Trade Wars, Currency Wars*

Stéphane Auray[†] Michael B. Devereux[‡] Aurélien Eyquem[§]

February 11, 2022

Keywords: Protectionism, Currency Wars, Trade Wars

JEL Category, F30, F40, F41

Abstract

Countries distort trade patterns ('trade wars') to gain strategic advantage relative to one another. At the same time, monetary policies are set independently and have spillover effects on partner countries ('currency wars'). We combine these two scenarios in simple open-economy model with sticky prices and show that they interact in deep and interesting ways. The stance of monetary policy has substantial effects on the degree of protection in a Nash equilibrium of the monetary and trade policy game. Trade wars lead to higher equilibrium inflation rates. Cooperation in monetary policy leads to both higher inflation and increased protection. By contrast, when monetary policy is constrained by pegged exchange rates or the zero lower bound on interest rates, equilibrium tariffs are lower. A country that issues the dominant currency in trade gains a large advantage in a trade war. Allowing for steady-state trade imbalances, we find that both inflation and tariffs depend critically upon the currency denomination of internationally traded bonds.

*We would like to thank Paul Bergin, Fabio Ghironi, Robert Johnson, Olivier Loisel, Philippe Martin, Isabelle Méjean and Dmitry Mukhin for their invaluable comments, as well as participants in many seminars and conferences for interesting comments. Devereux thanks SSHRC for research funding. Auray and Eyquem acknowledge the financial support of Projets Generique ANR 2015, Grant Number ANR-15-CE33-0001-01. Finally we acknowledge the financial support of the Europlace Institute of Finance.

[†]CREST-Ensai and Université du Littoral Côte d'Opale. ENSAI, Campus de Ker-Lann, Rue Blaise Pascal, BP 37203, 35172 BRUZ Cedex, France. stephane.auray@ensai.fr.

[‡]Vancouver School of Economics, University of British Columbia 6000, Iona Drive, Vancouver B.C. CANADA V6T 1L4, CEPR and NBER. michael.devereux@ubc.ca.

[§]Univ Lyon, Université Lumière Lyon 2, GATE L-SE UMR 5824, and Institut Universitaire de France. 93 Chemin des Mouilles, BP167, 69131 Ecully Cedex, France. aurelien.eyquem@univ-lyon2.fr.

1 Introduction

The question of cooperation in trade and monetary policy is a longstanding one, dating back at least to the discussions about the Gold Standard in the 30's. The Bretton Woods system of fixed exchange rates along with the GATT and then the WTO, or more recently the creation of a single currency in Europe are examples of cooperative policies. Indeed the interplay of trade and monetary policies was a crucial matter during the run-up to the implementation of EMU in Europe.[1] Relatedly, the Plaza Accord of 1985 can be looked at as an example of monetary cooperation to address imbalances that were resulting in protectionist pressures. After a long historical period of cooperation in the post-WWII era, the breakdown of the international monetary system opened the door for less cooperative monetary policies. Further, the recent accusations of currency manipulation and the retaliatory tariffs imposed by the U.S. administration illustrate how both dimensions of global economic policies are intertwined, and how non-cooperation can rapidly escalate.

In this paper, we analyze the strategic interaction between policy-makers in a model in which both trade and monetary policy decisions are simultaneously determined in a non-cooperative environment. There is a long literature on non-cooperative trade policy beginning at least with Johnson (1953).[2] These models analyze the way countries distort trade flows to gain a strategic advantage in a 'trade war'. Likewise, there are many studies of non-cooperative monetary policy in the international macro literature (references below). In recent years, the importance of international spillovers of monetary policy has sparked a debate on 'currency wars'.[3] We consider a global non-cooperative framework in which these two policies interact. In principle, both trade and monetary policy work by affecting relative prices, which have market and welfare spillovers across countries.[4] Monetary policy has real effects if some nominal prices are sticky. The impact of trade policy such as tariffs will depend on how prices adjust and the exchange rate responds. Our main insight is that the equilibrium degree of protection in a non-cooperative environment critically depends on the stance of monetary policy. Conversely, the equilibrium monetary policy choice depends on the outcome of trade wars between countries.

We explore the relationship between currency wars and trade wars within a standard two country New Keynesian open-economy model. Aside from the exploration of endogenous and non-cooperative trade policy, and the presence of trade in intermediate goods, the model is quite standard. Households consume and supply labor, trade goods and bonds, and monopolisticly

[1] Mario Draghi was very explicit on the connection between trade and monetary policy choices in his 2019 speech in Sintra: "The euro was introduced 20 years ago in response to repeated episodes of exchange-rate instability and the need to secure the Single Market against competitive devaluations". See https://www.ecb.europa.eu/press/key/date/2019/html/ecb.sp190618~ec4cd2443b.en.html.

[2] For a recent survey, see Bagwell and Staiger (2016).

[3] For instance, Mishra and Rajan (2018) argue that "Aggressive monetary policy actions by one country can lead to significant adverse cross-border spillovers on others, especially as countries contend with the zero lower bound. If countries do not internalize these spillovers, they may undertake policies that are collectively suboptimal."

[4] For recent evaluation of the policy issues, see for instance Fajgelbaum et al. (2019) or Eichengreen (2019).

competitive firms maximize profits subject to costs of price adjustment. For the baseline model, prices are set in producer currencies. Policy-makers in each country conduct monetary policy through the choice of inflation rates, but also determine trade protection by setting tariffs.[5]

We begin by deriving some analytical results in a simplified version of the model. We contrast the outcome of a joint 'trade and currency war' with the case where monetary policy is determined assuming zero tariffs (a currency war alone), and trade policy is derived assuming passive monetary policy (a trade war alone). In the currency and trade war case, both tariffs and inflation rates trade off internal distortions against external terms-of-trade manipulation. Tariffs follow an amended version of the optimal monopoly tariff formula, which depends on both the elasticity of foreign excess demand for domestic exports, as well as equilibrium domestic inflation rates. We find that both inflation and tariffs are higher in the joint trade and currency war equilibrium than when each is chosen separately. The presence of firm level markups is a key factor for the outcome of both tariffs and inflation. If markups are offset, the equilibrium inflation rate is zero. But a by-product of this is a large rise in protection, as tariff setters focus only on the terms-of-trade manipulation motive.

We further show that eliminating currency wars through monetary policy cooperation leads to a rise in both inflation and tariffs. The first result is familiar from Rogoff (1985), as monetary policy cooperation removes the incentive for competitive terms-of-trade appreciation through monetary policy. But the second result follows because trade wars become more intense when this role for monetary policy is eliminated. In fact, in contrast to Rogoff (1985), we find that monetary policy cooperation can be counterproductive in the trade and currency war game even in the absence of monopoly markups.[6]

We then proceed to solve a quantitative version of the model in a general case. The quantitative model confirms the analytical results. Both tariffs and inflation are higher in a setting where both trade and monetary policies are determined non-cooperatively. Cooperation in monetary policy alone leads to both higher tariffs and higher inflation, reducing welfare for all countries. In addition, tariffs are significantly higher in the currency and trade war than in an analogous economy with flexible prices and monetary neutrality.

We then explore a rich set of alternative environments, allowing for variations in country size, the degree of commitment in trade policy, the exchange rate regime, and the assumptions regarding the currency of invoicing of traded goods. We also explore the consequences of initial trade imbalances. In all cases, we find that the stance of monetary policy and the non-neutrality of the nominal environment due to sticky prices have major implications for the equilibrium

[5] We use the label 'currency wars' as a description of a situation where countries follow independent monetary policies which may have negative international spillovers. Other senses of the term may suggest that countries are attempting to target a level of the real exchange rate. That is not the case in this paper.

[6] Rogoff's model was based on a simple linear quadratic framework without explicit micro-foundations. Later studies expanded on Rogoff's results within New Open Economy models with microfounded distortions coming from monopoly pricing, as in our model. See in particular Betts and Devereux (2000), Canzoneri and Dale (1991), Tirelli (1993) or Carraro and Francesco (1998).

degree of trade protection.

Conventional wisdom suggests that self-oriented monetary policy-making by large countries imposes negative spillovers on smaller countries. In our model, focusing on monetary policy alone, spillovers from larger countries are actually lessened, since larger countries are less focused on manipulating inflation rates to improve the terms of trade. But when trade wars and currency wars are combined, the impact of country size is reversed, as larger countries are more aggressive in trade policy, and smaller countries suffer greater losses.

The baseline model assumes discretionary policy-making, tariffs and inflation are chosen without commitment to future policy actions. But we also consider the possibility that trade policy engenders some degree of commitment, still without any international cooperation. In this case, equilibrium tariff rates are much lower than the baseline case. With commitment, trade authorities take account of the impact of higher tariffs on inflation choices of the monetary authorities, tempering the incentive for terms-of-trade manipulation. Again, this is essentially due to the interaction of monetary policy and trade policy. Absent endogenous monetary policy, the quasi-commitment in tariffs would have no implications for equilibrium tariff rates.

In an economy with sticky prices, the ability to affect the terms of trade through changes in the nominal exchange rate is a key feature of the incentive to set tariffs and inflation. We show that optimal tariffs chosen under fixed exchange rate are much lower than under the baseline case of inflation targeting under flexible exchange rates. In this case, terms-of-trade manipulation can be done only by generating differences in inflation rates, which in itself imposes additional welfare costs. The result is a drop in the degree of equilibrium protection.

It is realistic to think that monetary policy may have limited traction due to the effective lower bound on interest rates. Interestingly, we find that non-cooperative tariffs are lower than the baseline case in such an environment. The intuitive explanation for this is that at the ZLB, inflation rates endogenously respond to tariffs instead of being chosen directly, and policy-makers take account of the spillover effect of tariffs on inflation.

We further extend the analysis to 'dominant currency pricing' (DCP hereafter), as in Mukhin (2018) and Gopinath et al. (2020) instead of the standard assumption of 'producer currency pricing' (PCP) as in the classic model of Galì and Monacelli (2005). In that case, all countries set their export prices in the Home country's currency (the dominant currency). This introduces a critical asymmetry in the analysis, since import prices in the Home country are no longer directly tied to exchange rate changes. Under DCP, where the Foreign country can only improve its terms of trade by generating costly inflation in its export goods prices, tariffs are a much less effective tool. As a result, the trade and currency war with DCP dramatically favours the dominant currency issuer.[7]

Finally we explore the implications of steady-state trade imbalances. Our general economy

[7] Again as in the previous examples, the currency of pricing would not matter at all in an economy with flexible prices and money neutrality.

has incomplete markets and trade in nominal bonds, and the baseline case assumes a zero net foreign assets (NFA) position. But extending the model to allow for non-zero NFA, we find that tariffs and inflation are sensitive both to the level and the currency denomination of NFA. Debtor countries whose currency is used in asset trade choose higher inflation rates than in the baseline case, in an attempt to partly expropriate creditors, but have lower tariff rates. In general, creditor countries, who run permanent trade deficits, are more protectionist than in the baseline case of zero NFA, whether or not their currency is used in bond trade. These results reinforce the overall message of the paper: the interaction between monetary policy and trade policy is critical for both outcomes in the strategic interaction between governments.

The rest of the paper is organized as follows. Section 2 offers an overview of the literature and highlights what we see as our contribution. Section 3 sets out a basic model with balanced trade, and establishes a number of analytical results. Section 4 presents an extended model with intermediate goods in production and asset trade, which is solved quantitatively, and develops the main results of the paper concerning currency and trade wars. There we explore the effect of cooperation in monetary policy, partial commitment in trade policy and country size. Section 5 analyzes the model under a variety of set-ups in which monetary policy is constrained due to either an exchange rate peg, the zero lower bound constraint on nominal interest rates, or due to the presence of dominant currency pricing. Finally, Section 6 looks at the implication of trade imbalances for joint currency and trade wars.

2 Literature

The interaction between trade policy, monetary policy, and macroeconomic outcomes has long been a subject of interest to economists (e.g. Eichengreen (1981) and Krugman (1982)). Our work and results also relate to a large number of studies from the 80's and 90's. On the issue of monetary policy cooperation and its impacts, Canzoneri and Dale (1991) note that if cooperation were indeed counterproductive, central banks should be able to choose cooperatively to implement the non-cooperative policies. Tirelli (1993) also contains many references such as Carraro and Francesco (1998). In this paper, they show that international policy coordination is not counterproductive and that international cooperation belongs to the central banks' dominant strategy. Betts and Devereux (2000) show that whether monetary policy cooperation is counterproductive or not may depend on the degree to which goods are priced in buyer's or seller's currency. Another example of how academics were thinking about strategic monetary-trade interactions in the late 1980s is Basevi, Denicolo, and Delbono (1990) which studies the relations between the United States, West Germany and Japan. Their analysis shows that, under certain assumptions, the outcome of the game converges to a cooperative-equivalent equilibrium, with zero tariffs and optimal monetary policies. Finally, on country size and policy incentives, Ghironi and Giavazzi (1998) show that the employment-inflation tradeoff facing a central bank depends on the size of the economy for which it sets monetary policy.

More recent events have led to a revival of interest in this area and an attempt to formalize the relationship within the modern macroeconomic toolkit. Barattieri, Cacciatore, and Ghironi (2021) investigate empirically the impact of exogenous changes in tariffs in an SVAR framework, and develop a small open economy model with firm entry and endogenous tradability that successfully rationalizes the empirical evidence. We adopt an alternative approach, considering tariffs as endogenous, exploring the consequences of alternative strategic settings for both monetary policy and tariff formation. Another paper by Erceg, Prestipino, and Raffo (2018) looks at the impact of trade policies in the form of import tariffs and export subsidies. They find that the effects critically depend on the response of the real exchange rate, and that in turn depends on the expectations about future policies and potential retaliation from trade partners. A recent paper by Furceri et al. (2019) examines the macroeconomic consequences of tariff shocks, and shows that these shocks are generally contractionary. Lindé and Pescatori (2019) study the conditions under which Lerner symmetry holds, and how this affects the macroeconomic costs of a trade war.

Among the recent contributions, Jeanne (2020) is closest in spirit to our paper. He explores the interaction between 'currency wars' and 'trade wars' in an analytical framework with a continuum of small open economies with downward nominal wage rigidity and, in some cases a global liquidity trap, and explores the benefits of international cooperation. By contrast, our study is focused on a more conventional two-country model, where countries are large, and focuses on a discretionary Ramsey approach to policy-making.[8] Bergin and Corsetti (2020) develop a rich multi-country DSGE model with global value chains and look at the optimal response of monetary policy to exogenous tariff shocks. In addition, they focus on cooperative determination of monetary policy and consider tariffs as exogenous. Another relevant paper is Caballero, Farhi, and Gourinchas (2015), who investigate the interaction between an environment of low interest rate, financial imbalances and currency wars. Our paper does consider a binding ZLB constraint as one of the possible cases but deals with more generic environments, and considers the joint endogenous formation of tariff and monetary policies. We thus consider our paper as an important complement. In particular, we find that international cooperation may be significantly welfare reducing in this environment.[9]

Focusing more closely on the endogenous determination of trade policies, we noted above that there is a large empirical literature investigating the link between trade restrictions and the economic cycle, and separately, the effect of real exchange rate undervaluation on trade policy

[8] This type of approach echoes the approach of Chari, Nicolini, and Teles (2018) or Auray, Eyquem, and Gomme (2018), although these papers focus on flex-price environments.

[9] Corsetti and Pesenti (2001) show how national welfare in open economies may depend on a terms-of-trade externality, using a two-country model with monopolistic competition. There are many papers analyzing optimal monetary policy in different open-economy frameworks, among them Benigno and Benigno (2003), Galì and Monacelli (2005), Faia and Monacelli (2008), de Paoli (2009), Bhattarai and Egorov (2016), Groll and Monacelli (2020), Fujiwara and Wang (2017), or more recently Egorov and Mukhin (2019).

(*e.g.* Oatley (2010), Gunnar and Francois (2006), Bown and Crowley (2013), among others). In a theoretical model Eaton and Grossman (1985) study optimal tariffs when international asset markets are incomplete and show that they can be used to partly compensate the lack of consumption insurance. Bergin and Corsetti (2008) also consider tariffs as policy instruments in addition to monetary policy but their focus is not specifically on tariffs, rather on the implications of monetary policy on the building of comparative advantages. Campolmi, Fadinger, and Forlati (2014) offer a detailed analysis of optimal non-cooperative policies with a large set of instruments, including tariffs.[10] In a rich model with endogenous location of firms and an extensive margin of trade, they show that the terms-of-trade externality remains the dominating incentive to apply positive tariffs. Bagwell and Staiger (2003) propose a trade model featuring potential terms-of-trade manipulation by governments, and trade agreements as means to restrict this policy option. Our paper is complementary to theirs. Most importantly, we incorporate endogenous tariff formation within a standard open-economy macroeconomic model, showing the importance of price stickiness, the exchange rate regime, the extent of cooperation, ZLB constraints on nominal interest rates or dominant currency pricing for the equilibrium degree of trade protection.

3 The Model

We first describe a simplified two-country model with balanced trade. The full quantitative model is described in Section 4 below. There are two countries, Home and Foreign, where agents supply labor and consume each other's goods. The world is populated with a unit mass of agents and for now, countries are equally sized. For now also, firms set prices in the currency of the producer (PCP), but are constrained by Rotemberg-style price adjustment costs.

3.1 Households

Agents in the Home country have preferences over consumption and hours given by

$$U_t = u(C_{ht}, C_{ft}) - \ell(H_t) \tag{1}$$

where u is a continuous, twice differentiable, and satisfies $u_{c_{ii}} < 0$ and $u_{c_{ij}} \geq 0$, for $i = h,\ f,\ i \neq j$. Consumption of the Home (Foreign) good is C_{ht} (C_{ft}). $\ell(.)$ is a continuous differentiable function of hours worked, satisfying $\ell'(.) > 0$, and $\ell''(.) > 0$.

The Home country budget constraint is:

$$P_{ht}C_{ht} + (1+\tau_t)S_t P^*_{ft} C_{ft} = W_t H_t + \Pi_t + TR_t \tag{2}$$

[10] More generally, our paper also relates to the literature on tax and structural reforms to manipulate the exchange, which includes Correia, Nicolini, and Teles (2008), Hevia and Nicolini (2013), Farhi, Gopinath, and Itskhoki (2014), Eggertsson, Ferrero, and Raffo (2014), Cacciatore et al. (2016), Auray, Eyquem, and Ma (2017) or Barbiero et al. (2019).

where P_{ht} (P^*_{ft}) is the Home (Foreign) goods price in Home (Foreign) currency, S_t is the exchange rate, τ_t is an import tariff imposed by the Home government, W_t is the Home nominal wage, Π_t represents the profits of the Home firm and TR_t is a lump-sum transfer from the Home government. Optimal choices over consumption and hours lead to the conditions:

$$u_{c_{ht}} \frac{(1+\tau_t)S_t P^*_{ft}}{P_{ht}} = u_{c_{ft}} \quad (3)$$

$$\ell'(H_t) = u_{c_{ht}} \frac{W_t}{P_{ht}} \quad (4)$$

3.2 Firms

Firms produce differentiated goods.[11] The aggregate Home good is a composite of these differentiated goods, where the elasticity of substitution between individual goods is denoted as $\epsilon > 1$. For now, assume that the firm's production depends only on domestic labor. Output of firm i in the Home country is:

$$Y_t(i) = A_t H_t(i) \quad (5)$$

where A_t is a measure of aggregate productivity. The profits of Home firm i are then represented as:

$$\Pi_t(i) = \left((1+s)P_{ht}(i) - \frac{W_t}{A_t} \right) Y_t(i) \quad (6)$$

where $\Pi_t(i)$ is the price set by firm i and s a sales subsidy. Firm i chooses its price to maximize the present value of its expected profits, net of price adjustment costs:

$$\mathbb{E}_t \sum_{j=0}^{\infty} \omega_{t+j} \left(\Pi_{t+j}(i) - \xi \left(\frac{P_{ht+j}(i)}{P_{ht+j-1}(i)} \right) P_{ht+j} Y_{t+j} \right) \quad (7)$$

where ω_t is the firm's nominal stochastic discount factor, and $\xi(.)$ represents a price adjustment cost function for the firm. We assume that $\xi'(.) > 0$ and $\xi''(.) > 0$. Price adjustment costs are proportional to the nominal value of Home output, to be consistent with the nominal profit objective function of the firm. The first-order condition for profit maximization gives:

$$\begin{aligned} (1+s)Y_t(i) &= \epsilon \left(P_{ht}(i)(1+s) - \frac{W_t}{A_t} \right) \frac{Y_t(i)}{P_{ht}(i)} + \xi' \left(\frac{P_{ht}(i)}{P_{ht-1}(i)} \right) \frac{1}{P_{ht}(i)} P_{ht} Y_t \\ &+ \mathbb{E}_t \omega_{t+1} \xi' \left(\frac{P_{ht+1}(i)}{P_{ht}(i)} \right) \frac{P_{ht+1}(i)}{P_{ht}(i)^2} P_{ht+1} Y_{t+1} = 0 \end{aligned} \quad (8)$$

3.3 Economic Policy

There are three separate levers of policy in this model. Fiscal policy may be used to subsidize monopoly firms (s). Trade policy may be used to levy tariffs on imports (τ_t and τ^*_t), and monetary

[11] We describe the situation of Home firms, noting that Foreign firms behave similarly.

policy may be used to either target inflation rates or exchange rates. In the case where firms are subsidized ($s \neq 0$), we assume that a fiscal authority sets the subsidy to offset the steady-state monopoly markup ($s = \frac{1}{\epsilon - 1}$). But we also allow for the possibility that the monopoly markup remains as a pre-existing distortion in the economy ($s = 0$). As is shown below, this may have an important implications for both optimal monetary policy and trade policy.

