(1-\gamma\right) \left(\frac{\left(1+\tau_t\right) S_t P_{ft}^*}{P_t} \right)^{-\lambda} C_t = \left(1-\gamma\right) \left(\frac{\mathcal{P}_t}{\left(1+\tau_t\right) \mathcal{S}_t} \right)^{\lambda} C_t, \tag{C.69}$$

where $\mathcal{P}_t = P_t/P_{ht} = \left(\gamma + (1-\gamma)\left((1+\tau_t)\,\mathcal{S}_t\right)^{1-\lambda}\right)^{\frac{1}{1-\lambda}}$ represents the relative price of the Home consumption good and $\mathcal{S}_t = S_t P^*_{ft}/P_{ht}$ denotes Home terms of trade. The first-order conditions of the Home household imply:

$$\beta \mathbb{E}_t \left\{ \frac{\mathcal{S}_{t+1} R^*_t \mathcal{P}_t C^\sigma_t}{\mathcal{S}_t \pi^*_{ft+1} \mathcal{P}_{t+1} C^\sigma_{t+1} \left(1 + \nu\left(\frac{S_t B^*_t}{P_t} - \frac{SB^*}{P}\right)\right)} \right\} = 1, \tag{C.70}$$

$$\beta \mathbb{E}_t \left\{ \frac{R_t \mathcal{P}_t C^\sigma_t}{\pi_{ht+1} \mathcal{P}_{t+1} C^\sigma_{t+1}} \right\} = 1, \tag{C.71}$$

$$\chi H^\psi_t C^\sigma_t = \frac{\mathcal{W}_t}{\mathcal{P}_t}, \tag{C.72}$$

where $\pi_{ht} = P_{ht}/P_{ht-1}$ and $\pi^*_{ft} = P^*_{ft}/P^*_{ft-1}$ are the gross rates of PPI inflation in the Home and Foreign country respectively, and $\mathcal{W}_t = W_t/P_{ht}$.

The Foreign representative household has a similar utility function, and its consumption bundle and price index are respectively:

$$C^*_t = \left(\gamma^{*1/\lambda} C^{*1-1/\lambda}_{ft} + (1-\gamma^*)^{1/\lambda} C^{*1-1/\lambda}_{ht}\right)^{\frac{1}{1-1/\lambda}}, \tag{C.73}$$

$$P^*_t = \left(\gamma^* P^{*1-\lambda}_{ft} + (1-\gamma^*)\left((1+\tau^*_t)\frac{P_{ht}}{S_t}\right)^{1-\lambda}\right)^{\frac{1}{1-\lambda}}, \tag{C.74}$$

and the corresponding demand functions are:

$$C^*_{ft} = \gamma^* \left(\frac{P_{ft}}{P^*_t}\right)^{-\lambda} = \gamma^* \mathcal{P}^{*\lambda}_t C^*_t, \tag{C.75}$$

$$C^*_{ht} = (1-\gamma^*)\left(\frac{(1+\tau^*_t)\,P_{ht}}{S_t P^*_t}\right)^{-\lambda} = (1-\gamma^*)\left(\frac{\mathcal{S}_t \mathcal{P}^*_t}{(1+\tau^*_t)}\right)^{\lambda} C^*_t, \tag{C.76}$$

where $\mathcal{P}^*_t = P^*_t/P^*_{ft} = \left(\gamma^* + (1-\gamma^*)\left((1+\tau^*_t)/\mathcal{S}_t\right)^{1-\lambda}\right)^{\frac{1}{1-\lambda}}$. The Foreign household faces a different budget constraint, as it only has access to local bonds without paying adjustment costs. Its labor supply equation is:

$$\chi C^{*\sigma}_t H^{*\psi}_t = \frac{W^*_t}{P^*_t} = \frac{\mathcal{W}^*_t}{\mathcal{P}^*_t}, \tag{C.77}$$

where $\mathcal{W}^*_t = W^*_t/P^*_{ft}$ and the Euler equation associated with Foreign bonds gives:

$$\beta \mathbb{E}_t \left\{ \frac{R^*_t \mathcal{P}^*_t C^{*\sigma}_t}{\pi^*_{ft+1} \mathcal{P}^*_{t+1} C^{*\sigma}_{t+1}} \right\} = 1. \tag{C.78}$$

C.2 Firms

A measure n of firms in the Home economy produce differentiated goods. The aggregate Home good is a composite of these differentiated goods, where the elasticity of substitution between individual goods is denoted $\epsilon > 1$. The production function for firm i in the Home country is

$$Y_t(i) = A_t H_t(i)^{1-\alpha} X_t(i)^{\alpha} \tag{C.79}$$

where A_t is an exogenous aggregate productivity term. Here, $X_t(i)$ represents the use of intermediate goods by the Home firm i and $H_t(i)$ the use of labor. Intermediate good inputs are composed of Home and Foreign goods in a different composition than that of the consumption aggregator. Namely,

$$X_t(i) = \left(\gamma_x^{\frac{1}{\lambda}} X_{ht}(i)^{\frac{\lambda-1}{\lambda}} + (1-\gamma_x)^{\frac{1}{\lambda}} X_{ft}(i)^{\frac{\lambda-1}{\lambda}} \right)^{\frac{\lambda}{\lambda-1}}, \tag{C.80}$$

where $X_{jt}(i)$ is the Home firm's use of inputs from country $j = \{h, f\}$. The profits of Home firm i are then:

$$\Pi_t(i) = (P_{ht}(i) - MC_t) Y_t(i), \tag{C.81}$$

where $MC_t = A_t^{-1}(1-\alpha)^{\alpha-1}\alpha^{-\alpha} W_t^{1-\alpha} P_{xt}^{\alpha}$ denotes the firm's nominal marginal cost, and where

$$P_{xt} = \left(\gamma_x P_{ht}^{1-\lambda} + (1-\gamma_x)((1+\tau_t) S_t P_{ft}^*)^{1-\lambda} \right)^{\frac{1}{1-\lambda}}, \tag{C.82}$$

is the price index relevant for the firm's use of intermediate inputs, and where τ_t is the tariff rate on imports. Cost minimization by the firm implies:

$$(1-\alpha)\frac{Y_t(i)}{H_t(i)} = \frac{W_t}{MC_t} \text{ and } \alpha \frac{Y_t(i)}{X_t(i)} = \frac{P_{xt}}{MC_t}, \tag{C.83}$$

with

$$X_{ht}(i) = \gamma_x \left(\frac{P_{ht}}{P_{xt}} \right)^{-\lambda} X_t(i) = \gamma_x \mathcal{P}_{xt}^{\lambda} X_t(i), \tag{C.84}$$

$$X_{ft}(i) = (1-\gamma_x) \left(\frac{(1+\tau_t) S_t P_{ft}^*}{P_{xt}} \right)^{-\lambda} X_t(i) = (1-\gamma_x) \left(\frac{\mathcal{P}_{xt}}{(1+\tau_t)\mathcal{S}_t} \right)^{\lambda} X_t(i), \tag{C.85}$$

where $\mathcal{P}_{xt}$ is the equivalent of $\mathcal{P}_t$ for intermediate goods.[14] The firm chooses its price to maximize the present value of expected profits, net of price adjustment costs:

$$\mathbb{E}_t \sum_{j=0}^{\infty} \omega_{t+j} \left(\Pi_{t+j}(i) - \frac{\phi}{2} \left(\frac{P_{ht+j}(i)}{P_{ht+j-1}(i)} - 1 \right)^2 P_{ht+j}(i) Y_{t+j}(i) \right), \tag{C.86}$$

where ω_t is the firm's nominal stochastic discount factor, and ϕ represents a price adjustment cost for the firm. Price adjustment costs are proportional to the nominal value of Home output, to be consistent with the nominal profit objective function of the firm. The first-order condition for profit maximization for the Home firm i takes into account the individual demand of good i,

[14] $\mathcal{P}_t$ and $\mathcal{P}_{xt}$ only differ by the presence of potentially different degrees of home bias.

i.e $Y_t^d(i) = (P_{ht}(i)/P_{ht})^{-\epsilon} Y_t$ and is the same for all producers so that $P_{ht}(i) = P_{ht}$ and $Y_t(i) = Y_t$ and that the i index can be dropped. It implies:

$$\theta + \phi\epsilon^{-1}\left(\pi_{ht}(\pi_{ht}-1) - \mathbb{E}_t\{\omega_{t+1}\pi_{ht+1}(\pi_{ht+1}-1)Y_{t+1}/Y_t\}\right) = \mathcal{MC}_t, \tag{C.87}$$

where $\theta = \frac{(1+s)(\epsilon-1)}{\epsilon}$, and:

$$\mathcal{MC}_t = MC_t/P_{ht} = \mathcal{MC}_t = \frac{\mathcal{W}_t^{1-\alpha}\mathcal{P}_{xt}^{\alpha}}{A_t\alpha^{\alpha}(1-\alpha)^{1-\alpha}} \text{ as well as } \omega_t = \beta\frac{C_{t-1}^{\sigma}\mathcal{P}_{t-1}}{C_t^{\sigma}\mathcal{P}_t}. \tag{C.88}$$

Using symmetry among producers, the factor demands can be rewritten as:

$$(1-\alpha)\mathcal{MC}_t Y_t = \mathcal{W}_t H_t, \text{ and } \alpha\mathcal{MC}_t Y_t = \mathcal{P}_{xt}X_t, \tag{C.89}$$

where $\mathcal{P}_{xt} = P_{xt}/P_{ht}$.