3.3.1 Monetary Policy

In the baseline model, it is assumed that the monetary authorities choose an inflation rate for the domestic good. Implicitly, we are assuming that the authorities can implement a desired inflation rate through an interest rate policy, but we abstract from the details of this policy. In a later section, we look at the case where monetary policy is represented by an exchange-rate target on the part of the Foreign government, leaving the Home country to independently choose an inflation rate.

3.3.2 Trade Policy

Trade policy is represented by tariffs chosen by each country. Tariff rates are chosen to maximize domestic welfare. In this scenario, countries engage in a 'trade war', where equilibrium tariff rates are determined in a Nash equilibrium. But in an economy with sticky prices and optimally determined monetary policy, an important determinant of the outcome of trade wars is the relationship between the domestic monetary authority and the trade authority. In the Nash equilibrium of the game between countries (as described below), we assume that both inflation and tariffs are chosen simultaneously by a domestic policy-maker to maximize domestic welfare. But the result would be the same if we thought of monetary and trade policy as determined (simultaneously) separately by a monetary and fiscal authority.

In our model, the only motive to levy tariffs is to affect the terms of trade. While the literature has explored many other reasons for countries to apply trade restrictions, Bagwell and Staiger (2010) argue that terms of trade manipulation is the most important and empirically relevant driver of tariffs.[12]

We also explore the implications of cooperation in monetary policy, assuming that tariffs are still chosen independently. [13] In all cases, regardless of the assumptions about trade and monetary policy, we assume that policy is discretionary. This means that policy-makers maximize *current* welfare, taking as given that future policy-makers will behave in a similar fashion.

[12] Broda, Limao, and Weinstein (2008) find that countries systematically set higher tariffs on imports with more inelastic supply schedules.

[13] This assumption is natural, since cooperative tariffs would always be zero in a symmetric equilibrium of our model, where trade policy to be determined jointly by policy-makers.

3.3.3 Government Budget constraint

While the assumptions about the stance of policy differs, the representation of the consolidated government budget constraint is the same in all situations. The government in each country balances its budget. Tariffs generate revenues, while subsidies represent a cost paid to domestic firms. The difference is rebated back to domestic households in the form of lump-sum transfers. Hence, for the Home country we have

$$TR_t = \tau_t S_t P^*_{ft} C_{ft} - sP_{ht} Y_t \tag{9}$$

where the last expression on the right hand side represents total subsidies paid to all domestic firms.

3.4 The Competitive Equilibrium

The full description of the competitive equilibrium is described in Appendix A. Monetary policy is represented by the PPI inflation rates set by each policy-maker, $\pi_{ht} = \frac{P_{ht}}{P_{ht-1}}$ and $\pi^*_{ft} = \frac{P^*_{ft}}{P^*_{ft-1}}$. In addition, we can define the terms of trade as $\mathcal{S}_t = \frac{S_t P^*_{ft}}{P_{ht}}$. Appendix A shows that, conditional on monetary policies $\left\{\pi_{ht}, \pi^*_{ft}\right\}$ and tariff policies $\{\tau_t, \tau^*_t\}$, the equilibrium can be written in the form of 7 equations in the variables H_t, H^*_t, C_{ht}, C_{ft} C^*_{ht}, C^*_{ft} and $\mathcal{S}_t$.

$$\begin{aligned}
\text{Balance of Payments} &: \quad C^*_{ht} = \mathcal{S}_t C_{ft} && (10)\\
\text{Home Market clearing} &: \quad A_t H_t (1 - \frac{\phi}{2}(\pi_{ht} - 1)^2) = C_{ht} + C^*_{ht} && (11)\\
\text{Foreign Market clearing} &: \quad A_t H^*_t (1 - \frac{\phi}{2}(\pi^*_{ft} - 1)^2) = C_{ft} + C^*_{ft} && (12)\\
\text{Home labor market} &: \quad \ell'(H_t) = A_t u_{c_{ht}} \mathbb{E}_t \Psi(\pi_{ht}, \pi_{ht+1}, \theta) && (13)\\
\text{Foreign labor market} &: \quad \ell'(H^*_t) = A^*_t u_{c^*_{ft}} \mathbb{E}_t \Psi(\pi^*_{ft}, \pi^*_{ft+1}, \theta^*) && (14)\\
\text{Foreign optimal spending} &: \quad u_{c^*_{ht}} \mathcal{S}_t = (1 + \tau^*_t) u_{c^*_{ft}} && (15)\\
\text{Home optimal spending} &: \quad u_{c_{ht}} (1 + \tau_t) \mathcal{S}_t = u_{c_{ft}} && (16)
\end{aligned}$$

where we have used the assumption that the quadratic cost of price adjustment is $\frac{\phi}{2}(\pi_{ht} - 1)^2$ for the Home country and analogously for the Foreign country. We define $\Psi_t = \theta + \phi\pi_{ht}(\pi_{ht} - 1) - \phi\beta\pi_{ht+1}(\pi_{ht+1} - 1)$ (and analogously for Foreign), which represents the impact of price adjustment costs on the firm's profit maximization, and $\theta = \frac{(1+s)(\epsilon-1)}{\epsilon}$ is a subsidy-adjusted measure of the monopoly distortion, with $\theta = 1$ if an optimal subsidy $s = \frac{1}{\epsilon-1}$ is in place. If current and future inflation is zero, and the optimal subsidy is in place, then $\Psi = 1$.[14]

[14] Here we simplify by assuming the firm's discount factor for the expected future inflation cost is constant at β. This makes little difference to the example.

3.5 Optimal Monetary Policy: Currency Wars

We first analyze the determination of optimal inflation rates in a discretionary Nash equilibrium. Each government chooses its inflation rate to maximize domestic welfare, subject to competitive equilibrium conditions, taking the inflation rate of the other government as given. Without trade in financial assets, there are no endogenous state variables in the model, so a discretionary (time-consistent) Nash equilibrium can be described simply by each government's choice of current-valued variables, taking future inflation rates, consumption levels, output levels and terms of trade as given (assuming $\tau_t = \tau_t^* = 0$).[15] Let the current state be defined as $\mathcal{Z}_t = (A_t, A_t^*)$, representing aggregate TFP. Define the firm's value function as $v(\mathcal{Z}_t)$. In a discretionary Nash equilibrium in monetary policy the Home government chooses $\{C_t, C_t^*, H_t, H_t^*, \mathcal{S}_t, \pi_{ht}\}$ to maximize

$$v(\mathcal{Z}_t) = u(C_{ht}, C_{ft}) - \ell(H_t) + \beta \mathbb{E}_t v(\mathcal{Z}_{t+1}), \tag{17}$$

subject to Equations (10)-(16), while the Foreign firm chooses $\{C_t, C_t^*, H_t, H_t^*, \mathcal{S}_t, \pi_{ft}^*\}$ to maximize

$$v^*(\mathcal{Z}_t) = u(C_{ht}^*, C_{ft}^*) - \ell(H_t^*) + \beta \mathbb{E}_t v^*(\mathcal{Z}_{t+1}), \tag{18}$$

subject to Equations (10)-(16). Since the model is symmetric, we state results for the Home country alone. The Nash equilibrium implies equivalent results for the Foreign country. Let $\xi_{1,t}$ .. $\xi_{7,t}$ denote the Home country Lagrange multipliers on constraints (10)-(16) respectively. Then Appendix A shows that, in a steady-state Nash equilibrium of the currency war game, the inflation rate of the Home government is implicitly characterized by the condition:

$$\Psi = \frac{1 - \frac{\phi}{2}(\pi_h - 1)^2 - \frac{(\pi_h - 1)}{(2\pi_h - 1)} \psi \Psi}{1 - \frac{AH(\pi_h - 1)}{u_{c_h}(2\pi_h - 1)} u_{c_{hh}} \Psi - \frac{\xi_7}{\xi_2}(\mathcal{S} u_{c_{hh}} - u_{c_{hf}})} \tag{19}$$

where $\psi = \frac{H\ell''(H)}{\ell'(H)}$ is the inverse of the Frisch elasticity of labor supply, and $u_{c_{hh}}$ indicates the second derivative of the utility function and $\Psi = \theta + \phi \pi_h (\pi_h - 1)(1 - \beta)$. From this expression, we obtain the following results (detailed in Appendix A):

Proposition 1. *The Nash equilibrium in the currency war game between countries will generically not imply price stability (zero inflation).*

Proof. Appendix A shows that $\pi_h = 1$, is a generically not a solution to Equation (19).

Proposition 2. *When $s = \frac{1}{\epsilon - 1}$, the steady-state Nash equilibrium of the currency war game implies that both countries choose negative inflation rates.*

[15] In the extensive description of the model, nominal price levels are state variables. But since monetary policy is implemented by the choice of inflation rates, current policy makers take future inflation as chosen by future policy, so current price levels have no relevance for the evaluation of future welfare.

Proof. When $s = \frac{1}{\epsilon - 1}$, Appendix A shows that $\frac{\xi_7}{\xi_2} > 0$. Then the only solution to (19) must involve $\pi_h < 1$ (and therefore also $\pi_f^* < 1$).

This case recalls well-known results in the open economy macro literature. Corsetti and Pesenti (2001) and Clarida, Gali, and Gertler (2002) characterize the optimal monetary policy in an open economy as a tension between the desire to eliminate domestic distortions associated with monopoly pricing on the one hand, and the desire to manipulate the terms of trade for strategic advantage on the other. These conflicting objectives could result in either a positive or a negative inflation rate. When the optimal subsidy is in place, each policy maker focuses only on the terms-of-trade objective, and inflation is negative in a Nash equilibrium of the currency war game.

This example is also reminiscent of Rogoff (1985), who shows that international policy cooperation can reduce welfare in the case of monopoly distortions, since it eliminates the terms-of-trade externality, and may lead to excessive Nash inflation in the currency war game.[16] In the quantitative analysis below, we confirm this result. With an optimal subsidy in place, however, international cooperation eliminates disinflation and raises welfare relative to the currency war Nash equilibrium. It is straightforward to extend Result 2 to show that in a symmetric cooperative equilibrium with optimal subsidies inflation rates are zero in each country.

3.6 Optimal Monetary and Trade Policy: Trade Wars and Currency Wars

When governments choose both inflation and tariffs, the problem of the Home government is to choose $\{C_t, C_t^*, H_t, H_t^*, \mathcal{S}_t, \pi_{ht}, \tau_t\}$ to maximize (17) subject to Equations (10)-(16).

In this extended environment, Appendix A shows that the determination of tariffs and inflation in the Home economy may be represented by the conditions:

$$\frac{1}{1+\tau_t} = \frac{1 - \frac{A_t H_t (\pi_{ht}-1)}{u_{c_{ht}} (2\pi_{ht}-1)} u_{c_{hht}} \Psi(\pi_{ht}, \mathbb{E}_t \pi_{ht+1})}{\frac{\eta_t}{\eta_t - 1} - \frac{A_t H_t (\pi_{ht}-1)}{\mathcal{S}_t u_{c_{ht}} (2\pi_{ht}-1)} u_{c_{hft}} \Psi(\pi_{ht}, \mathbb{E}_t \pi_{ht+1})} \tag{20}$$

$$\Psi(\pi_{ht}, \mathbb{E}_t \pi_{ht+1}) = \frac{1 - \frac{\phi}{2}(\pi_{ht} - 1)^2 - \frac{(\pi_{ht}-1)}{(2\pi_{ht}-1)} \psi \Psi(\pi_{ht}, \mathbb{E}_t \pi_{ht+1})}{1 - \frac{A_t H_t (\pi_{ht}-1)}{u_{c_{ht}} (2\pi_{ht}-1)} u_{c_{hht}} \Psi(\pi_{ht}, \mathbb{E}_t \pi_{ht+1})} \tag{21}$$

where η_t is the foreign country's general equilibrium elasticity of demand for home goods.[17]

[16] As noted above, **?** shows this within a simplified non-microfounded open economy model, but the result generalizes to more recent new open economy DSGE models.

[17] In particular,

$$\eta_t = \frac{\frac{(u^*_{c_{fft}}(1+\tau_t^*) - u^*_{c_{hft}} \mathcal{S}_t) c^*_{ht}}{u^*_{c_{ht}} \mathcal{S}_t^2 \left(1 - \frac{A_t^{*2}(1-\varphi_t^*)\Psi_t^* u^*_{c_{fft}}}{\ell''(H_t^*)}\right)} - 1}{\frac{(u^*_{c_{fft}}(1+\tau_t^*) - u^*_{c_{hft}} \mathcal{S}_t) c^*_{ht}}{u^*_{c_{ht}} \mathcal{S}_t^2 \left(1 - \frac{A_t^{*2}(1-\varphi_t^*)\Psi_t^* u^*_{c_{fft}}}{\ell''(H_t^*)}\right)} \left(1 - \mathcal{S}_t \frac{A_t^{*2}(1-\varphi_t^*)\Psi_t^* u^*_{c_{hft}}}{\ell''(H_t^*)}\right) + \frac{(u^*_{c_{hht}} \mathcal{S}_t - u^*_{c_{hft}}(1+\tau_t^*)) c^*_{ht}}{u^*_{c_{ht}} \mathcal{S}_t}}$$

Equation (20), shows that the optimal tariff follows an amended 'monopoly tariff' formula. The pure monopoly tariff would be set at $\tau_t = \frac{1}{\eta_t - 1}$, the reciprocal of the demand elasticity minus one. But in the presence of inflation and monopoly distortions in domestic production, the tariff in general will be lower than this value. If inflation is positive, then $\tau_t < \frac{1}{\eta_t - 1}$, since positive inflation distorts output, and tariffs creates further distortions. Equations (20) and (21) then imply:

Proposition 3. *In a steady-state Nash equilibrium of the trade and currency war game, when* $s = s^* = \frac{1}{\eta - 1}$*, the Home and Foreign country will set inflation rates to zero and tariffs are given by the pure monopoly tariff formula* $\frac{1}{\eta - 1}$ *and* $\frac{1}{\eta^* - 1}$.

Proof. From Equation (21), with $s = \frac{1}{\epsilon - 1}$, we obtain $\pi_h = 1$ in a steady state. Then from Equation (20) we obtain $\tau = \frac{1}{\eta - 1}$ and likewise for the Foreign country.

Proposition 4. *In the presence of monopoly distortions, the stationary Nash equilibrium of the trade and currency war game exhibits positive inflation rates and tariffs rates lower than the monopoly tariff.*

Proof. When $s = 0$, the left-hand side of Equation (21) is less than unity in a steady state with $\pi_h = 1$, while the right-hand side equals unity. Since the left-hand side (right-hand side) is increasing (decreasing) in π_h, the solution then must involve $\pi_h > 1$. Then from Equation (20), since $u_{c_{hh}} < 0$ and $u_{c_{hf}} \geq 0$, the right-hand side must be greater than $\frac{1}{\frac{\eta}{\eta - 1}}$, so $\tau < \frac{1}{\eta - 1}$, and analogously for the Foreign country.

Result 4 above points out the interrelationship between trade policy and monetary policy in a distorted economy. Monopoly distortions tend to reduce the degree of protectionism, while increasing the inflation rate. By contrast to the currency war example, eliminating monopoly distortions fully removes deflation bias and leads to zero inflation but a by-product is that equilibrium tariff rates increase as shown by Result 3. In the quantitative analysis below, we show that the rise in tariffs following the removal of domestic distortions may be large.

The above analysis suggests that trade wars lead to higher equilibrium inflation rates in a currency war. When tariffs are used to exploit terms of trade externalities, inflation rates focus more on domestic distortions. We can also ask how trade wars would play out with an alternative scenario for monetary policy. We explore this by assuming that monetary authorities follow a passive policy of zero inflation, limiting strategic interaction to trade policy alone. This case is analyzed in Appendix A. There we establish the following.

Proposition 5. *When monetary policy is limited to price stability,* ($\pi_{ht} = \pi^*_{ft} = 1$)*, the optimal tariff in the steady-state Nash equilibrium of the trade war game is described by:*

$$\frac{1}{1 + \tau} = \frac{1 + \Omega u_{c_{hh}} \theta}{\frac{\eta_t}{\eta_t - 1} + \Omega u_{c_{hf}} \theta} \tag{22}$$

where $\varphi^*_t \equiv \frac{\phi}{2}(\pi^*_{ft} - 1)^2$.

where $\Omega = \frac{(\theta-1)A}{\frac{\ell'}{A} - u_{c_{hh}} A\theta^2}$, *and* $\theta = \frac{(1+s)(\epsilon-1)}{\epsilon} \leq 1$.

Proof. See Appendix A.

From Equation (22) we conclude that, in a distorted economy where $\theta < 1$ and without active monetary policy, the steady-state Nash equilibrium tariff in the currency war game will be less than the pure monopoly tariff rate. Intuitively, this is because policy-makers take account of the distortionary impacts of the tariff on domestic production, which is inefficiently low when $\theta < 1$. In the quantitative analysis below, we compare the equilibrium tariff rates in the model with constrained monetary policies with those of an economy where policy-makers choose inflation rates optimally. We find that tariffs are significantly lower in the constrained case. When tariffs are the only instrument to exploit terms-of-trade externalities and respond to domestic distortions, the optimal degree of protection is reduced.[18]

In the case of the currency war above, we noted that international monetary policy cooperation would raise welfare in the absence of monopoly distortions. We find that this is not the case in the currency and trade war equilibrium. Indeed, we may state the following.

Proposition 6. *In a stationary Nash equilibrium of the currency and trade war game when each country uses a subsidy to offset monopoly distortions, international monetary policy cooperation will lead to positive rates of inflation and tariff rates above the monopoly tariff level.*

Proof. Appendix A shows that the inflation rate of the Home (and Foreign) country in the steady state of a symmetric cooperative equilibrium is characterized by the condition

$$\Psi = \frac{1 - \frac{\phi}{2}(\pi_h - 1)^2 - \frac{(\pi_h-1)}{(2\pi_h-1)}\psi\Psi}{1 - \frac{AH(\pi_h-1)}{u_{c_{ht}}(2\pi_h-1)} u_{c_{hh}}\Psi - \frac{\xi_6}{\xi_2}(u_{c_{hh}}(1+\tau) - u_{c_{hf}})} \tag{23}$$

where

$$\frac{\xi_6}{\xi_2} = \frac{u_{c_h}\tau}{\xi_2(u_{c_{hh}}(1+\tau) + u_{c_{ff}} - 2u_{c_{hf}})} + \frac{(u_{c_{hh}} - u_{c_{hf}})\Psi AH(\pi_h - 1)}{u_{c_h}(2\pi_h - 1)} \tag{24}$$

with $\xi_2 = \frac{\ell'(H)}{A(1-\frac{\phi}{2}(\pi_h-1)^2) - \frac{\ell''(H)H(\pi_h-1)}{u_{c_h}(2\pi_h-1)}}$.

Let us start with inflation. Assume that $\theta = 1$, so that the optimal subsidy is applied to correct the monopoly distortion. If $\pi_h = 1$ the left-hand side of Equation (23) is unity, while the right-hand side is greater than unity, using Equation (24), as long as there is a positive tariff rate, *i.e.* $\tau > 0$. Since the left-hand side is increasing in π_h and the right-hand side is decreasing in π_h, it must be that the equilibrium cooperative inflation rate is greater than zero when $\theta = 1$ and $\tau > 0$.

[18] This case is actually equivalent to that of a flexible price economy (where $\phi = 0$), since zero inflation rates in this model replicate the flexible price equilibrium.

Now focus on tariffs. Appendix A shows that in a symmetric steady state with international cooperation in monetary policy, the Home (and Foreign) tariff rate is given by:

$$\frac{1}{1+\tau} = \frac{1 + \frac{\xi_4}{\xi_2} u_{c_{hh}} \Psi}{\frac{\eta}{\eta - 1} + \frac{\xi_4}{\xi_2} u_{c_{hf}} \Psi} \tag{25}$$

where again, the Foreign demand elasticity is η. In addition, it is shown that $\xi_4 = \frac{\ell'(H) - Au_{c_h}(1-\frac{\phi}{2}(\pi_h - 1)^2)}{\frac{\ell'(H)}{A} - u_{c_{hh}} \Psi A (1 - \frac{\phi}{2}(\pi_h - 1)^2)}$ and $\xi_2 = u_{c_h} - u_{c_{hh}} \Psi \frac{(\ell'(H) - Au_{c_h}(1-\frac{\phi}{2}(\pi_h-1)^2))}{\frac{\ell'(H)}{A} - u_{c_{hh}} \Psi A (1-\frac{\phi}{2}(\pi_h-1)^2)}$. When $\theta = 1$, then from Equation (14), in a steady state, we have $\ell'(H_t) - A_t u_{c_{ht}}(1 - \frac{\phi}{2}(\pi_{ht-1})^2) = Au_{c_h}\left(\phi \pi_h (\pi_h - 1) + \frac{\phi}{2}(\pi_h - 1)^2\right) > 0$. Since $\frac{\xi_4}{\xi_2} > 0$ it follows from Equation (25) that in the case $\theta = 1$, and monetary policy is determined cooperatively, the tariff rate exceeds the monopoly tariff rate.

To follow the intuition for Result 6, look at Equation (14). When $\theta = 1$ and inflation is zero, output is determined by $\ell'(h) = Au_{c_h}$ in the Home country and similarly in the Foreign country. The presence of tariffs distort the pattern of consumption in both countries, reducing output, for a given inflation rate. The cooperative policy-makers increase inflation above zero, raising equilibrium output. *Ceteris paribus* however, this tends to reduce the terms of trade for each country, and non-cooperative tariff authorities respond by increasing tariff rates.

Result 6 represents an interesting addition to the classic Rogoff (1985) result. Result 3 and 4 above showed that with an optimal monopoly subsidy, the Nash equilibrium implied zero inflation and tariff rates equal to the monopoly tariff level. But if the policy choice is separated so that monetary policy is chosen cooperatively, policy-makers will increase inflation rates to offset the distortion generated by tariffs. Simultaneously, acting individually, tariff setters will increase their tariffs since the perceived external monopoly strength increases as inflation is higher. In the quantitative analysis below, we show that, even in the case of an optimal domestic monopoly subsidy, welfare may be reduced by international monetary policy cooperation, since it leads to higher tariffs and higher inflation rates.

4 Quantitative Results

We now extend the analysis to a more general model allowing for CES preferences, production using intermediate goods, trade in intermediate goods, home bias, potential differentials in country size and trade in bonds. In particular, we assume that period utility is now

$$U_t = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{\chi}{1+\psi} H_t^{1+\psi} \tag{26}$$

where:

$$C_t = \left(\varepsilon^{\frac{1}{\lambda}} C_{ht}^{1-\frac{1}{\lambda}} + (1-\varepsilon)^{\frac{1}{\lambda}} C_{ft}^{1-\frac{1}{\lambda}} \right)^{\frac{1}{1-\frac{1}{\lambda}}} \tag{27}$$

where $\varepsilon \geq n$, representing the possibility of home bias in preferences.[19] Given this, the true price index for the Home consumer becomes

$$P_t = \left(\varepsilon P_{ht}^{1-\lambda} + (1-\varepsilon)((1+\tau_t) S_t P_{ft}^*)^{1-\lambda} \right)^{1/(1-\lambda)}. \tag{28}$$

Firms now use domestic and imported intermediate goods in production, so the production function for home firm i becomes:

$$Y_t(i) = A_t H_t(i)^{1-\alpha} X_t(i)^{\alpha}. \tag{29}$$

Here, $X_{i,t}$ represents the use of intermediate goods on the part of the Home firm i and $H_{i,t}$ the use of labor. We allow that intermediate good inputs are composed of Home and Foreign goods in a different composition than that of the consumption aggregator. Namely

$$X_t(i) = \left(\varepsilon_x^{\frac{1}{\lambda}} X_{ht}(i)^{1-\frac{1}{\lambda}} + (1-\varepsilon_x)^{\frac{1}{\lambda}} X_{ft}(i)^{1-\frac{1}{\lambda}} \right)^{\frac{1}{1-\frac{1}{\lambda}}}. \tag{30}$$

The full description of the competitive equilibrium is set out in Appendix B. Using the notation of Section 3 above, and the definition of the true price index, we define $\frac{P_{ht}}{P_t} = \frac{1}{\mathcal{P}_t}$, where $\mathcal{P}_t = \left(\varepsilon + (1-\varepsilon)((1+\tau_t)\mathcal{S}_t)^{1-\lambda}\right)^{\frac{1}{1-\lambda}}$, and likewise $\mathcal{P}_{x,t} = \left(\varepsilon_x + (1-\varepsilon_x)((1+\tau_t)\mathcal{S}_t)^{1-\lambda}\right)^{\frac{1}{1-\lambda}}$. Then, Appendix B shows that, conditional on monetary policies $\left\{\pi_{ht}, \pi_{ft}^*\right\}$ and tariff policies $\{\tau_t, \tau_t^*\}$, the equilibrium can be written in the form of 7 equations in the 7 variables $Y_t,\ Y_t^*,\ C_t,\ C_t^*, b_t, b_t^*$ and

[19] Letting $0 \leq x \leq 1$ represents the degree of home bias in preferences, where $x = 0$ ($x = 1$) represents zero (full) home bias, we can define $\varepsilon = n + x(1-n)$.

$\mathcal{S}_t$. These are expressed as follows:

$$
\begin{aligned}
(1+s)(1-\epsilon)+\epsilon\mathcal{MC}_t-\phi\left(\pi_{ht}(\pi_{ht}-1)-\mathbb{E}_t\left\{\omega_{t+1}\pi_{ht+1}(\pi_{ht+1}-1)\frac{Y_{t+1}}{Y_t}\right\}\right) &= 0 \quad (31)\\
(1+s)(1-\epsilon)+\epsilon\mathcal{MC}^*_t-\phi\left(\pi^*_{ft}\left(\pi^*_{ft}-1\right)-\mathbb{E}_t\left\{\omega^*_{t+1}\pi^*_{ft+1}\left(\pi^*_{ft+1}-1\right)\frac{Y^*_{t+1}}{Y^*_t}\right\}\right) &= 0 \quad (32)\\
Y_t\left(1-\frac{\phi}{2}(\pi_{ht}-1)^2\right)-D_t-D^*_{xt} &= 0 \quad (33)\\
Y^*_t\left(1-\frac{\phi}{2}\left(\pi^*_{ft}-1\right)^2\right)-D^*_t-D_{xt} &= 0 \quad (34)\\
nb_t+(1-n)\frac{\mathcal{S}_t\mathcal{P}^*_t}{\mathcal{P}_t}b^*_t &= 0 \quad (35)\\
\mathbb{E}_t\left\{\frac{\mathcal{S}_{t+1}\omega_{t+1}}{\mathcal{S}_t\omega^*_{t+1}(1+\nu(b_t-b))}-1\right\} &= 0 \quad (36)\\
b_t-\frac{\mathcal{S}_t\mathcal{P}_{t-1}}{\mathcal{S}_{t-1}\mathcal{P}_t\omega^*_t}b_{t-1}-\mathcal{P}_t^{-1}\left(D^*_{xt}-\frac{1-n}{n}\mathcal{S}_tD_{xt}\right) &= 0 \quad (37)
\end{aligned}
$$

where

$$
D_t=\varepsilon\mathcal{P}_t^{\lambda}(C_t+\Lambda_t)+\varepsilon_x\mathcal{P}_{xt}^{\lambda}X_t\ ;\ D_{xt}=\frac{n}{1-n}\left(\frac{\mathcal{S}_t^{-1}}{1+\tau_t}\right)^{\lambda}\left((1-\varepsilon)\mathcal{P}_t^{\lambda}(C_t+\Lambda_t)+(1-\varepsilon_x)\mathcal{P}_{xt}^{\lambda}X_t\right) \quad (38)
$$

$$
D^*_t=\varepsilon^*\mathcal{P}_t^{*\lambda}C^*_t+\varepsilon^*_x\mathcal{P}_{xt}^{*\lambda}X^*_t\ ;\ D^*_{xt}=\frac{1-n}{n}\left(\frac{\mathcal{S}_t}{1+\tau^*_t}\right)^{\lambda}\left((1-\varepsilon^*)\mathcal{P}_t^{*\lambda}C^*_t+(1-\varepsilon^*_x)\mathcal{P}_{xt}^{*\lambda}X^*_t\right) \quad (39)
$$

stand for the internal and external demands for final and intermediate goods, and where $\omega_t=\beta\frac{C^{\sigma}_{t-1}\mathcal{P}_{t-1}}{C^{\sigma}_t\mathcal{P}_t}$; $\omega^*_t=\beta\frac{C^{*\sigma}_{t-1}\mathcal{P}^*_{t-1}}{C^{*\sigma}_t\mathcal{P}^*_t}$. Equations (31) and (32) are the Home and Foreign inflation equations, where the marginal cost functions can be expressed as

$$
\mathcal{MC}_t=\frac{\left(\mathcal{P}_t\chi H_t^{\psi}C_t^{\sigma}\right)^{1-\alpha}\mathcal{P}_{xt}^{\alpha}}{A_t\alpha^{\alpha}(1-\alpha)^{1-\alpha}},\mathcal{MC}^*_t=\frac{\left(\mathcal{P}^*_t\chi H_t^{*\psi}C_t^{*\sigma}\right)^{1-\alpha}\mathcal{P}_{xt}^{*\alpha}}{A_t\alpha^{\alpha}(1-\alpha)^{1-\alpha}} \quad (40)
$$

with

$$
H_t=\left(\frac{(1-\alpha)\left(\mathcal{P}_t\chi C_t^{\sigma}\right)^{-\alpha}\mathcal{P}_{xt}^{\alpha}Y_t}{A_t\alpha^{\alpha}(1-\alpha)^{1-\alpha}}\right)^{\frac{1}{1+\alpha\psi}},\ H^*_t=\left(\frac{(1-\alpha)\left(\mathcal{P}^*_t\chi C_t^{*\sigma}\right)^{-\alpha}\mathcal{P}_{xt}^{*\alpha}Y^*_t}{A^*_t\alpha^{\alpha}(1-\alpha)^{1-\alpha}}\right)^{\frac{1}{1+\psi\alpha}} \quad (41)
$$

$$
X_t=\frac{\alpha\left(\mathcal{P}_t\chi H_t^{\psi}C_t^{\sigma}\right)^{1-\alpha}\mathcal{P}_{xt}^{\alpha-1}Y_t}{A_t\alpha^{\alpha}(1-\alpha)^{1-\alpha}},\ X^*_t=\frac{\alpha\left(\mathcal{P}^*_t\chi H_t^{*\psi}C_t^{*\sigma}\right)^{1-\alpha}\mathcal{P}_{xt}^{*\alpha-1}Y^*_t}{A^*_t\alpha^{\alpha}(1-\alpha)^{1-\alpha}} \quad (42)
$$

Equations (33) and (34) are the goods market clearing conditions for the two countries. Equation (35) represents the equilibrium on international bonds market, Equation (36) is the modified IUP condition while Equation (37) is the balance of payment equation that gives the dynamics of net foreign assets.

4.1 Calibration

We now derive the solution to the optimal policy games in the full model. The model is calibrated to an annual frequency. The discount factor of households is $\beta = 0.96$, consistent with a real interest rate of 4% *per annum*. Both countries are of similar size in the baseline calibration so that $n = 0.5$. Further, we assume a home bias parameter $x = 0.7$ which implies $\varepsilon = \varepsilon_x = (1 - \varepsilon^*) = (1 - \varepsilon_x^*) = 0.85$. With zero tariffs, this number is associated with a 30% total trade openness ratio, as in U.S. data. We consider a baseline value of $\sigma = 1$, implying a log utility for consumption, but also examine alternative values of σ. The Frisch elasticity is $\psi^{-1} = 1$ and we normalize $\chi = 1$. The elasticity of substitution between varieties is $\epsilon = 6$, consistent with a 20% steady-state price-cost mark-up when not corrected by a steady-state subsidy and the Rotemberg parameter is $\phi = 40$. Following Itskhoki and Mukhin (2021), we consider a $\alpha = 0.5$ share of intermediate goods in production. Last, the trade elasticity is $\lambda = 3$. This is on the higher end of the range estimated by Feenstra et al. (2018), but is more appropriate for the evaluation of trade policy. The bond adjustment cost parameter suggested by Ghironi and Melitz (2005) is 0.0025 in a quarterly set-up which, in our annual set-up, implies $\nu = 0.01$. Finally, the baseline results are derived under the assumption that trade is balanced in the steady state, *i.e.* $b = b^*$, an assumption that will be relaxed later on.

4.2 Currency Wars

Table 1 describes the steady state outcome of the Nash and Cooperative equilibrium where policy-makers choose only inflation rates. As described above, in the Nash equilibrium each country faces a trade-off. A positive rate of inflation reduces the monopoly pricing distortion, while disinflation reduces output and appreciates the terms of trade, thus partly substituting for the absence of direct trade policy instruments. For the particular calibration in Table 1, the first motive dominates, and the Nash equilibrium inflation rate is 4.02 percent. By contrast, with cooperative monetary policy the terms of trade motive is eliminated, and each country chooses a much higher positive rate of inflation of 4.84 percent. As expected, monetary policy cooperation is welfare reducing.

If subsidies are in place to offset the monopoly distortion, then Table 1 confirms Result 2 above in Section 3. Each country follows a deflationary monetary policy, since the terms-of-trade motive then fully dominates the incentives for inflation in each country. By contrast, if optimal subsidies are in place, and monetary policy is chosen cooperatively, inflation rates are zero, then the equilibrium is first-best, since all distortions are eliminated.

These results support the contention 'currency wars' may be either good or bad. If there is a pre-existing monopoly distortion, cooperation in monetary policy may be undesirable, whereas

Table 1: Currency wars

	Currency war							
	No subsidy ($s = 0$)				Subsidy ($s = 1/(\epsilon - 1)$)			
	Base.	Coop M.	Flex. P	Fixed XR	Base.	Coop M.	Flex. P	Fixed XR
$\pi_h = \pi_f^*$	1.040	1.049	–	1.049	0.9898	1.0000	–	1.0000
$\tau = \tau^*$	–	–	–	–	–	–	–	–
$\mathcal{S}$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$C = C^*$	0.203	0.201	0.205	0.201	0.249	0.250	0.250	0.250
$H = H^*$	0.880	0.894	0.845	0.894	0.999	1.000	1.000	1.000
Home/For. welf. loss (%)	9.265	11.326	5.214	11.326	0.419	0.000	0.000	0.000

Note: Base. denotes the baseline Nash equilibrium, Coop M. the equilibrium where monetary policy is chosen cooperatively and Flex. P the allocations under flexible prices.

with optimal subsidies in place, cooperation supports the first best outcome.

4.3 Currency Wars and Trade Wars

Table 2 illustrates the allocations and welfare effects of the combined currency war and trade war, where both inflation and tariffs are chosen in a Nash equilibrium. We start with the unilateral case of the Table, which shows the outcome where both countries choose inflation rates non-cooperatively, but the Home country chooses an optimal tariff unilaterally. Under the baseline calibration, without subsidy, the Home country chooses a tariff rate of 41 percent. This generate a 18.7 percent appreciation in its terms of trade and raises Home welfare at the expense of Foreign welfare, comparing the 4th column of the left panel of Table 2 with the corresponding currency war in Table 1. In addition, the tariff raises relative Home consumption, but reduces relative Home output.

Table 2: Trade and Currency Wars

	No subsidy ($s = 0$)					Subsidy ($s = 1/(\epsilon - 1)$)			
	Base.	Coop M.	Flex P.	Unil.	Commit.	Base.	Coop M.	Flex P.	Unil.
π_h	1.047	1.053	–	1.047	1.045	1.000	1.007	–	1.000
π_f^*	1.047	1.053	–	1.043	1.045	1.000	1.007	–	0.994
τ	0.405	0.418	0.354	0.410	0.168	0.518	0.521	0.518	0.525
τ^*	0.405	0.418	0.354	0.000	0.168	0.518	0.521	0.518	0.000
S	1.000	1.000	1.000	0.813	1.000	1.000	1.000	1.000	0.775
C	0.190	0.188	0.195	0.200	0.196	0.234	0.235	0.234	0.249
C^*	0.190	0.188	0.195	0.192	0.196	0.234	0.235	0.234	0.234
H	0.873	0.884	0.829	0.864	0.874	0.976	0.979	0.976	0.962
H^*	0.873	0.884	0.829	0.884	0.874	0.976	0.979	0.976	0.999
Home welf. loss (%)	14.438	16.116	8.666	9.255	11.619	3.994	4.199	3.994	−3.252
Foreign welf. loss (%)	14.438	16.116	8.666	14.473	11.619	3.994	4.199	3.994	6.129

Note: Base. denotes the baseline Nash equilibrium, Coop M. the equilibrium where monetary policy is chosen cooperatively, Flex. P the allocations under flexible prices, Unil. stands for the case with non-cooperative monetary policy and the Home Nash policy-maker sets its tariff while the Foreign tariff is nul. Commit. denotes the case where tariffs are set non-cooperatively but taking into account their effects on monetary policy.

The remaining columns of Table 2 show the results for a trade war, where both countries choose an optimal tariff rate, in addition to an optimal inflation rate, in a discretionary Nash equilibrium. Without optimal subsidies, the trade war leads to mutual tariff rates of 40.5 percent. In the symmetric Nash equilibrium, there is no change in the terms of trade, but the rise in domestic prices leads to a shift back in labor supply which reduces equilibrium employment and output. At the same time, the fall in consumption of imported goods distorts the composition of consumption and leads to a fall in aggregate consumption in both countries. Thus, the trade war has large negative effects on real activity.

Table 2 also shows, however, that the trade war causes a change in equilibrium inflation rates. Absent the trade war, Nash equilibrium inflation rates were 4.02 percent (see Table 1), which as described above, represented a balance between the desire to eliminate monopoly distortions and the desire to improve the national terms of trade. When countries engage in the trade war, optimal tariffs focus on the second objective – terms-of-trade manipulation – and monetary authorities redirect inflation rates towards the first objective. As a result, inflation rates rise to 4.7 percent in the equilibrium with both trade and currency wars.