C.3 The Competitive Equilibrium

Given that Rotemberg costs are paid in units of local goods and using the demand functions for intermediate and final goods, the goods market clearing conditions are given by:

$$Y_t\left(1 - \frac{\phi}{2}(\pi_{ht}-1)^2\right) = D_t + D_{xt}^*, \tag{C.90}$$

$$Y_t^*\left(1 - \frac{\phi}{2}\left(\pi_{ft}^*-1\right)^2\right) = D_t^* + D_{xt}, \tag{C.91}$$

where

$$D_t = \gamma\mathcal{P}_t^{\lambda}(C_t + \Lambda_t) + \gamma_x\mathcal{P}_{xt}^{\lambda}X_t, \tag{C.92}$$

$$D_{xt} = \frac{n}{1-n}\left(\frac{\mathcal{S}_t^{-1}}{1+\tau_t}\right)^{\lambda}\left((1-\gamma)\mathcal{P}_t^{\lambda}(C_t+\Lambda_t) + (1-\gamma_x)\mathcal{P}_{xt}^{\lambda}X_t\right), \tag{C.93}$$

$$D_t^* = \gamma^*\mathcal{P}_t^{*\lambda}C_t^* + \gamma_x^*\mathcal{P}_{xt}^{*\lambda}X_t^*, \tag{C.94}$$

$$D_{xt}^* = \frac{1-n}{n}\left(\frac{\mathcal{S}_t}{1+\tau_t^*}\right)^{\lambda}\left((1-\gamma^*)\mathcal{P}_t^{*\lambda}C_t^* + (1-\gamma_x^*)\mathcal{P}_{xt}^{*\lambda}X_t^*\right). \tag{C.95}$$

The labor-market clearing conditions are:

$$(1-\alpha)\mathcal{MC}_t A_t H_t^{-\alpha}X_t^{\alpha} = \chi\mathcal{P}_t C_t^{\sigma}H_t^{\psi}, \tag{C.96}$$

$$(1-\alpha)\mathcal{MC}_t^* A_t^* H_t^{*-\alpha}X_t^{*\alpha} = \chi\mathcal{P}_t^* C_t^{*\sigma}H_t^{*\psi}. \tag{C.97}$$

Finally, Home bonds are in zero-net supply so that $B_t = 0$ and the clearing condition on the market for Foreign bonds writes:

$$nB_t^* + (1-n)B_t^{**} = 0. \tag{C.98}$$

Defining $b_t = \frac{S_t B_t^*}{P_t}$ and $b_t^* = \frac{B_t^{**}}{P_t^*}$ as the real per-capita net foreign asset positions, Equation (C.98) implies:

$$nb_t + (1-n)\frac{\mathcal{S}_t \mathcal{P}_t^*}{\mathcal{P}_t} b_t^* = 0. \tag{C.99}$$

Further, the modified uncovered interest rate parity condition stemming from the combination of Home and Foreign Euler Equations writes:

$$\mathbb{E}_t \left\{ \frac{\mathcal{S}_{t+1}\omega_{t+1}}{\mathcal{S}_t \omega_{t+1}^* \left(1 + \nu \left(b_t - b\right)\right)} - 1 \right\} = 0, \tag{C.100}$$

where, remember, $\omega_t = \beta \frac{C_{t-1}^{\sigma} \mathcal{P}_{t-1}}{C_t^{\sigma} \mathcal{P}_t}$ and $\omega_t^* = \beta \frac{C_{t-1}^{*\sigma} \mathcal{P}_{t-1}^*}{C_t^{*\sigma} \mathcal{P}_t^*}$. Last, the consolidation of the Home household budget constraint with other equilibrium and market clearing conditions gives:

$$b_t = \frac{\mathcal{S}_t \mathcal{P}_{t-1}}{\mathcal{S}_{t-1}\mathcal{P}_t \omega_t^*} b_{t-1} + \mathcal{P}_t^{-1}\left(D_{xt}^* - \frac{1-n}{n}\mathcal{S}_t D_{xt}\right). \tag{C.101}$$