Table 2 further indicates that the trade war has major implications for welfare. Comparing the Nash discretionary equilibrium of the combined trade and currency wars with that of the Nash equilibrium under the currency war alone (Table 2 compared with similar cases in Table 1) leads to a fall in welfare. Without subsidies, the welfare losses from a currency war are 9.26% of first-best equivalent consumption while the welfare losses from combined trade and currency wars jump to 14.44 percent, a difference of more than 5 percentage points of first-best consumption equivalent.

The second column of Table 2, still in the case of zero subsidies, documents the outcome where policy-makers cooperate on monetary policy, but follow a trade war in the choice of tariffs. As we would anticipate, given the results of Table 1, monetary policy cooperation is again counter-productive. But this is now for two reasons. First, as before, the equilibrium inflation rates increase from 4.7 percent to 5.3 percent, as monetary policy focuses only on eliminating domestic distortions and ignores the impact on the terms of trade. Second, this adjustment in the focus of monetary policy leads to a redirection of tariffs: the trade war becomes more intense, as independent policy-makers increase tariffs to more fully exploit a terms-of-trade advantage. Tariff rates increase to 41.8 percent – against 40.5 percent when monetary policy is non-cooperative – and aggregate consumption falls by 1 percent. We conclude again that eliminating currency wars is undesirable, not just due to higher inflation, but because it also leads to an increase in trade protection.

4.3.1 Equilibrium with zero markups

When markups are removed by a production subsidy, Result 1 in the simple model of Section 3 showed that inflation rates in the currency and trade war Nash equilibrium were zero. This is confirmed in Table 2: imposing an optimal subsidy full eliminates inflation. But the consequence is a substantial increase in protection, as removing the markup distortion leads to a rise in equilibrium tariffs from 40.5 percent to 51.8 percent. Instead of using deflationary monetary policy as in the currency wars case in Table 1, governments now increase tariff rates, consistently with the analytical results of Section 3. Intuitively, in the distorted economy, tariffs are set as a compromise between improving the terms of trade and limiting the distortionary effects on domestic output, which is already inefficiently low due to the presence of markups. Removing markups means that both governments implement the monopoly tariff level as shown in Section 3.

The 7th column of Table 2 quantitatively echoes the conclusions of Result 6 above. Monetary policy cooperation reduces welfare when tariffs are determined non-cooperatively, *even* in the presence of optimal subsidies. Indeed, comparing the 6th and 7th column of Table 2, cooperation in monetary policy leads to a rise in equilibrium inflation rates, and a rise in equilibrium tariff rates. As shown in Section 3 , inflation rates rise as cooperative policy-makers attempt to offset the distortion in the composition of global consumption generated by tariffs. But at the same time, this would reduce the terms of trade for each country and thus leads individual tariff setters to raise their tariff rates in a Nash equilibrium. Thus, eliminating the currency war (without eliminating the trade war) is counter-productive, even in the absence of monopoly pricing distortions.

4.3.2 Flexible price equilibrium

Proposition 4 above showed that in the simplified model, when monetary policy was constrained to stabilize prices, tariffs were lower than the monopoly tariff rate if there were positive monopoly markups.

The 3rd column of Table 2 shows that without production subsidies, equilibrium Nash tariffs are substantially lower under zero-inflation monetary policy. We noted that this outcome is identical to one without any price rigidities (*i.e.* $\phi = 0$), since in this model, the equilibrium under zero inflation is equivalent to a flexible price economy.

With monopoly markup distortions, and when prices are fully flexible, inflation has no traction in either reducing distortions or affecting the terms of trade. Hence, tariffs must be used as a compromise between the two objectives, and in a Nash equilibrium protection is less than in an economy with sticky prices. Again, this observation highlights the main theme of the paper; the critical interaction between monetary policy and trade policy in a non-cooperative environment with nominal price stickiness.

By contrast, without the markup distortion, tariff rates are exactly the same whether prices are flexible or sticky. In this case, as shown in Propositions 3 and 4, tariffs are entirely focused on the terms-of-trade externality.

These results indicate that trade wars imply very high rates of protection. How relevant is the analysis given that, in recent history, observed tariffs among advanced economies have been much lower. For instance, the average size of trade restrictions (including both tariff and non-tariff barriers) reported by UNCTAD (2013) for advanced economies is approximately 10 percent. These observations are taken from a period where WTO rules and other bilateral agreements governed the size of tariffs. The interpretation we follow here is to explore the consequences of a full scale breakdown of cooperation in trade policy. In this case, the tariff rates may not be so unrealistic. In fact, in a calibrated multi-country trade model, Ossa (2014) finds that average tariffs would be over 60 percent in a full-scale world 'tariff war'. In addition, we note that in the case of US China trade, average US tariff rates as measured by Bown (2019) rose from 8 percent in early 2018 to 26 percent at the end of 2019.

4.4 Commitment in trade policy

So far it has been assumed that both inflation and tariffs are chosen simultaneously by domestic policy-makers to maximize national welfare. A central assumption is that policy is discretionary, so that policy-makers cannot bind the hands of future policy-makers, rather take these future actions as given. But it could be argued that trade policy embodies more commitment than monetary policy. Trade policy is typically enacted by legislation, and this is not as easily changed as monetary policy decisions, which can be altered at the whim of an independent central bank.

In this subsection, we analyze a simplified game where trade policy is determined in a non-cooperative game between policy-makers, but assuming that the trade policy-makers can commit to their tariff choices. The general case where trade policy is made with commitment and monetary policy is discretionary in the two-country setting involves a complicated dynamic interaction. We focus instead on a simplified setting where trade authorities commit to a single tariff rate that remains constant. Moreover, we assume that in choosing tariffs, the trade authorities internalize the endogenous response of inflation rates to tariffs in the currency war game between monetary authorities.

Therefore, in the initial period trade authorities choose a tariff rate, taking the tariff rate of the other authority as given, but taking into account the equilibrium of the monetary policy game played by the monetary authorities, within each period. We focus on a steady state of this tariff game with commitment. Given the initial tariff rate, monetary authorities choose their inflation rate in a currency war, without commitment. With constant tariff rates, which are equal in a symmetric equilibrium, inflation rates are constant over time, and also equal across countries.

The optimal tariff rates for this game can be chosen simply as a Nash equilibrium in τ and τ^* where each trade authority chooses to maximize one-period domestic utility, taking account of the competitive equilibrium, and internalizing the response of inflation in both countries to their tariff rate, but taking as given the tariff rate of the other country.

More formally, define $V(\tau_t, \tau_t^*)$ and $V^*(\tau_t, \tau_t^*)$ as follows:

$$V(\tau_t, \tau_t^*) = \text{Max}_{\{C_t, C_t^*, Y_{ht}, Y_{ft}^*, b_t, b_t^*, \mathcal{S}_t, \pi_{ht}\}} \frac{C_t^{1-\sigma} - 1}{1-\sigma} - \frac{\chi H_t^{1+\psi}}{1+\psi} \tag{43}$$

subject to (31)-(37).

$$V^*(\tau_t, \tau_t^*) = \text{Max}_{\{C_t, C_t^*, Y_{ht}, Y_{ft}^*, b_t, b_t^*, \mathcal{S}_t, \pi_{ft}^*\}}, \frac{C_t^{*1-\sigma} - 1}{1-\sigma} - \frac{\chi H_t^{*1+\psi}}{1+\psi} \tag{44}$$

subject to (31)-(37).

Then a Nash equilibrium with commitment in tariff policy, τ_t^N, τ_t^{*N} is defined by the equilibrium to the conditions:

$$\text{Max}_{\tau_t} V(\tau_t, \tau_t^{*N}) \tag{45}$$

$$\text{Max}_{\tau_t^*} V^*(\tau_t^N, \tau_t^*) \tag{46}$$

The 5th column of Table 2 illustrates the equilibrium of this game when markups are not offset by subsidies. The most striking feature of the corresponding results is that tariff rates are significantly lower than those in the baseline case of the simultaneous-move game. The Nash tariff rates for the calibrated model are 16.8 percent, compared with 40.5 percent in the baseline model. At the same time, the equilibrium inflation rates are lower, and consumption, output, and welfare for each country are higher.

What accounts for the difference between the commitment equilibrium and the baseline case? The key factor is that the trade authorities take account of the endogenous increase in inflation that will follow from a higher round of tariffs facing the monetary policy-makers in the second stage of the game. Because this inflation will be costly due to price adjustment costs, but have little benefit in terms of higher output, the trade authorities endogenously choose lower equilibrium tariff rates. Individually, monetary authorities choose a rate of inflation taking future inflation rates as given. In a steady-state equilibrium, the future inflation rate is equal to the current inflation rate, so that from the firm's first order condition in the Home country, we have

$$(1+s) - \epsilon\left((1+s) - \mathcal{MC}\right) - \xi'\left(\pi_h\right)\pi_h + \beta\xi'\left(\pi_h\right)\pi_h = 0$$

Since the trade authorities take account of the sequence of their tariff choices on π_h, they individually choose a lower degree of protection than they would in the tariff game without commitment, where both tariffs and inflation are taken as given.

This example highlights the implications of a loss of commitment in trade policy. Even in the absence of any international trade agreements, when tariffs are chosen without commitment, at the same frequency as monetary policy, there may be significant losses in welfare. Again however, if prices were fully flexible and monetary policy was neutral, this commitment would

Table 3: Effects of country size on combined Trade and Currency Wars

	Currency war		Curr.+Trade war	
	Base.	$n = 0.75$	Base.	$n = 0.75$
π_h	1.0402	1.0445	1.0474	1.0479
π_f^*	1.0402	1.0357	1.0474	1.0467
τ	0.0000	0.0000	0.4053	0.4179
τ^*	0.0000	0.0000	0.4053	0.3934
S	1.0000	1.0009	1.0000	0.9809
C	0.2026	0.2016	0.1900	0.1955
C^*	0.2026	0.2036	0.1900	0.1840
H	0.8799	0.8869	0.8733	0.8837
H^*	0.8799	0.8732	0.8733	0.8629
Home loss (%)	9.2647	10.2871	14.4376	12.7529
Foreign loss (%)	9.2647	8.2732	14.4376	16.3782

be irrelevant, and tariffs would be identical to those in the flexible price economy.

4.5 Country-size effects and alternative parameter values

All the previous derivations assume equally-sized countries. But discussions of currency and trade wars are often focused on the role of large countries relative to small countries. Particularly in the discussion of monetary policy spillovers, it is often argued that smaller countries are more exposed to the negative effects of policy spillovers from larger countries.

In the baseline model without endogenous policy choice, country size is irrelevant for real outcomes such as consumption, output, terms of trade or welfare.[20] But size may matter when countries engage in currency wars or trade wars. Table 3 illustrates the importance of large versus small countries in the case of currency wars, and currency and trade wars.[21]

The first two panels on the left-hand side show the effect of an increase in the Home country from 50 percent to 75 percent of the world economy in the case of a currency war alone, with zero tariffs. Contrary to received wisdom, the Home (large) country actually suffers relative to the equal size benchmark. The reason is again related to the trade-off between terms-of-trade manipulation and inflation. When the Home country is larger, it behaves more like a closed economy and focuses more on inflationary stimulus to offset the monopoly distortion. In a discretionary equilibrium, this leaves the Home country worse off. The Foreign country, by contrast, focuses more on terms-of-trade manipulation. In equilibrium, the Home's inflation rate rises, and Foreign's falls. So, in the currency war, country size is welfare reducing.

The two right-hand panels of Table 3 now look at the case of combined currency and trade wars. Relative to the equal-size Nash equilibrium, the Home tariff rises and Foreign's falls. Because the larger country's consumption basket is more weighted towards its own goods, the cost of a tariff on domestic consumption is less, while conversely, that for the Foreign country is

[20]This is because as country size varies, so also does the range of goods that each country produces, so size has no implications for the terms of trade.

[21]Here too it is assumed that trade is balanced in the steady state.

greater. The result is that the (large) Home country is more protectionist, obtains a significant terms-of-trade advantage, and gains in welfare relative to the Foreign country. Country size is thus an advantage in the combined currency and trade war environment, but a disadvantage in the currency war alone.

Table 7 in Appendix B illustrates the outcome under alternative parameter values for the trade and currency wars. For the degree of protection, not surprisingly the most important parameter is the trade elasticity. Our calibration uses $\lambda = 3$, which is on the high side of the trade elasticities used in the aggregate macro literature. But elasticities in the trade literature tend to be higher. For a value of $\lambda = 6$ we find that the symmetric Nash equilibrium of the current and trade war implies a tariff rate of 16.4 percent, substantially lower than that of Table 2. The consequent welfare impacts of the trade war are then less. But the main qualitative implications are the same as above.

5 Constraints on monetary policy

In this section we explore the effects of three situations where monetary policy is subject to some kind of constraint, and the interaction between monetary and trade policy susceptible of being significantly altered. First we consider a situation of fixed exchange rates, in which the Foreign economy loses its monetary policy independence by pegging its currency to the Home economy. Second, we investigate how trade and monetary policies are affected when both economies are hit by a large discount factor shock that leads both economies to hit the zero lower bound (ZLB) on nominal interest rates. In this case, the determination of both inflation rates becomes endogenous. Third, we consider a world economy with dominant currency pricing (DCP), and where the Home economy issues the dominant currency.

5.1 Fixed Exchange Rates

Now assume that the Foreign economy has an exchange-rate target, so it cedes control over its domestic inflation rate, leaving the Home country to independently choose an inflation rate. In this case, only the Home policy-maker has an independent monetary instrument. If the Foreign country targets the nominal exchange rate, we must have

$$\pi_{ht} = \pi^*_{ft} \frac{\mathcal{S}_{t-1}}{\mathcal{S}_t}. \tag{47}$$

This adds an additional state variable to the model – in addition to net foreign assets – in the form of the lagged terms of trade. Since the nominal exchange rate is pegged, the terms of trade can adjust *only* via differences in inflation rates. In addition, because the Foreign country is pegging the nominal exchange rate, it loses control of π^*_{ft}, so the Home country takes (47) as a constraint in its choice of π_{ht}.

Table 4: Trade war under fixed exchange rates

	Currency war				Curr.+Trade war			
	No subsidy ($s=0$)		Subsidy ($s=1/(\epsilon-1)$)		No subsidy ($s=0$)		Subsidy ($s=1/(\epsilon-1)$)	
	Flex ER	Fixed ER	Flex ER	Fixed ER	Flex ER	Fixed ER	Flex ER	Fixed ER
$\pi_h=\pi_f^*$	1.0402	1.0484	0.9898	1.0000	1.047	1.053	1.000	1.007
$\tau=\tau^*$	0.0000	0.0000	0.0000	0.0000	0.405	0.292	0.518	0.410
S	1.0000	1.0000	1.0000	1.0000	1.000	1.000	1.000	1.000
$C=C^*$	0.2026	0.2005	0.2488	0.2500	0.190	0.191	0.234	0.237
$H=H^*$	0.8799	0.8841	0.9994	1.0000	0.873	0.885	0.976	0.979
Home/For. loss (%)	9.2647	11.326	0.4189	0.0000	14.438	14.904	3.994	3.183

Under a fixed exchange rate regime, we must explicitly account for the initial conditions faced by the policy-makers in the form of the lagged terms of trade $\mathcal{S}_{t-1}$. Since the peg itself represents the monetary policy of the Foreign country, we describe a fixed exchange rate problem as the problem of the Home country. In this case, the Home country will choose π_{ht} to maximize its value $v(\mathcal{S}_{t-1})$. The problem can be stated as

$$\text{Max}_{\{C_t,C_t^*,Y_t,Y_t^*,b_t,b_t^*,\mathcal{S}_t,\pi_{ht},\pi_{ft}^*\}} \quad v(\mathcal{S}_{t-1},b_{t-1})=\frac{C_t^{1-\sigma}-1}{1-\sigma}-\frac{\chi H_t^{1+\psi}}{1+\psi}+\mathbb{E}_t\beta v(\mathcal{S}_t,b_t) \tag{48}$$

subject to (31)-(37) and (47).

Table 4 shows the outcome of the currency and trade war in the case of fixed exchange rates. When exchange rates are pegged by the Foreign country, only the Home country has an independent monetary policy. Absent tariffs, the left panel shows that the Home country will choose an inflation rate of 4.84 percent under our calibration, and the equilibrium is perfectly symmetric. Given unitary initial terms of trade, so that $\mathcal{S}_{-1}=1$, and zero initial net foreign assets $b_{-1}=0$, the Home country can only improve its terms of trade by choosing a higher rate of inflation, relative to the Foreign country. This contrasts with the flexible exchange rate case, where, for a given Foreign rate of inflation, the terms of trade can be improved by a contractionary monetary policy and an exchange rate appreciation, giving rise to a downward bias in inflation rates in both countries. With a fixed exchange rate, the Home country instead focuses on removing the monopoly distortion for a given terms of trade. This leads to a symmetric equilibrium where both countries inflation rates are positive, and the terms of trade is unchanged. In fact, in comparing Table 4 with Table 1, we see that the fixed exchange rate case is identical to the equilibrium of the currency war with cooperation in monetary policy. This then implies that in welfare terms, the currency war equilibrium dominates the equilibrium with fixed exchange rates, absent the trade war.

The right panel of Table 4 compares trade wars under an exchange rate peg to the flexible exchange rate case. This panel also identifies a fully symmetric outcome, where the existing terms of trade facing each policy-maker is unity. The Home country chooses its inflation rate and its tariff rate, and the Foreign country chooses only its tariff rate. In a symmetric equilibrium both inflation rates and tariff rates are equal. What is most striking about this outcome is the large difference between non-cooperative tariff rates relative to the flexible exchange rate case. In the Nash equilibrium tariff rates in each country are only 29.2 percent, compared to 40.5 percent in the flexible exchange rate equilibrium.

Why are tariffs under a fixed exchange rate so different from the baseline case? This can be best explained by focusing on Equation (47), repeated here.

$$\pi_{ht} = \pi^*_{ft} \frac{\mathcal{S}_{t-1}}{\mathcal{S}_t} \tag{49}$$

Under the fixed exchange rate regime, the Home country is choosing both its tariff rate and its own inflation rate. If it chooses its tariff rate to appreciate the terms of trade, then this implies, given $\mathcal{S}_{t-1}$, that it must be increasing its inflation rate, relative to the Foreign country inflation rate. But the fact that the authority is simultaneously choosing π_{ht} subject to the costs of inflation adjustment effectively reduces the benefits of an appreciated terms of trade. In a symmetric equilibrium where $\mathcal{S}_{-1} = 1$ and $b_{-1} = 0$ these factors exactly offset, so that it chooses an inflation rate identical to the Foreign rate, and a tariff rate identical to the Foreign tariff rate. For our calibration, the reduced benefit of tariff hikes under a peg leads to a substantially lower equilibrium tariff rates. Moreover, in welfare terms, there is little difference between fixed and flexible exchange rates under the trade and currency war, given the lower rate of protection in the former, while, as noted, the currency war outcome under fixed exchange rates is significantly worse in welfare terms.[22]

5.2 The Zero Lower Bound

One of the principal sources of the debate on currency wars was the fall in policy interest rates in the US and Europe following the Great Financial Crisis. Caballero, Farhi, and Gourinchas (2015) and Jeanne (2020) develop models of trade and currency wars at the zero lower bound (ZLB) of nominal interest rates. We now address this issue within the context of our model. We assume that monetary policy is temporarily constrained, and inflation rates are determined endogenously, given expectations about future monetary policy as well as the current stance of trade policy. In this case, tariffs are the only policy tool available during the zero-bound period.[23]

In order to capture the ZLB constraint, we have to explicitly incorporate monetary policy as

[22] There is an important caveat to these results. Indeed, there exists a continuum of equilibrium Nash tariff rates conditioned on different values of $\mathcal{S}_{-1}$. If we take an initial value $\mathcal{S}_{-1} < 1$, then the Home country will choose a tariff rate higher than that of the Foreign country, so that in equilibrium $\mathcal{S}_t = \mathcal{S}_{t-1} < 1$, and equilibrium inflation rates are, again, equalized. Likewise for $\mathcal{S}_{-1} > 1$, then the Home country chooses a lower tariff rate than the Foreign country, and again $\mathcal{S}_t = \mathcal{S}_{t-1} > 1$, with identical inflation rates. Thus, there is a continuum of Nash equilibrium tariff rates in which the Home country is more or less protectionist than the Foreign country, and each delivers a more or less appreciated terms of trade for the Home country. See Auray, Devereux, and Eyquem (2020) for a further analysis of this case.

[23] Since we assume that policy-makers lack commitment, we do not explore the consequences of Forward Guidance in monetary policy announcements.

an interest rate rule. In the case of the Home economy, defining R_t as the gross nominal interest rate, the Euler equation is:

$$1 = \beta \exp(-\zeta_t) \mathbb{E}_t \left\{ \frac{C_t^{\sigma} P_t}{C_{t+1}^{\sigma} P_{t+1}} R_t \right\} \tag{50}$$

We assume that outside of the ZLB, the interest rate is determined by the Taylor rule:

$$R_t^{Taylor} = \frac{1}{\beta} \exp(\zeta_t) \left(\frac{\pi_t}{\bar{\pi}} \right)^{\sigma_\pi} \tag{51}$$

where $\bar{\pi}$ is a CPI target rate of inflation, which is set to mimic the steady state of the Nash equilibrium in the policy game defined above, and ζ_t is a time preference shock. We also assume that $\sigma_\pi > 1$. We will assume an 'MIT' shock process for the ζ_t shock. Initially, $\zeta = 0$, but then $\zeta < 0$ occurs without anticipation, and continues with probability μ, while it reverts to zero with probability $1 - \mu$. We assume identical ζ_t shocks in each country. We focus on a $\zeta_t < 0$ that is large enough in absolute value that, without a ZLB constraint, $R_t < 1$ would be required to satisfy (50) and (51). In this case, we need to impose the interest rate non-negativity constraint:

$$R_t = \max\left(R_t^{Taylor}, 1 \right). \tag{52}$$

Table 5 illustrates the impact of the zero lower bound on the trade war. In our numerical computation, the ZLB is generated by a 15% fall in the subjective discount rate of the private sector and we assume this persists with probability 0.5. As discussed above, in this case, the monetary authority has no control of current rates of inflation, and inflation is determined by aggregate demand, given forward looking consumers and the expectation that the economy will revert to the Nash equilibrium of the currency and trade war as described in Table 2. In the absence of trade policy, the ZLB outcome leads to an equilibrium with large deflation rates, with consumption and output significantly below the Nash equilibrium of the currency war levels.

As shown in Table 5, when countries engage in a trade war under the zero lower bound, the outcome is substantially worse. Each country levies tariffs in the Nash equilibrium, but this leads to essentially unchanged inflation rates, but results in lower levels of consumption, output, and welfare. Although the trade war worsens the conditions of the ZLB, the equilibrium tariff rates are actually lower; 35.3 percent compared to 40.5 percent in the baseline case with active monetary policy and flexible exchange rates. The reasoning behind this is similar to the example of commitment in trade policy discussed above. In the environment of the ZLB, trade policy-makers take account of their choice of tariffs on the endogenous rates of inflation in the two countries. This leads them to limit the size of their tariff choices relative to the case where inflation and tariff rates are chosen simultaneously. ZLB constraints thus make trade wars less rather than more intense.

Table 5: Trade and Currency Wars at the ZLB

	Curr. war		Curr.+Trade war	
	Base.	ZLB	Base.	ZLB
$\pi_h = \pi_f^*$	1.0402	0.9818	1.047	0.982
$\tau = \tau^*$	0.0000	0.0000	0.405	0.353
S	1.0000	1.0000	1.000	1.000
$C = C^*$	0.2026	0.1773	0.190	0.169
$H = H^*$	0.8799	0.7693	0.873	0.756
Home/For. welf. loss (%)	9.265	13.010	14.438	16.466

5.3 Dominant Currency Pricing

Recent evidence has pointed to the role of the US dollar as an invoice currency for pricing exports for a large share of the world economy (see Gopinath et al. (2020) and Mukhin (2018)). In terms of our model, this would imply that one country (the Home country) sets the price of both its exports and domestic sales in its own currency, while the Foreign country sets its domestic sales price in its own currency, but sets its export price in the currency of the Home country. Gopinath et al. (2020) denote this practice as one of dominant currency pricing (DCP). In this section we explore the implications of DCP for the currency and trade war equilibrium.

The model under DCP differs in only a few features, as explained in details in Appendix E. The nominal exchange rate is still flexible, but the impact of exchange rate changes on the terms of trade is muted, in particular for the Home country, since both its exports and imports are priced in its own currency.

The true price index for the Home consumers under DCP now becomes:

$$P_t = \left(\varepsilon P_{ht}^{1-\lambda} + (1-\varepsilon)((1+\tau_t)P_{ft})^{1-\lambda}\right)^{1/(1-\lambda)} \tag{53}$$

where P_{ft} is the price of the Foreign good set in Home currency. By contrast, the price index for the Foreign economy is unchanged compared to the baseline PCP model, since the Home country firm sets all prices in Home currency.

The optimal pricing policy of Home firms is as before, but Foreign firms charge separate prices to the domestic (in Foreign currency) and Home (in Home currency) firms and households respectively buying intermediate and final goods. The profits of the Foreign firm i are then represented as

$$\Pi_t^*(i) = (1+s^*(i))\left(P_{ft}^*(i)\,Y_{ft}^*(i) + S_t^{-1}P_{ft}(i)\,Y_{ft}(i)\right) - MC_t^*\left(Y_{ft}^*(i) + Y_{ft}(i)\right) \tag{54}$$

where $MC_t^* = A_t^{*-1}(1-\alpha)^{\alpha-1}\alpha^{-\alpha}W_t^{*1-\alpha}P_{xt}^{*\alpha}$ and where $Y_{ft}(i) = D_{xt}(i)$ and $Y_{ft}^*(i) = D_t^*(i)$ in equilibrium. Foreign firm i maximizes

$$\mathbb{E}_t \left\{ \sum_{j=0}^{\infty} \omega_{t+j}^* \left(\begin{array}{c} \Pi_{t+j}^*(i) - \frac{\phi}{2}\left(\frac{P_{ft+j}^*(i)}{P_{ft+j-1}^*(i)} - 1\right)^2 P_{ft+j}^*(i)\, Y_{ft+j}^*(i) \\ -\frac{\phi}{2}\left(\frac{P_{ft+j}(i)}{P_{ft+j-1}(i)} - 1\right)^2 S_{t+j}^{-1} P_{ft+j}(i)\, Y_{ft+j}(i) \end{array} \right) \right\} \tag{55}$$

Note that the Foreign firm incurs costs of price adjustment for sales to the Home country that are separate from those pertaining to sales to the domestic consumers and firms. Profit maximization now yield two (not one) New Keynesian inflation equations for Foreign goods depending on whether they are consumed locally or exported. The dynamics of Foreign prices for goods sold locally are:

$$(1+s^*)(1-\epsilon) + \epsilon\left(\mathcal{S}_t^*/\mathcal{S}_t\right)\mathcal{MC}_t^* - \phi\pi_{ft}\left(\pi_{ft}-1\right) + \mathbb{E}_t\left\{\omega_{t+1}^* \frac{D_{xt+1}}{D_{xt}} \frac{\mathcal{S}_t^*/\mathcal{S}_t}{\mathcal{S}_{t+1}^*/\mathcal{S}_{t+1}} \phi\pi_{ft+1}\left(\pi_{ft+1}-1\right)\right\} = 0 \tag{56}$$

while the condition for exported Foreign goods is

$$(1+s^*)(1-\epsilon) + \epsilon\mathcal{MC}_t^* - \phi\pi_{ft}^*\left(\pi_{ft}^*-1\right) + \mathbb{E}_t\left\{\omega_{t+1}^* \frac{D_{t+1}^*}{D_t^*}\phi\pi_{ft+1}^*\left(\pi_{ft+1}^*-1\right)\right\} = 0 \tag{57}$$

The essential new element that DCP brings to the analysis relates to the terms of trade. In fact, we now have two separate terms of trade. For the Home country, the relative price of imports to exports is now $\mathcal{S}_t = \frac{P_{ft}}{P_{ht}}$, where both prices are expressed in Home currency. The terms of trade for the Foreign country is expressed as before; $\mathcal{S}_t^* = \frac{S_t P_{ft}^*}{P_{ht}}$. The two measures may differ due to deviations of the law of one price for the foreign good, since in general with price adjustment costs, P_{ft} will not always equal $S_t P_{ft}^*$. More critically, $\mathcal{S}_t$ can be adjusted only through nominal price adjustment, while $\mathcal{S}_t^*$ adjusts to nominal exchange rate changes for given nominal prices. This effectively means that the Home country terms of trade $\mathcal{S}_t$ displays the same type of persistence as in the case of fixed exchange rates. Since $\mathcal{S}_t = \frac{P_{ft}}{P_{ht}}$, we have

$$\mathcal{S}_t = \mathcal{S}_{t-1}\frac{\pi_{ft}}{\pi_{ht}}. \tag{58}$$

Thus, the home terms of trade adjusts according to the differential between the Foreign export price inflation and the Home inflation rate.

In the analysis so far, we have assumed that the monetary policy instrument for each country is the PPI inflation rate. In the case of DCP we continue to assume that each country targets its PPI inflation rate in domestic currency. But then from Equation (58), the Foreign exported goods inflation rate π_{ft} is an endogenous variable.

The policy game under DCP is defined in the same way as before, where in the currency war game the Home and Foreign policy-makers choose π_{ht} and π_{ft}^* respectively, and with both trade

Table 6: Trade and Currency Wars under Dominant Currency Pricing (DCP)

	Curr. war		Curr.+Trade war	
	Base.	DCP	Base.	DCP
π_h	1.0402	1.0515	1.0474	1.0570
π_f^*	1.0402	1.0337	1.0474	1.0302
π_f	1.0402	1.0515	1.0474	1.0570
τ	0.0000	0.0000	0.4053	0.2571
τ^*	0.0000	0.0000	0.4053	0.0062
$\mathcal{S}^*$	1.0000	1.0043	1.0000	0.8754
$\mathcal{S}$	1.0000	1.0009	1.0000	0.8796
C	0.2026	0.1996	0.1900	0.1972
C^*	0.2026	0.1941	0.1900	0.1855
H	0.8799	0.9002	0.8733	0.8907
H^*	0.8799	0.9156	0.8733	0.9210
Home welf. loss (%)	9.2647	12.2367	14.4376	12.5286
Foreign welf. loss (%)	9.2647	15.8156	14.4376	19.9628

and currency wars they choose both inflation rates and tariff rates.

Table 6 describes the equilibrium of the policy game under DCP. First, focusing on the currency war outcome, we see that the equilibrium is asymmetric, with the Home policy-maker choosing a larger inflation rate (5.15%) than in the PCP case (4.02%), and the Foreign country choosing a smaller inflation rate (3.37%). For the currency war these results very much look like the PCP case with asymmetric country size but for a different reason: running up inflation is the only way for the Home country to improve its terms of trade. This policy gives the dominant currency issuer an edge in terms of welfare compared to the Foreign country, but both countries are worse off than in the case of a currency war under flexible exchange rates and PCP.

When we allow for both currency wars and trade wars under the DCP specification, Table 6 shows a more substantial asymmetry. The Foreign country sets a very low tariff, around 1%, while the Home country imposes a 26% tariff, much larger than the Foreign country but lower than the PCP tariff. This leads to an equilibrium where the terms of trade are substantially in favour of the Home country. The logic behind this follows from the fact that for the Foreign country to improve its terms of trade *via* a tariff, it must engage in costly inflation in its exported goods price. But in the Nash equilibrium of the trade and currency war, inflation is already high. Increasing exported goods inflation even further would be self-defeating. In fact, it is optimal to moderate inflation through a very small tariff. This leads to a terms-of-trade benefit for the Home country. Then, for the Home country, given an equilibrium terms of trade substantially in its favour, there is little benefit in levying a large tariff. As a result, the presence of DCP leads to a significant asymmetry in welfare outcomes in favour of the dominant currency issuer.[24]

[24] It is important to note that this effect of DCP is purely due to the currency of price-setting and the presence of sticky prices. If prices were fully flexible, then both countries would levy tariffs in a Nash equilibrium at an equal rate given by Table 2.

6 Initial trade imbalances

Up to now, we have assumed a zero level of initial net foreign assets (NFA), *i.e.* $b_{-1} = 0$. We now relax this assumption. We explore the impacts of an initial non-zero net foreign assets on the trade and currency war. We focus on the model without production subsidies and so distorted steady states. In addition, we concentrate on the equilibria with sticky prices, PCP and flexible exchange rates.

A country with positive net foreign assets can sustain higher levels of household consumption for a given labor and production effort. Even before considering the effects of monetary and trade policies, the presence of home bias in final and intermediate goods therefore means that creditor countries will have a more favourable terms of trade, as well as lower overall welfare losses. Debtor countries are in the opposite situation, with lower consumption for a given labor and production effort, depreciated terms of trade and larger welfare losses.

In the model, this 'natural' advantage interacts with the strategic motives already highlighted in the previous sections and with a new wealth motive that relates to the real returns on net foreign assets. As we show below, a key determinant of both non-cooperative tariffs and inflation is the currency denomination of internationally traded bonds.

6.1 Baseline case

We first consider the baseline case, where bonds are denominated in the Foreign currency. Following this, we will show how the outcomes differ when bonds are Home currency denominated. In Appendix G, we show an alternative specification with index linked bonds, where the the outcome is fully symmetric and the currency of denomination is irrelevant.

In general, policy-makers will consider both the current-period equation that governs net foreign asset dynamics, but also the next-period dynamics, introducing net foreign assets as a state variable to the policy problem. While policy-making is discretionary, policy-makers will internalize the effect of their choices on the future value of net foreign assets. This net foreign asset equation enters differently in the current and in future periods because nominal returns R^*_{t-1} are determined before policy choices are made at time t. Net foreign asset dynamics that enter the Lagrangian of policy-makers at time t are

$$b_t = \frac{\mathcal{S}_t \mathcal{P}_{t-1} R^*_{t-1}}{\mathcal{S}_{t-1} \mathcal{P}_t \pi^*_{ft}} b_{t-1} + \mathcal{P}_t^{-1} \left(D^*_{xt} - \frac{1-n}{n} \mathcal{S}_t D_{xt} \right) \tag{59}$$

Since bonds are denominated in the Foreign currency, the Foreign policy-maker can manipulate the real returns on Home (and Foreign) net foreign assets by choosing a larger or lower inflation rate π^*_{ft}. Increasing Foreign inflation for a given level of nominal returns reduces the real returns on Home net foreign assets, reducing the size of the permanent trade deficit the Home country can sustain for a given level of initial net foreign assets. Similarly, when the Foreign country is creditor, reducing Foreign inflation raises the real returns on past net assets and increases the wealth of the Foreign country, allowing the Foreign country to run larger permanent trade deficits. For a given level of net foreign assets, this expropriation incentive is tempered by the fact that inflation is itself costly due to price adjustment costs for domestic firms. Hence it will never be desirable to completely inflate away the value of net foreign debt – unless prices are fully flexible. Moreover, in a rational expectations equilibrium the next-period nominal interest rate on foreign debt will adjust to take account of the *ex-post* optimal inflation set by the Foreign policy-maker.

Figure 1 reports the resulting steady-state tariff rates, inflation rates, terms of trade $\mathcal{S}$ and welfare losses with respect to the first best equilibrium when varying the steady-state level of the Home trade balance. The calibration is the same as in Section 4.

When the Home country is creditor and sustains permanent trade deficits (left part of each panel), the Foreign country raise its inflation rate quite substantially above the zero net foreign asset baseline case, in a bid to devalue the existing stock of their external debt. By contrast, given that debt is denominated in Foreign currency, the Home country does not gain from changing its inflation rate relative to the baseline case.

The implication for tariffs is quite different. When the Home country is a creditor, and sustains a trade balance deficit, it raises its tariff relative to the baseline zero NFA case, and more-so the larger its net external credit (the larger its trade deficit). Intuitively, with a larger net asset position, the Home country consumers have a larger effect on world relative prices, and the Home policy-maker takes advantage of its greater strategic power by raising its tariff rate relative to the baseline case.

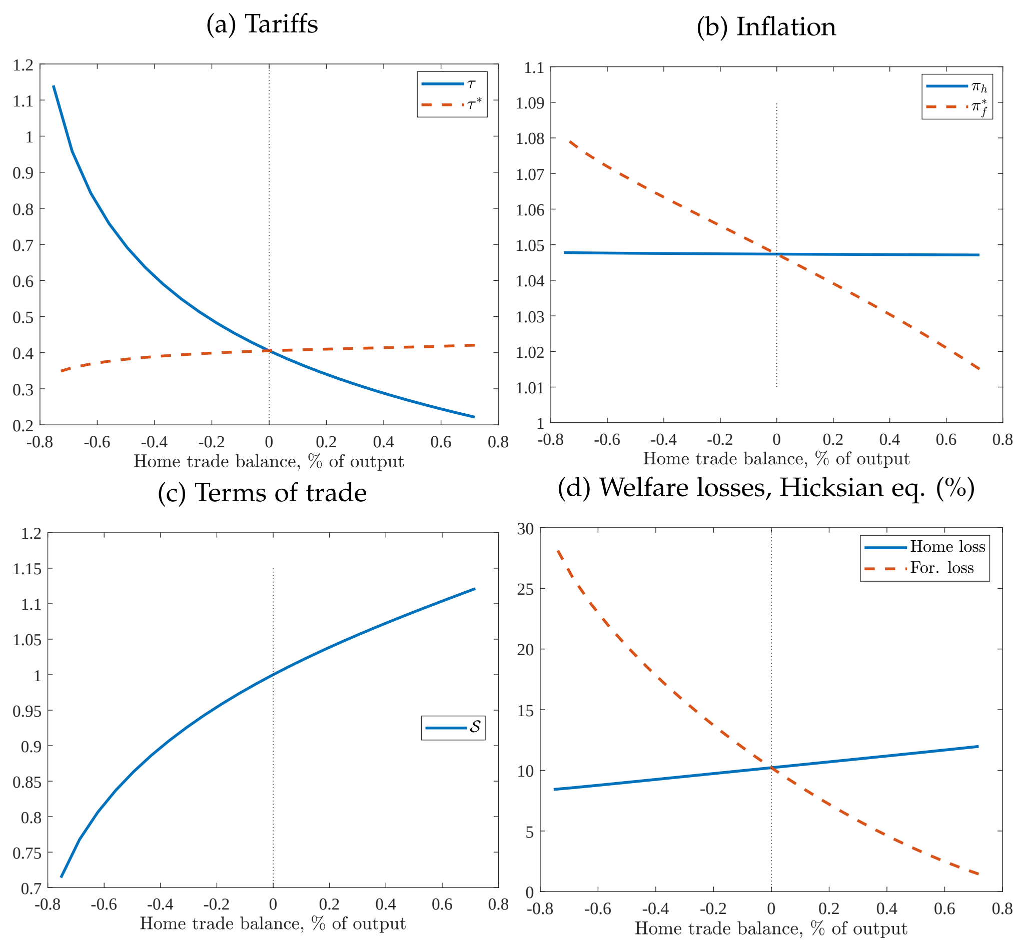
The situation reverses itself if the Foreign currency is a creditor. In that case, the Foreign policy-maker *reduces* inflation relative to the zero NFA baseline case, and simultaneously raises its tariff in order to exploit its greater strategic power. But strikingly, the sensitivity of the Foreign tariff to the net foreign position is much less than that of the Home country. The intuition is that with Foreign currency denominated assets, the Foreign policy-maker exploits its additional market power through manipulating inflation so that, in equilibrium, it relies less on the use of tariffs to improve its terms of trade as its net foreign position increases.

The lower two panels of Figure 1 show the implications for the terms of trade and welfare. As the Home country's net foreign asset position rises (its trade deficit increases), its terms of trade progressively improve, and its welfare losses relative to the zero NFA baseline case fall, while those of the Foreign country increase.