C.4 Reduced-form equilibrium conditions

Using appropriate substitutions, equilibrium conditions can be summarized as:

$$\theta + \phi\epsilon^{-1}\left(\pi_{ht}\left(\pi_{ht}-1\right) - \mathbb{E}_t\left\{\omega_{t+1}\pi_{ht+1}\left(\pi_{ht+1}-1\right)\frac{Y_{t+1}}{Y_t}\right\}\right) = \mathcal{MC}_t, \tag{C.102}$$

$$\theta + \phi\epsilon^{-1}\left(\pi_{ft}^*\left(\pi_{ft}^*-1\right) - \mathbb{E}_t\left\{\omega_{t+1}^*\pi_{ft+1}^*\left(\pi_{ft+1}^*-1\right)\frac{Y_{t+1}^*}{Y_t^*}\right\}\right) = \epsilon\mathcal{MC}_t^*, \tag{C.103}$$

$$Y_t\left(1 - \frac{\phi}{2}\left(\pi_{ht}-1\right)^2\right) - D_t - D_{xt}^* = 0, \tag{C.104}$$

$$Y_t^*\left(1 - \frac{\phi}{2}\left(\pi_{ft}^*-1\right)^2\right) - D_t^* - D_{xt} = 0, \tag{C.105}$$

$$nb_t + (1-n)\frac{\mathcal{S}_t\mathcal{P}_t^*}{\mathcal{P}_t}b_t^* = 0, \tag{C.106}$$

$$\mathbb{E}_t\left\{\frac{R_t\omega_{t+1}}{\pi_{ht+1}}\right\} = 1, \tag{C.107}$$

$$\mathbb{E}_t\left\{\frac{R_t^*\omega_{t+1}^*}{\pi_{ft+1}^*}\right\} = 1, \tag{C.108}$$

$$\mathbb{E}_t\left\{\frac{\mathcal{S}_{t+1}\omega_{t+1}}{\mathcal{S}_t\omega_{t+1}^*\left(1+\nu\left(b_t-b\right)\right)} - 1\right\} = 0, \tag{C.109}$$

$$b_t - \frac{\mathcal{S}_t\mathcal{P}_{t-1}}{\mathcal{S}_{t-1}\mathcal{P}_t\omega_t^*}b_{t-1} - \mathcal{P}_t^{-1}\left(D_{xt}^* - \frac{1-n}{n}\mathcal{S}_t D_{xt}\right) = 0. \tag{C.110}$$

where:

$$\omega_t = \beta \frac{C_{t-1}^{\sigma} \mathcal{P}_{t-1}}{C_t^{\sigma} \mathcal{P}_t}, \tag{C.111}$$

$$\omega_t^* = \beta \frac{C_{t-1}^{*\sigma} \mathcal{P}_{t-1}^*}{C_t^{*\sigma} \mathcal{P}_t^*} \tag{C.112}$$

$$\mathcal{MC}_t = \frac{\left(\mathcal{P}_t \chi H_t^{\psi} C_t^{\sigma}\right)^{1-\alpha} \mathcal{P}_{xt}^{\alpha}}{A_t \alpha^{\alpha} (1-\alpha)^{1-\alpha}}, \tag{C.113}$$

$$\mathcal{MC}_t^* = \frac{\left(\mathcal{P}_t^* \chi H_t^{*\psi} C_t^{*\sigma}\right)^{1-\alpha} \mathcal{P}_{xt}^{*\alpha}}{A_t \alpha^{\alpha} (1-\alpha)^{1-\alpha}} \tag{C.114}$$

with:

$$H_t = \left(\frac{(1-\alpha) \left(\mathcal{P}_t \chi C_t^{\sigma}\right)^{-\alpha} \mathcal{P}_{xt}^{\alpha} Y_t}{A_t \alpha^{\alpha} (1-\alpha)^{1-\alpha}} \right)^{\frac{1}{1+\alpha\psi}}, \; H_t^* = \left(\frac{(1-\alpha) \left(\mathcal{P}_t^* \chi C_t^{*\sigma}\right)^{-\alpha} \mathcal{P}_{xt}^{*\alpha} Y_t^*}{A_t^* \alpha^{\alpha} (1-\alpha)^{1-\alpha}} \right)^{\frac{1}{1+\psi\alpha}}, \tag{C.115}$$

$$X_t = \frac{\alpha \left(\mathcal{P}_t \chi H_t^{\psi} C_t^{\sigma}\right)^{1-\alpha} \mathcal{P}_{xt}^{\alpha-1} Y_t}{A_t \alpha^{\alpha} (1-\alpha)^{1-\alpha}}, \; X_t^* = \frac{\alpha \left(\mathcal{P}_t^* \chi H_t^{*\psi} C_t^{*\sigma}\right)^{1-\alpha} \mathcal{P}_{xt}^{*\alpha-1} Y_t^*}{A_t^* \alpha^{\alpha} (1-\alpha)^{1-\alpha}}. \tag{C.116}$$