6.2 Bonds denominated in the Home currency

When bonds are denominated in the Home currency, Appendix F shows that most equilibrium condition are unchanged, except the equation describing the dynamics of Home net foreign

Figure 1: Trade and currency war with trade imbalances



assets. The latter becomes

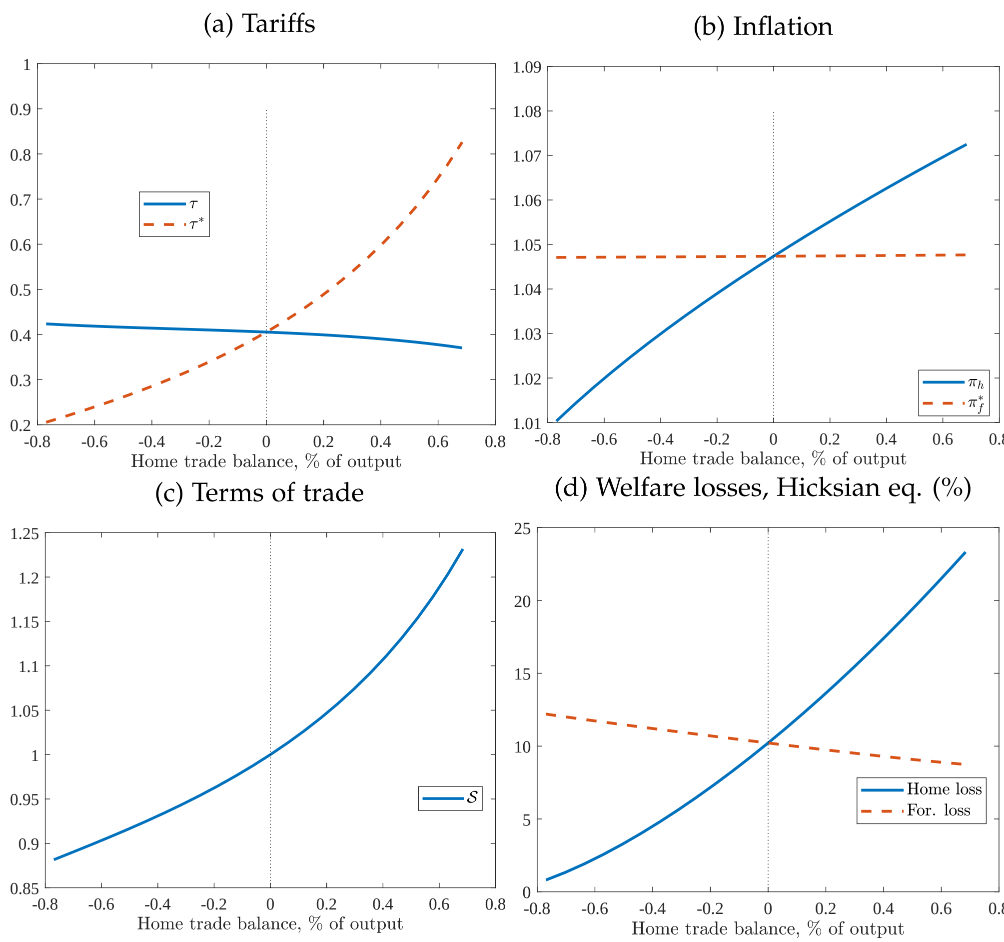
$$b_t = \frac{\mathcal{P}_{t-1} R_{t-1}}{\mathcal{P}_t \pi_{ht}} b_{t-1} + \mathcal{P}_t^{-1} \left(D_{xt}^* - \frac{1-n}{n} \mathcal{S}_t D_{xt} \right) \tag{60}$$

in period t from the perspective of policy-makers, with R_{t-1} predetermined, and

$$b_t = \frac{\mathcal{P}_{t-1}}{\mathcal{P}_t \omega_t} b_{t-1} + \mathcal{P}_t^{-1} \left(D_{xt}^* - \frac{1-n}{n} \mathcal{S}_t D_{xt} \right) \tag{61}$$

from period $t+1$ onwards. The possibility of manipulating real returns on net foreign assets is now in the hands of the Home policy-makers, and Figure 2 shows that the resulting equilibria are essentially a mirror image of the case with Foreign currency denominated bonds.

Figure 2: Trade and currency war with trade imbalances when bonds are denominated in Home currency



When the Foreign country is creditor, sustains trade deficits and the Home economy sustains trade surpluses (right panels of Figure 2), Home policy-makers choose to partly expropriate Foreign creditors by running up inflation. The Foreign country has no incentive to change inflation relative to the zero NFA benchmark, but is more aggressive in setting tariffs, exploiting its greater strategic power in influencing world relative prices.

When the Home country is creditor and sustains trade deficits (left panels of Figure 2), the natural terms-of-trade advantage combines with the bonds-denomination advantage, Home policymakers choose lower rates of inflation to raise real returns on their net foreign assets, which further appreciates Home terms of trade. But as in the reverse case discussed above, because the combined use of inflation and tariffs as instruments, they apply only moderately higher tariffs. In this case Home households experience large welfare gains and Foreign households suffer moderate welfare losses.

Overall, what can be learned from these results? First, being a net creditor gives a natural terms-of-trade advantage in trade and currency wars. Second, considering trade and financial imbalances introduces an additional partial expropriation motive when choosing inflation, that essentially dominates the motives discussed in the baseline case with zero NFA. Third, trade imbalances introduce strong asymmetries in trade and currency wars, as the cross-country differences in tariffs and inflation rates increases with the size of trade imbalances. Fourth, being a net creditor (with trade deficits) in its own currency is so advantageous that the need to use tariffs to ameliorate its terms of trade is less important. As a corollary, creditor countries are more protectionist when the corresponding positive net foreign assets are denominated in the currency of trade partners.

7 Conclusions

This paper is primarily a theoretical exploration of the links between trade policy and monetary policy from the point of view of international strategic policy interaction. There is a large literature both on international macroeconomic policy coordination/non-coordination on the one hand and the determinants of trade policy and tariff setting in strategic environments on the other hand. In our labeling, we denote the first topic as pertaining to 'currency wars', and the second related to 'trade wars'. Our paper represents a first pass at combining 'currency wars' and 'trade wars' within a simple New Keynesian open-economy framework. In the introduction, we argued that contemporary developments in global economic policy made the interaction of these two dimensions of policy-making of much greater relevance than in the past. The results of our analysis show that in many ways, currency wars and trade wars are very closely linked to one another, and differences in policy settings can lead to major differences in macroeconomic outcomes, the overall degree of trade protection, and welfare.

References

Auray, Stéphane, Aurélien Eyquem, and Paul Gomme. 2018. "Ramsey-optimal Tax Reforms and Real Exchange Rate Dynamics." *Journal of International Economics* 115 (C):159–169.

Auray, Stéphane, Aurélien Eyquem, and Xiaofei Ma. 2017. "Competitive Tax Reforms in a Monetary Union with Endogenous Entry and Tradability." *European Economic Review* 98 (C):126–143.

Auray, Stéphane, Michael B. Devereux, and Aurélien Eyquem. 2020. "Trade Wars, Currency Wars." NBER Working Paper 27460.

Bagwell, Kyle and Robert W. Staiger. 2003. "Protection and the Business Cycle." *The B.E. Journal of Economic Analysis & Policy* 3 (1):1–45.

———. 2010. "The World Trade Organization: Theory and Practice." *Annual Review of Economics* 2 (1):223–256.

———. 2016. "Handbook of Commercial Policy." *North Holland* .

Barattieri, Alessandro, Matteo Cacciatore, and Fabio Ghironi. 2021. "Protectionism and the Business Cycle." *Journal of International Economics* 129:103417.

Barbiero, Omar, Emmanuel Farhi, Gita Gopinath, and Oleg Itskhoki. 2019. "The Macroeconomics of Border Taxes." *NBER Macroeconomics Annual* 33 (1):395–457.

Basevi, Giorgio, Vincenzo Denicolo, and Flavio Delbono. 1990. "International Monetary Cooperation under Tariff Threats." *Journal of International Economics* 28 (1):1–23.

Benigno, Gianluca and Pierpaolo Benigno. 2003. "Price Stability in Open Economies." *The Review of Economic Studies* 70 (4):743–764.

Bergin, Paul R. and Giancarlo Corsetti. 2008. "The Extensive Margin and Monetary Policy." *Journal of Monetary Economics* 55 (7):1222–1237.

———. 2020. "The Macroeconomic Stabilization of Tariff Shocks: What is the Optimal Monetary Response?" *NBER Working Paper 26995* .

Betts, Caroline and Michael B. Devereux. 2000. "International Monetary Coordination and Competitive Depreciation: A Re-Evaluation." *Journal of Money Credit and Banking* 32:722–746.

Bhattarai, Saroj and Konstantin Egorov. 2016. "Optimal Monetary and Fiscal Policy at the Zero Lower Bound in a Small Open Economy." Working Paper 260, Globalization and Monetary Policy Institute.

Bown, Chad P. 2019. "US-China Trade War: The Guns of August." *Peterson Institute for International Economics* .

Bown, Chad P. and Meredith A. Crowley. 2013. "Import Protection, Business Cycles, and Exchange Rates: Evidence from the Great Recession ." *Journal of International Economics* 90:50–64.

Broda, Christian, Nuno Limao, and David E. Weinstein. 2008. "Optimal Tariffs and Market Power: The Evidence." *American Economic Review* 98:2032–2065.

Caballero, Ricardo, Emmanuel Farhi, and Pierre-Olivier Gourinchas. 2015. "Global Imbalances and Currency Wars at the ZLB." CEPR Discussion Papers 10905.

Cacciatore, Matteo, Romain Duval, Giuseppe Fiori, and Fabio Ghironi. 2016. "Market Reforms in the Time of Imbalance." *Journal of Economic Dynamics and Control* 72 (C):69–93.

Campolmi, Alessia, Harald Fadinger, and Chiara Forlati. 2014. "Trade Policy: Home Market Effect versus Terms-of-trade Externality." *Journal of International Economics* 93 (1):92–107.

Canzoneri, Matthew B. and W. Henderson Dale. 1991. *Monetary Policy in Interdependent Economies: A Game-Theoretic Approach*, *MIT Press Books*, vol. 1. The MIT Press.

Carraro, Carlo and Giavazzi Francesco. 1998. "Can International Policy Coordination Really Be Counterproductive." NBER Working Paper 2669.

Chari, V. V., Juan Pablo Nicolini, and Pedro Teles. 2018. "Ramsey Taxation in the Global Economy." CEPR Discussion Paper 12753.

Clarida, Richard, Jordi Gali, and Mark Gertler. 2002. "A Simple Framework for International Monetary Policy Analysis." *Journal of Monetary Economics* 49 (5):879–904.

Correia, Isabel, Juan Pablo Nicolini, and Pedro Teles. 2008. "Optimal Fiscal and Monetary Policy: Equivalence Results." *Journal of Political Economy* 116 (1):141–170.

Corsetti, Giancarlo and Paolo Pesenti. 2001. "Welfare and Macroeconomic Interdependence." *The Quarterly Journal of Economics* 116 (2):421–445.

de Paoli, Bianca. 2009. "Monetary Policy and Welfare in a Small Open Economy." *Journal of International Economics* 77 (1):11–22.

Eaton, Jonathan and Gene M. Grossman. 1985. "Tariffs as Insurance: Optimal Commercial Policy When Domestic Markets Are Incomplete." *Canadian Journal of Economics* 18 (2):258–272.

Eggertsson, Gauti, Andrea Ferrero, and Andrea Raffo. 2014. "Can Structural Reforms Help Europe?" *Journal of Monetary Economics* 61 (C):2–22.

Egorov, Konstantin and Dmitry Mukhin. 2019. "Optimal Monetary Policy under Dollar Pricing." 2019 Meeting Papers 1510, Society for Economic Dynamics.

Eichengreen, Barry. 1981. "A Dynamic Model of Tariffs, Output and Employment Under Flexible Exchange Rates." *Journal of International Economics* 11:341–59.

———. 2019. "Trade Policy and the Macroeconomy." *IMF Economic Review* 67:4–23.

Erceg, Christopher, Andrea Prestipino, and Andrea Raffo. 2018. "The Macroeconomic Effect of Trade Policy." 2018 Meeting Papers 221, Society for Economic Dynamics.

Faia, Ester and Tommaso Monacelli. 2008. "Optimal Monetary Policy in a Small Open Economy with Home Bias." *Journal of Money, Credit and Banking* 40 (4):721–750.

Fajgelbaum, Pablo D., Pinipoli K. Goldberg, Patrick K. Kennedy, and Amit K. Khandelwal. 2019. "The Return to Protectionism." *NBER Working Paper 25638* .

Farhi, Emmanuel, Gita Gopinath, and Oleg Itskhoki. 2014. "Fiscal Devaluations." *Review of Economic Studies* 81 (2):725–760.

Feenstra, Robert C., Philip Luck, Maurice Obstfeld, and Katheryn N. Russ. 2018. "In Search of the Armington Elasticity." *Review of Economics and Statistics* 100.

Fujiwara, Ippei and Jiao Wang. 2017. "Optimal Monetary Policy in Open Economies Revisited." *Journal of International Economics* 108 (C):300–314.

Furceri, Davide, Swarnali A. Hannan, Jonathan D. Ostry, and Andrew K. Rose. 2019. "Macroeconomic Consequences of Tariffs." IMF Working Paper 9.

Galì, Jordi and Tommaso Monacelli. 2005. "Monetary Policy and Exchange Rate Volatility in a Small Open Economy." *The Review of Economic Studies* 72 (3):707–734.

Ghironi, Fabio and Francesco Giavazzi. 1998. "Currency Areas, International Monetary Regimes, and the Employment-Inflation Tradeoff." *Journal of International Economics* 45:259–296.

Ghironi, Fabio and Marc J. Melitz. 2005. "International Trade and Macroeconomic Dynamics with Heterogeneous Firms." *The Quarterly Journal of Economics* 120 (3):865–915.

Gopinath, Gita, Emine Boz, Camila Casas, Federico J. Diez, Pierre-Olivier Gourinchas, and Plagborg-Mollerm Mikkel. 2020. "Dominant Currency Paradigm." *American Economic Review* 110 (3):677–719.

Groll, Dominik and Tommaso Monacelli. 2020. "The Inherent Benefit of Monetary Unions." *Journal of Monetary Economics* 111 (C):63–79.

Gunnar, Niels and Joseph Francois. 2006. "Business Cycles, the Exchange Rate, and Demand for Antidumping Protection in Mexico." *Review of Development Economics* 10:388–399.

Hevia, Constantino and Juan Pablo Nicolini. 2013. "Optimal Devaluations." *IMF Economic Review* 61 (1):22–51.

Itskhoki, Oleg and Dmitry Mukhin. 2021. "Exchange Rate Disconnect in General Equilibrium." *Journal of Political Economy* 129 (8):2183–2232.

Jeanne, Olivier. 2020. "Currency Wars, Trade Wars, and Global Demand." *Mimeo Johns Hopkins University* .

Johnson, Harry G. 1953. "Optimum Tariffs and Retaliation." *Review of Economic Studies* 21 (2):142–153.

Krugman, Paul. 1982. "The Macroeconomics of Protection with a Floating Exchange Rate." *Carnegie Rochester Conference Series on Public Policy* 16:41–82.

Lindé, Jesper and Andrea Pescatori. 2019. "The Macroeconomic Effects of Trade Tariffs: Revisiting the Lerner Symmetry Result." *Journal of International Money and Finance* 95 (C):52–69.

Mishra, Prachi and Raghuram Rajan. 2018. "Rules of the Monetary Game." *Hoover Institute mimeo* .

Mukhin, Dmitry. 2018. "An Equilibrium Model of the International Price System." 2018 Meeting Papers 1510, Society for Economic Dynamics.

Oatley, Thomas. 2010. "Real Exchange Rates and Trade Protectionism." *Business and Politics* 12 (2):1–17.

Ossa, Ralph. 2014. "Trade Wars and Trade Talks with Data." *American Economic Review* 104 (12):4104–46.

Rogoff, Kenneth. 1985. "Can International Monetary Cooperation be Counterproductive?" *Journal of International Economics* 18:199–217.

Tirelli, Pattricio. 1993. *Monetary and Fiscal Policy, the Exchange Rate and Foreign Wealth*. Palgrave macmillan.

UNCTAD. 2013. "NON-TARIFF MEASURES TO TRADE: Economic and Policy Issues for Developing Countries." *United Nations* .

A The Analytical Model

Using Equations (2), (6) and (9) of the main text, we obtain the balance of payments condition

$$C^*_{ht} = \mathcal{S}_t C_{ft} \tag{A.1}$$

Goods market clearing conditions for the home and foreign country are represented by:

$$A_t H_t (1 - \frac{\phi}{2}(\pi_{ht} - 1)^2) = C_{ht} + C^*_{ht} \tag{A.2}$$

$$A^*_t H^*_t (1 - \frac{\phi}{2}(\pi^*_{ft} - 1)^2) = C_{ft} + C^*_{ft} \tag{A.3}$$

Using Equations (4), (8) and the equivalent for the Foreign country, we obtain the labor market equilibrium conditions:

$$\frac{\ell'(H_t)}{A_t} = u_{c_{ht}} \mathbb{E}_t \Psi(\pi_{ht}, \pi_{ht+1}, \theta) \tag{A.4}$$

$$\frac{\ell'(H^*_t)}{A^*_t} = u_{c^*_{ft}} \mathbb{E}_t \Psi(\pi^*_{ft}, \pi^*_{ft+1}, \theta^*) \tag{A.5}$$

Finally, using Equation (3) and the equivalent for the Foreign country we obtain:

$$\frac{u_{c^*_{ht}}}{u_{c^*_{ft}}} = \frac{(1+\tau^*_t)}{\mathcal{S}_t} \tag{A.6}$$

$$\frac{u_{c_{ht}}}{u_{c_{ft}}} = \frac{1}{(1+\tau_t)\mathcal{S}_t} \tag{A.7}$$

Equations (A.1)-(A.7) represent the competitive equilibrium of the simplified model, conditional on monetary and tariff policy, which can be implicitly solved for H_t, H^*_t, C_{ht}, C_{ft} C^*_{ht}, C^*_{ft}, and $\mathcal{S}_t$.

A.1 Currency Wars: Optimal inflation choice

The policy-maker in the Home economy chooses inflation, taking the actions of both Foreign policy-maker and future policy-makers (both domestic and foreign) as given. We take the firm's production subsidy as given and constant. In this problem, we abstract from tariffs altogether, and assume that there is free trade, so that $\tau_t = \tau^*_t = 0$. The point is to show that the inflation choice of governments will partly attempt to manipulate the terms of trade in the absence of tariffs.

Define the terms of trade as $\mathcal{S}_t = \frac{S_t P^*_{ft}}{P_{ht}}$. The policy problem for the Home government is defined in the form of a value function:

$$v(\mathcal{Z}_t) = \text{Max}_{\{C_{ht}, C_{ft},\ H_t, \mathcal{S}_t, \pi_t, C^*_{ht}, C^*_{ft}, H^*_t\}}\ u(C_{ht}, C_{ft}) - \ell(H_t) + \mathbb{E}_t \beta v(\mathcal{Z}_{t+1}) \tag{A.8}$$

subject to (A.1)-(A.7). Let $\xi_{1,t}$, ...,$\xi_{7,t}$ denote the Lagrange multipliers on the constraints (A.1)-(A.7). The first-order conditions for the discretionary policy-maker are then listed as:

$$C_{ht} \quad : \quad u_{c_{ht}} = \xi_{2,t} + \xi_{4,t} u_{c_{hht}} \Psi_t - \xi_{7,t}(\mathcal{S}_t u_{c_{hht}} - u_{c_{hft}}) \tag{A.9}$$

$$C_{ft} \quad : \quad u_{c_{ft}} = \xi_{1,t}\mathcal{S}_t + \xi_{3,t} + \xi_{4,t} u_{c_{hft}} \Psi_t - \xi_{7,t}(\mathcal{S}_t u_{c_{hft}} - u_{c_{fft}}) \tag{A.10}$$

$$H_t \quad : \quad \ell'(H_t) = \xi_{2,t} A_t \left(1 - \frac{\phi}{2}(\pi_{ht} - 1)^2\right) + \xi_{4,t} \frac{\ell''(H_t)}{A_t} \tag{A.11}$$

$$\mathcal{S}_t \quad : \quad -\xi_{1,t} \frac{C^*_{ht}}{\mathcal{S}_t} + \xi_{6,t} u^*_{c_{ht}} + \xi_{7,t} u_{c_{ht}} = 0 \tag{A.12}$$

$$\pi_{ht} \quad : \quad -\xi_{2,t} A_t H_t \phi(\pi_{ht} - 1) - \xi_{4,t} u_{c_{ht}} \phi(2\pi_{ht} - 1) = 0 \tag{A.13}$$

$$C^*_{ht} \quad : \quad \xi_{1,t} - \xi_{2,t} - \xi_{5,t} u^*_{c_{hft}} \Psi^*_t + \xi_{6,t}(\mathcal{S}_t u^*_{c_{hht}} - u^*_{c_{hft}}) = 0 \tag{A.14}$$

$$C^*_{ft} \quad : \quad -\xi_{3,t} + \xi_{6,t}(u^*_{c_{hft}} \mathcal{S}_t - u^*_{c_{fft}}) - \xi_{5,t} u^*_{c_{fft}} \Psi^*_t = 0 \tag{A.15}$$

$$H^*_t \quad : \quad \xi_{3,t} A^*_t \left(1 - \frac{\phi}{2}(\pi^*_{ft} - 1)^2\right) + \xi_{5,t} \frac{\ell''(H^*_t)}{A^*_t} = 0 \tag{A.16}$$

Using Equations (A.9) and (A.11) along with Equation (A.4), we can obtain:

$$\Psi_t = \frac{1 - \frac{\phi}{2}(\pi_{ht} - 1)^2 - \frac{(\pi_{ht}-1)}{(2\pi_{ht}-1)} \psi \Psi_t}{1 - \frac{A_t H_t (\pi_{ht}-1)}{u_{c_{ht}}(2\pi_{ht}-1)} u_{c_{hht}} \Psi_t - \frac{\xi_{7,t}}{\xi_{2,t}}(\mathcal{S}_t u_{c_{hht}} - u_{c_{hft}})} \tag{A.17}$$

where $\psi = \frac{H\ell''(H)}{\ell'(H)}$ is the inverse of the Frisch elasticity of labor supply.

Proof of Result 1. Assume that in a steady state, $\pi_h = 1$. Then from Equation (A.17) it must be that:

$$\theta = \frac{1}{1 - \frac{\xi_{7,t}}{\xi_{2,t}}(\mathcal{S}_t u_{c_{hht}} - u_{c_{hft}})} \tag{A.18}$$

(where $\theta = \frac{(1+s)(\epsilon-1)}{\epsilon} \leq 1$) which in a symmetric equilibrium implies a unique particular value of $\frac{\xi_{7,t}}{\xi_{2,t}}$. But this is generally inconsistent with the solution of Equations (A.9)-(A.16).

Proof of Result 2. Assume that $\theta = 1$ (so the optimal subsidy is applied). Then in a symmetric equilibrium $u_{c_h} = u_{c_f}$ so that from Equations (A.9) and (A.10) we have:

$$1 - \frac{\xi_{1,t}\mathcal{S}_t + \xi_{3,t}}{\xi_{2,t}} + \frac{\xi_{4,t}}{\xi_{2,t}}(u_{c_{hht}} - u_{c_{hft}})\Psi_t = \frac{\xi_{7,t}}{\xi_{2,t}}(\mathcal{S}_t u_{c_{hht}} + u_{c_{fft}} - 2u_{c_{hft}}). \tag{A.19}$$

The expression $\frac{\xi_{1,t}\mathcal{S}_t + \xi_{3,t}}{\xi_{2,t}}$ is a measure of the Foreign elasticity of demand for the Home good (see below) which is greater than unity by assumption. If $\pi_h = 0$, then $\xi_4 = 0$ and from (A.19) we must have $\xi_7 > 0$. When $\theta = 1$ this must imply that beginning at $\pi_h = 0$, the left-hand side of Equation (A.17) falls, so π_h must fall to ensure that (A.17) is satisfied.

A.2 Optimal policy with both tariffs and inflation as instruments.

The policy problem for the home government is defined in the form of a value function:

$$v(\mathcal{Z}_t) = \text{Max}_{\{C_{ht}, C_{ft}, H_t, \mathcal{S}_t, \pi_{ht}, C^*_{ht}, C^*_{ft}, H^*_t, \tau_t\}} u(C_{ht}, C_{ft}) - \ell(H_t) + \mathbb{E}_t \beta v(\mathcal{Z}_{t+1}) \tag{A.20}$$

subject to

$$\begin{aligned}
\text{Balance of Payments} &: \quad C^*_{ht} = \mathcal{S}_t C_{ft} &\text{(A.21)}\\
\text{Market clearing Home} &: \quad A_t H_t \left(1 - \frac{\phi}{2}(\pi_{ht} - 1)^2\right) = C_{ht} + C^*_{ht} &\text{(A.22)}\\
\text{Market clearing Foreign} &: \quad A^*_t H^*_t \left(1 - \frac{\phi}{2}(\pi^*_{ft} - 1)^2\right) = C_{ft} + C^*_{ft} &\text{(A.23)}\\
\text{Labor market equilibrium Home} &: \quad \frac{\ell'(H_t)}{A_t} = u_{c_{ht}} \mathbb{E}_t \Psi(\pi_{ht}, \pi_{ht+1}, \theta) &\text{(A.24)}\\
\text{Labor market equilibrium Foreign} &: \quad \frac{\ell'(H^*_t)}{A^*_t} = u_{c^*_{ft}} \mathbb{E}_t \Psi(\pi^*_{ft}, \pi^*_{ft+1}, \theta^*) &\text{(A.25)}\\
\text{Optimal spending Foreign} &: \quad \frac{u_{c^*_{ht}}}{u_{c^*_{ft}}} = \frac{1+\tau^*_t}{\mathcal{S}_t} &\text{(A.26)}\\
\text{Optimal spending Home} &: \quad \frac{u_{c_{ht}}}{u_{c_{ft}}} = \frac{1}{\mathcal{S}_t(1+\tau_t)} &\text{(A.27)}
\end{aligned}$$

Since the policy-maker has free choice over τ_t, constraint (A.27) will not bind in equilibrium, so we can ignore it in the policy problem. Denote $\xi_{1,t}$,...,$\xi_{6,t}$ as the Lagrange multipliers on the constraints (A.21)-(A.26) respectively. The first-order conditions for the discretionary policy-maker are then listed as:

$$\begin{aligned}
C_{ht} &: \quad u_{c_{ht}} = \xi_{2,t} + \xi_{4,t} u_{c_{hht}} \Psi_t &\text{(A.28)}\\
C_{ft} &: \quad u_{c_{ft}} = \xi_{1,t}\mathcal{S}_t + \xi_{3,t} + \xi_{4,t} u_{c_{hft}} \Psi_t &\text{(A.29)}\\
H_t &: \quad \ell'(H_t) = \xi_{2,t} A_t \left(1 - \frac{\phi}{2}(\pi_{ht} - 1)^2\right) + \xi_{4,t}\frac{\ell''(H_t)}{A_t} &\text{(A.30)}\\
\mathcal{S}_t &: \quad -\xi_{1,t}\frac{c^*_{ht}}{\mathcal{S}_t} + \xi_{6,t} u^*_{c_{ht}} = 0 &\text{(A.31)}\\
\pi_{ht} &: \quad -\xi_{2,t} A_t H_t \phi(\pi_{ht} - 1) - \xi_{4,t} u_{c_{ht}} \phi(2\pi_{ht} - 1) &\text{(A.32)}\\
C^*_{ht} &: \quad \xi_{1,t} - \xi_{2,t} - \xi_{5,t} u^*_{c_{hft}} \Psi^*_t + \xi_{6,t}(\mathcal{S}_t u^*_{c_{hht}} - u^*_{c_{hft}}(1+\tau^*_t)) = 0 &\text{(A.33)}\\
C^*_{ft} &: \quad -\xi_{3,t} + \xi_{6,t}(u^*_{c_{hft}}\mathcal{S}_t - u^*_{c_{fft}}(1+\tau^*_t)) - \xi_{5,t} u^*_{c_{fft}} \Psi^*_t &\text{(A.34)}\\
H^*_t &: \quad \xi_{3,t} A^*_t \left(1 - \frac{\phi}{2}(\pi^*_{ft} - 1)^2\right) + \xi_{5,t}\frac{\ell''(H^*_t)}{A^*_t} = 0 &\text{(A.35)}
\end{aligned}$$

From Equation (A.31), we have:

$$\xi_{6,t} = \xi_{1,t}\frac{c^*_{ht}}{\mathcal{S}_t u^*_{c_{ht}}} \tag{A.36}$$

and from Equation (A.35):

$$\xi_{5,t} = -\xi_{3,t} \frac{A_t^* \left(1 - \frac{\phi}{2}(\pi_{ft}^* - 1)^2\right)}{\frac{\ell''(H_t^*)}{A_t^*}} \tag{A.37}$$

Use these in Equation (A.34) to get:

$$-\xi_{3,t} + \xi_{1,t} \frac{c_{ht}^*}{\mathcal{S}_t u_{c_{ht}}^*}(u_{c_{hft}}^* \mathcal{S}_t - u_{c_{fft}}^*(1+\tau_t^*)) + \xi_{3,t} \frac{A_t^* \left(1 - \frac{\phi}{2}(\pi_{ft}^* - 1)^2\right)}{\frac{\ell''(H_t^*)}{A_t^*}} u_{c_{fft}}^* \Psi_t^* = 0 \tag{A.38}$$

which gives:

$$\xi_{3,t} = \frac{\xi_{1,t} \frac{c_{ht}^*}{\mathcal{S}_t u_{c_{ht}}^*}(u_{c_{hft}}^* \mathcal{S}_t - u_{c_{fft}}^*(1+\tau_t^*))}{1 - \frac{A_t^* \left(1 - \frac{\phi}{2}(\pi_{ft}^* - 1)^2\right)}{\frac{\ell''(H_t^*)}{A_t^*}} u_{c_{fft}}^* \Psi_t^*} \tag{A.39}$$

From Equations (A.33) and (A.36), we have:

$$\begin{aligned}
\xi_{2,t} &= \xi_{1,t} + \xi_{1,t} \frac{c_{ht}^*}{\mathcal{S}_t u_{c_{ht}}^*}(\mathcal{S}_t u_{c_{hht}}^* - u_{c_{hft}}^*(1+\tau_t^*)) - \xi_{5,t} u_{c_{hft}}^* \Psi_t^* \\
&= \xi_{1,t} + \xi_{1,t} \frac{c_{ht}^*}{\mathcal{S}_t u_{c_{ht}}^*}(\mathcal{S}_t u_{c_{hht}}^* - u_{c_{hft}}^*(1+\tau_t^*)) + \xi_{3,t} \frac{A_t^* \left(1 - \frac{\phi}{2}(\pi_{ft}^* - 1)^2\right)}{\frac{\ell''(H_t^*)^*}{A_t}} u_{c_{hft}}^* \Psi_t^* \\
&= \xi_{1,t} + \xi_{1,t} \frac{c_{ht}^*}{\mathcal{S}_t u_{c_{ht}}^*}(\mathcal{S}_t u_{c_{hht}}^* - u_{c_{hft}}^*(1+\tau_t^*)) + \frac{\xi_{1,t} \frac{c_{ht}^*}{\mathcal{S}_t u_{c_{ht}}^*}(u_{c_{hft}}^* \mathcal{S}_t - u_{c_{fft}}^*(1+\tau_t^*))}{1 - \frac{A_t^* \left(1 - \frac{\phi}{2}(\pi_{ft}^* - 1)^2\right)}{\frac{\ell''(H_t^*)}{A_t^*}} u_{c_{fft}}^* \Psi_t^*} \frac{A_t^* \left(1 - \frac{\phi}{2}(\pi_{ft}^* - 1)^2\right)}{\frac{\ell''(H_t^*)}{A_t^*}} u_{c_{hft}}^* \Psi_t^*
\end{aligned} \tag{A.40}$$

From Equation (A.39), we have:

$$\xi_{1,t}\mathcal{S}_t + \xi_{3,t} = \xi_{1,t}\mathcal{S}_t + \frac{\xi_{1,t} \frac{c_{ht}^*}{\mathcal{S}_t u_{c_{ht}}^*}(u_{c_{hft}}^* \mathcal{S}_t - u_{c_{fft}}^*(1+\tau_t^*))}{1 - \frac{A_t^* \left(1 - \frac{\phi}{2}(\pi_{ft}^* - 1)^2\right)}{\frac{\ell''(H_t^*)}{A_t^*}} u_{c_{fft}}^* \Psi_t^*} \tag{A.41}$$

So, we get:

$$\begin{aligned}
\frac{\xi_{1,t}\mathcal{S}_t + \xi_{3,t}}{\xi_{2,t}} &= \frac{\mathcal{S}_t + \frac{\frac{c_{ht}^*}{\mathcal{S}_t u_{c_{ht}}^*}(u_{c_{hft}}^* \mathcal{S}_t - u_{c_{fft}}^*(1+\tau_t^*))}{1 - \frac{A_t^* \left(1 - \frac{\phi}{2}(\pi_{ft}^* - 1)^2\right)}{\frac{\ell''(H_t^*)}{A_t^*}} u_{c_{fft}}^* \Psi_t^*}}{1 + \frac{c_{ht}^*}{\mathcal{S}_t u_{c_{ht}}^*}(\mathcal{S}_t u_{c_{hht}}^* - u_{c_{hft}}^*(1+\tau_t^*)) + \frac{\frac{c_{ht}^*}{\mathcal{S}_t u_{c_{ht}}^*}(u_{c_{hft}}^* \mathcal{S}_t - u_{c_{fft}}^*(1+\tau_t^*))}{1 - \frac{A_t^* \left(1 - \frac{\phi}{2}(\pi_{ft}^* - 1)^2\right)}{\frac{\ell''(H_t^*)}{A_t^*}} u_{c_{fft}}^* \Psi_t^*} \frac{A_t^* \left(1 - \frac{\phi}{2}(\pi_{ft}^* - 1)^2\right)}{\frac{\ell''(H_t^*)}{A_t^*}} u_{c_{hft}}^* \Psi_t^*} \\
&= \mathcal{S}_t \frac{\eta_t}{\eta_t - 1}
\end{aligned} \tag{A.42}$$

where η_t is the Foreign country's general equilibrium elasticity of demand for Home goods, which is:

$$\eta_t = \frac{\frac{(u^*_{c_{fft}}(1+\tau^*_t)-u^*_{c_{hft}}\mathcal{S}_t)c^*_{ht}}{u^*_{c_{ht}}\mathcal{S}^2_t(1-\frac{A^{*2}_t(1-\varphi^*_t)\Psi^*_t u^*_{c_{fft}}}{\ell''(H^*_t)})} - 1}{\frac{(u^*_{c_{fft}}(1+\tau^*_t)-u^*_{c_{hft}}\mathcal{S}_t)c^*_{ht}}{u^*_{c_{ht}}\mathcal{S}^2_t(1-\frac{A^{*2}_t(1-\varphi^*_t)\Psi^*_t u^*_{c_{fft}}}{\ell''(H^*_t)})}(1-\mathcal{S}_t\frac{A^{*2}_t(1-\varphi^*_t)\Psi^*_t u^*_{c_{hft}}}{\ell''(H^*_t)}) + \frac{(u^*_{c_{hht}}\mathcal{S}_t-u^*_{c_{hft}}(1+\tau^*_t))c^*_{ht}}{u^*_{c_{ht}}\mathcal{S}_t}} \tag{A.43}$$

and $\varphi^*_t \equiv \frac{\phi}{2}(\pi^*_{ft}-1)^2$. From Equation (A.32) we have:

$$\frac{\xi_{4,t}}{\xi_{2,t}} = -\frac{A_t H_t \phi(\pi_{ht}-1)}{u_{c_{ht}}\phi(2\pi_{ht}-1)} \tag{A.44}$$

So that using Equations (A.28) and (A.29) we have:

$$\begin{aligned} \frac{u_{c_{ht}}}{u_{c_{ft}}} &= \frac{\xi_{2,t}+\xi_{4,t}u_{c_{hht}}\Psi_t}{\xi_{1,t}\mathcal{S}_t+\xi_{3,t}+\xi_{4,t}u_{c_{hft}}\Psi_t} \\ &= \frac{1+\frac{\xi_{4,t}}{\xi_{2,t}}u_{c_{hht}}\Psi_t}{\frac{\xi_{1,t}\mathcal{S}_t+\xi_{3,t}}{\xi_{2,t}}+\frac{\xi_{4,t}}{\xi_{2,t}}u_{c_{hft}}\Psi_t} \\ &= \frac{1-\frac{A_t H_t\phi(\pi_{ht}-1)}{u_{c_{ht}}\phi(2\pi_{ht}-1)}u_{c_{hht}}\Psi_t}{\frac{\mathcal{S}_t\eta_t}{\eta_t-1}-\frac{A_t H_t\phi(\pi_{ht}-1)}{u_{c_{ht}}\phi(2\pi_{ht}-1)}u_{c_{hft}}\Psi_t} \end{aligned} \tag{A.45}$$

Then, using the competitive equilibrium condition:

$$\frac{u_{c_{ht}}}{u_{c_{ft}}} = \frac{1}{\mathcal{S}_t(1+\tau_t)} \tag{A.46}$$

we have:

$$\frac{1}{1+\tau_t} = \frac{1-\frac{A_t H_t\phi(\pi_{ht}-1)}{u_{c_{ht}}\phi(2\pi_{ht}-1)}u_{c_{hht}}\Psi_t}{\frac{\eta_t}{\eta_t-1}-\frac{A_t H_t\phi(\pi_{ht}-1)}{\mathcal{S}_t u_{c_{ht}}\phi(2\pi_{ht}-1)}u_{c_{hft}}\Psi_t} \tag{A.47}$$

From Equation (A.32) we have:

$$\xi_{4,t} = -\xi_{2,t}\frac{A_t h_t\phi(\pi_{ht}-1)}{u_{c_{ht}}\phi(2\pi_{ht}-1)} \tag{A.48}$$

Using this with (A.28) and (A.30) we arrive at the following description for the labor market condition:

$$\frac{\ell'(H_t)}{u_{c_{ht}}} = \frac{\left(A_t\left(1-\frac{\phi}{2}(\pi_{ht}-1)^2\right)-\frac{A_t H_t\phi(\pi_{ht}-1)}{u_{c_{ht}}\phi(2\pi_{ht}-1)}\frac{\ell''(H_t)}{A_t}\right)}{\left(1-\frac{A_t H_t\phi(\pi_{ht}-1)}{u_{c_{ht}}\phi(2\pi_{ht}-1)}u_{cc_{ht}}\Psi_t\right)} \tag{A.49}$$

which, from Equation (A.24) gives:

$$\Psi(\pi_{ht}, \mathbb{E}_t\pi_{ht+1}) = \frac{\left(\left(1 - \frac{\phi}{2}(\pi_{ht} - 1)^2\right) - \frac{\phi(\pi_{ht}-1)}{\phi(2\pi_{ht}-1)}\frac{H_t\ell''(h_t)}{\ell'(H_t)}\Psi_t\right)}{\left(1 - \frac{A_tH_t\phi(\pi_{ht}-1)}{u_{c_{ht}}\phi(2\pi_{ht}-1)}u_{cc_{ht}}\Psi_t\right)} \tag{A.50}$$

This implicitly determines the inflation rate in the Home country.

Also, this analysis pertains only to the Home country's tariff decisions. The other country's decision is exactly analogous. Then tariffs will be determined simultaneously in the Markov Nash game between countries.

Results 3 and 4 may be obtained from Equations (A.47) and (A.50). If monopoly distortions are zero, then inflation will be zero and the tariffs will follow the optimal monopoly tariff rule.

Proof of Result 5. Either in the case of zero inflation, or purely flexible prices, we can set $\phi = 0$, and from Equation (A.30) together with Equations (A.28) and (A.29) and the definition of η_t from above, we obtain the implicit tariff formula as:

$$\frac{1}{1+\tau_t} = \frac{1 + \Omega_t u_{c_{hht}}\Psi_t}{\frac{\eta_t}{\eta_t - 1} + \Omega_t u_{c_{hft}}\Psi_t} \tag{A.51}$$

where $\Omega_t = \frac{(\theta-1)A_t}{\frac{\ell''(H_t)}{A_t} - u_{c_{hht}}A_t\theta^2}$, and $\theta = \frac{(1+s)(\epsilon-1)}{\epsilon} \leq 1$. From Equation (A.51), we conclude that in a distorted economy, where $\theta < 1$ with flexible prices ($\phi = 0$), the Nash equilibrium tariff in the currency war game will be less than the pure monopoly tariff rate. Intuitively, this is because policy-makers take account of the distortionary impacts of the tariff on domestic production, which is inefficiently low when $\theta < 1$.

Proof of Result 6 - Part 1

Let inflation be determined cooperatively and tariffs non-cooperatively. We define the terms of trade as $\mathcal{S}_t = \frac{S_tP^*_{ft}}{P_{ht}}$. The policy problem for the cooperative government is defined in the form of a value function:

$$v(\mathcal{Z}_t) = \text{Max}_{\{C_{ht},C_{ft},H_t,H^*_t,C^*_{ht},C^*_{ft},\mathcal{S}_t,\pi_{ht},\pi^*_{ft}\}} u(C_{ht}, C_{ft}) - \ell(H_t) + u(C^*_{ht}, C^*_{ft}) - \ell(H^*_t) + \mathbb{E}_t\beta v(\mathcal{Z}_{t+1}) \tag{A.52}$$

subject to

$$\text{Balance of Payments} \quad : \quad C^*_{ht} = \mathcal{S}_t C_{ft} \tag{A.53}$$

$$\text{Market clearing Home} \quad : \quad A_t H_t \left(1 - \frac{\phi}{2}(\pi_{ht} - 1)^2\right) = C_{ht} + C^*_{ht} \tag{A.54}$$

$$\text{Market clearing Foreign} \quad : \quad A^*_t H^*_t \left(1 - \frac{\phi}{2}(\pi^*_{ft} - 1)^2\right) = C_{ft} + C^*_{ft} \tag{A.55}$$

$$\text{Labor market equilibrium Home} \quad : \quad \frac{\ell'(H_t)}{A_t} = u_{c_{ht}} \mathbb{E}_t \Psi(\pi_{ht}, \pi_{ht+1}, \theta) \tag{A.56}$$

$$\text{Labor market equilibrium Foreign} \quad : \quad \frac{\ell'(H^*_t)}{A^*_t} = u_{c^*_{ft}} \mathbb{E}_t \Psi(\pi^*_{ft}, \pi^*_{ft+1}, \theta^*) \tag{A.57}$$

$$\text{Optimal spending Home} \quad : \quad \frac{u_{c_{ht}}}{u_{c_{ft}}} = \frac{1}{\mathcal{S}_t(1+\tau_t)} \tag{A.58}$$

$$\text{Optimal spending Foreign} \quad : \quad \frac{u_{c^*_{ht}}}{u_{c^*_{ft}}} = \frac{1+\tau^*_t}{\mathcal{S}_t} \tag{A.59}$$

The first-order conditions for the discretionary policy-maker are then listed as:

$$C_{ht} \quad : \quad u_{c_{ht}} = \xi_{2,t} + \xi_{4,t} u_{c_{hht}} \Psi_t - \xi_{6,t}(u_{c_{hht}} \mathcal{S}_t (1+\tau_t) - u_{c_{hft}}) \tag{A.60}$$

$$C_{ft} \quad : \quad u_{c_{ft}} = \xi_{1,t}\mathcal{S}_t + \xi_{3,t} + \xi_{4,t} u_{c_{hft}} \Psi_t + \xi_{6,t}(u_{c_{fft}} - u_{c_{hft}}(1+\tau_t)\mathcal{S}_t) \tag{A.61}$$

$$H_t \quad : \quad \ell'(H_t) = \xi_{2,t} A_t \left(1 - \frac{\phi}{2}(\pi_{ht}-1)^2\right) + \xi_{4,t}\frac{\ell''(H_t)}{A_t} \tag{A.62}$$

$$\mathcal{S}_t \quad : \quad -\xi_{1,t}\frac{c^*_{ht}}{\mathcal{S}_t} + \xi_{6,t} u_{c_{ht}}(1+\tau_t) + \xi_{7,t} u^*_{c_{ht}} = 0 \tag{A.63}$$

$$\pi_{ht} \quad : \quad -\xi_{2,t} A_t H_t \phi(\pi_{ht}-1) - \xi_{4,t} u_{c_{ht}} \phi(2\pi_{ht}-1) = 0 \tag{A.64}$$

$$C^*_{ht} \quad : \quad u^*_{c_{ht}} + \xi_{1,t} - \xi_{2,t} - \xi_{5,t} u^*_{c_{hft}} \Psi^*_t + \xi_{7,t}(\mathcal{S}_t u^*_{c_{hht}} - u^*_{c_{hft}}(1+\tau^*_t)) = 0 \tag{A.65}$$

$$C^*_{ft} \quad : \quad u^*_{c_{ft}} - \xi_{3,t} + \xi_{7,t}(u^*_{c_{hft}}\mathcal{S}_t - u^*_{c_{fft}}(1+\tau^*_t)) - \xi_{5,t} u^*_{c_{fft}} \Psi^*_t = 0 \tag{A.66}$$

$$H^*_t \quad : \quad -\ell'(H^*_t) + \xi_{3,t} A^*_t \left(1 - \frac{\phi}{2}(\pi^*_{ft}-1)^2\right) + \xi_{5,t}\frac{\ell''(H^*_t)}{A^*_t} = 0 \tag{A.67}$$

$$\pi^*_{ft} \quad : \quad -\xi_{3,t} A^*_t H^*_t \phi(\pi^*_{ft}-1) - \xi_{5,t} u^*_{c_{ft}} \phi(2\pi^*_{ft}-1) = 0 \tag{A.68}$$

Result 6 states that, when tariffs are positive, the cooperative policy-maker will depart from a zero inflation policy even when the monopoly distortion in prices is offset by an optimal subsidy. To prove it, start by assuming the opposite. Say the cooperative policy-maker sets inflation to zero in both countries, *i.e.* $\pi_{ht} = \pi^*_{ft} = 0$. Then by Equation (A.64) and Equation (A.68) we must have $\xi_4 = \xi_5 = 0$. But if the firms receive an optimal subsidy, then we must have: $\ell'(H_t) = A_t u_{c_{ht}}$ and $\ell'(H^*_t) = A^*_t u^*_{c_{ft}}$, so that from Equations (A.62) and (A.67), we must have $\xi_{2,t} = u_{c_{ht}}$ and $\xi_{3,t} = u^*_{c_{ft}}$. Then from Equations (A.60) and (A.66), we must have $\xi_{6,t} = \xi_{7,t} = 0$. And from Equation (A.63), we must have $\xi_{1,t} = 0$. This then implies from Equations (A.60) and (A.65), and also from Equations (A.61) and (A.66), that $u_{c_{ht}} = u^*_{c_{ht}}$ and $u_{c_{ft}} = u^*_{c_{ft}}$. But this violates the optimal spending equations (A.58) and (A.59), which together imply

$$\frac{u_{c_{ht}}}{u^*_{c_{ht}}} = \frac{u_{c_{ft}}}{u^*_{c_{ft}}} \frac{1}{(1+\tau_t)(1+\tau^*_t)} \tag{A.69}$$

Thus, we have a contradiction. So cooperative policy-making with non-cooperative tariff setting will not close the output gap, even if an optimal subsidy is in place.

Intuitively, the cooperative planner will depart from zero inflation if tariffs are positive, because there is a distortion preventing full consumption risk-sharing across countries. We can see this more clearly as follows.

We may show more directly how this impacts on the equilibrium rate of inflation. Using Equations (A.63), (A.62), and (A.60) we obtain:

$$\frac{\ell'(H_t)}{u_{c_{ht}}} = \frac{A_t\left(1-\frac{\phi}{2}(\pi_{ht}-1)^2\right) - \frac{\ell''(H_t)H_t(\pi_{ht}-1)}{u_{c_{ht}}(2\pi_{ht}-1)}}{1-\frac{A_tH_t(\pi_{ht}-1)}{(2\pi_{ht}-1)}u_{c_{hht}}\Psi_t - \frac{\xi_{6,t}}{\xi_{2,t}}u_{c_{hht}}} \tag{A.70}$$

Using Equation (A.56), we can write this as an equation determining the inflation rate (also imposing a symmetric equilibrium with $\mathcal{S}_t = 1$):

$$\Psi_t = \frac{1-\frac{\phi}{2}(\pi_{ht}-1)^2 - \frac{\ell''(H_t)H_t(\pi_{ht}-1)}{A_tu_{c_{ht}}(2\pi_{ht}-1)}}{1-\frac{A_tH_t(\pi_{ht}-1)}{u_{c_{ht}}(2\pi_{ht}-1)}u_{c_{hht}}\Psi_t - \frac{\xi_{6,t}}{\xi_{2,t}}(u_{c_{hht}}(1+\tau_t)-u_{c_{hft}})} \tag{A.71}$$

where in a symmetric equilibrium it can be shown that:

$$\frac{\xi_{6,t}}{\xi_{2,t}} = \frac{u_{c_{ht}}\tau_t}{\xi_{2,t}(u_{c_{hht}}(1+\tau_t)+u_{c_{fft}}-2u_{c_{hft}})} + \frac{(u_{c_{hht}}-u_{c_{hft}})\Psi_tA_tH_t(\pi_{ht}-1)}{u_{c_{ht}}(2\pi_{ht}-1)} \tag{A.72}$$

with $\xi_{2,t} = \frac{\ell''(H_t)}{A_t\left(1-\frac{\phi}{2}(\pi_{ht}-1)^2\right)-\frac{\ell''(H_t)H_t(\pi_{ht}-1)}{u_{c_{ht}}(2\pi_{ht}-1)}}$

Now take Equation (A.71), and impose a steady state. Assume that $\theta = 1$, and then assume that inflation was zero, so $\pi_h = 1$. Then the left-hand side of Equation (A.71) is unity, while the right-hand side is greater than unity, using Equation (A.72) as long as there is a positive tariff rate, *i.e.* $\tau > 0$. Since the left-hand side is increasing in π_h and the right-hand side is decreasing in π_h, it must be that the equilibrium cooperative inflation rate is greater than zero when $\theta = 1$ and $\tau > 0$.

Proof of Result 6 - part 2

The policy problem for the home tariff setter when inflation is chosen by the cooperative planner is:

$$v(\mathcal{Z}_t) = \text{Max}_{\{C_{ht},C_{ft},H_t,\mathcal{S}_t,C^*_{ht},C^*_{ft},H^*_t,\ \tau_t\}}u(C_{ht},C_{ft}) - \ell(H_t) + \mathbb{E}_t\beta v(\mathcal{Z}_{t+1}) \tag{A.73}$$

subject to

$$\text{Balance of Payments} \quad : \quad C^*_{ht} = \mathcal{S}_t C_{ft} \tag{A.74}$$

$$\text{Market clearing Home} \quad : \quad A_t H_t (1 - \frac{\phi}{2}(\pi_{ht} - 1)^2) = C_{ht} + C^*_{ht} \tag{A.75}$$

$$\text{Market clearing Foreign} \quad : \quad A^*_t H^*_t (1 - \frac{\phi}{2}(\pi^*_{ft} - 1)^2) = C_{ft} + C^*_{ft} \tag{A.76}$$

$$\text{Labor market equilibrium Home} \quad : \quad \frac{\ell'(H_t)}{A_t} = u_{c_{ht}} \mathbb{E}_t \Psi(\pi_{ht}, \pi_{ht+1}, \theta) \tag{A.77}$$

$$\text{Labor market equilibrium Foreign} \quad : \quad \frac{\ell'(H^*_t)}{A^*_t} = u_{c^*_{ft}} \mathbb{E}_t \Psi(\pi^*_{ft}, \pi^*_{ft+1}, \theta^*) \tag{A.78}$$

$$\text{Optimal spending Foreign} \quad : \quad \frac{u_{c^*_{ht}}}{u_{c^*_{ft}}} = \frac{1 + \tau^*_t}{\mathcal{S}_t} \tag{A.79}$$

$$\text{Optimal spending Home} \quad : \quad \frac{u_{c_{ht}}}{u_{c_{ft}}} = \frac{1}{\mathcal{S}_t (1 + \tau_t)} \tag{A.80}$$

Again, Equation (A.80) will not bind. The first-order conditions for the discretionary policy-maker are then listed as:

$$C_{ht} \quad : \quad u_{c_{ht}} = \xi_{2,t} + \xi_{4,t} u_{c_{hht}} \Psi_t \tag{A.81}$$

$$C_{ft} \quad : \quad u_{c_{ft}} = \xi_{1,t} \mathcal{S}_t + \xi_{3,t} + \xi_{4,t} u_{c_{hft}} \Psi_t \tag{A.82}$$

$$H_t \quad : \quad \ell'(H_t) = \xi_{2,t} A_t \left(1 - \frac{\phi}{2}(\pi_{ht} - 1)^2\right) + \xi_{4,t} \frac{\ell''(H_t)}{A_t} \tag{A.83}$$

$$\mathcal{S}_t \quad : \quad -\xi_{1,t} \frac{c^*_{ht}}{\mathcal{S}_t} + \xi_{6,t} u^*_{c_{ht}} = 0 \tag{A.84}$$

$$C^*_{ht} \quad : \quad \xi_{1,t} - \xi_{2,t} - \xi_{5,t} u^*_{c_{hft}} \Psi^*_t + \xi_{6,t} (\mathcal{S}_t u^*_{c_{hht}} - u^*_{c_{hft}} (1 + \tau^*_t)) = 0 \tag{A.85}$$

$$C^*_{ft} \quad : \quad -\xi_{3,t} + \xi_{6,t} (u^*_{c_{hft}} \mathcal{S}_t - u^*_{c_{fft}} (1 + \tau^*_t)) - \xi_{5,t} u^*_{c_{fft}} \Psi^*_t = 0 \tag{A.86}$$

$$H^*_t \quad : \quad \xi_{3,t} A^*_t \left(1 - \frac{\phi}{2}(\pi^*_{ft} - 1)^2\right) + \xi_{5,t} \frac{\ell''(H^*_t)}{A^*_t} = 0 \tag{A.87}$$

Following the steps from Equation (A.42) we have:

$$\frac{u_{c_{ht}}}{u_{c_{ft}}} = \frac{1 + \frac{\xi_{4,t}}{\xi_{2,t}} u_{c_{hht}} \Psi_t}{\frac{\xi_{1,t}\mathcal{S}_t + \xi_{3,t}}{\xi_{2,t}} + \frac{\xi_{4,t}}{\xi_{2,t}} u_{c_{hft}} \Psi} \tag{A.88}$$

where $\frac{\xi_{1,t}\mathcal{S}_t + \xi_{3,t}}{\xi_{2,t}}$ is the same as in Equation (A.42), with

$$\xi_{4,t} = \frac{\ell'(H_t) - A_t u_{c_{ht}} \left(1 - \frac{\phi}{2}(\pi_{ht} - 1)^2\right)}{\frac{\ell''(H_t)}{A_t} - u_{c_{hht}} \Psi_t A_t \left(1 - \frac{\phi}{2}(\pi_{ht} - 1)^2\right)}$$

and

$$\xi_{2,t} = u_{c_{ht}} - u_{c_{hht}} \Psi_t \frac{\ell'(H_t) - A_t u_{c_{ht}} \left(1 - \frac{\phi}{2}(\pi_{ht} - 1)^2\right)}{\frac{\ell''(H_t)}{A_t} - u_{c_{hht}} \Psi_t A_t \left(1 - \frac{\phi}{2}(\pi_{ht} - 1)^2\right)}$$

If we assume $\theta = 1$, then from from Equation (A.77), in a steady state, we have

$$\ell'(H_t) - A_t u_{c_{ht}} \left(1 - \frac{\phi}{2}(\pi_{ht} - 1)^2\right) = A u_{c_h} \left(\phi \pi_h (\pi_h - 1) + \frac{\phi}{2}(\pi_h - 1)^2\right) > 0 \tag{A.89}$$

Then $\frac{\xi_4}{\xi_2} > 0$, and from Equation (A.42) ,we can describe the optimal home tariff by the condition:

$$\frac{1}{1+\tau} = \frac{1 + \frac{\xi_4}{\xi_2} u_{c_{hh}} \Psi}{\frac{\eta}{\eta - 1} + \frac{\xi_4}{\xi_2} u_{c_{hf}} \Psi} \tag{A.90}$$

where we have used the notation for the steady-state Foreign demand elasticity η. Since $\frac{\xi_4}{\xi_2} > 0$ it follows that in the case $\theta = 1$, and monetary policy is determined cooperatively, the tariff rate exceeds the monopoly tariff rate.

B General Model derivation

We describe a two country model, denoted Home and Foreign, where agents supply labor and consume goods from both countries. The world is populated with a unit mass of agents and Home has share n of these, with Foreign share $1-n$. We assume that firms set prices in domestic currency (PCP), and adjust prices constrained by Rotemberg-style price adjustment costs. Agents in the Home country have preferences over consumption and hours given by

$$U_t = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{\chi H_t^{1+\psi}}{1+\psi} \tag{B.91}$$

We assume trade in bonds across countries.

B.1 Households

The representative Home household maximizes its welfare index

$$\mathbb{E}_t \left\{ \sum_{j=0}^{\infty} \beta^j \left(\frac{C_{t+j}^{1-\sigma}}{1-\sigma} - \chi \frac{H_{t+j}^{1+\psi}}{1+\psi} \right) \right\} \tag{B.92}$$

subject to the following budget constraint:

$$S_t B_t^* + B_t + P_{ht} C_{ht} + (1+\tau_t) S_t P_{ft}^* C_{ft} + P_t \Lambda_t = S_t B_{t-1}^* R_{t-1}^* + B_{t-1} R_{t-1} + W_t H_t + \Pi_t + TR_t \tag{B.93}$$

where B_t^* and B_t are the amounts of Foreign and Home currency-denominated bonds bought by Home households, paying returns R_t^* and R_t between t and $t+1$. Buying Foreign bonds incurs the payment of a small adjustment cost $\Lambda_t = \frac{\nu}{2} \left(\frac{S_t B_t^*}{P_t} - \frac{SB^*}{P} \right)^2$, proportional to the deviation of real Foreign bonds to their steady-state value. The bundle structure of adjustment costs mimics that of final goods. The representative household in the Home economy consumes local goods in quantity C_{ht} at the price P_{ht} and foreign goods in quantity C_{ft} at the price $(1+\tau_t) S_t P_{ft}^*$. The consumption bundle is

$$C_t = \left(\varepsilon^{1/\lambda} C_{ht}^{1-1/\lambda} + (1-\varepsilon)^{1/\lambda} C_{ft}^{1-1/\lambda} \right)^{\frac{1}{1-1/\lambda}} \tag{B.94}$$

where $\varepsilon = n + x(1-n)$, with x denoting Home bias, and the aggregate consumption price index is

$$P_t = \left(\varepsilon P_{ht}^{1-\lambda} + (1-\varepsilon) \left((1+\tau_t) S_t P_{ft}^* \right)^{1-\lambda} \right)^{\frac{1}{1-\lambda}} \tag{B.95}$$

so that $P_{ht}C_{ht} + (1+\tau_t) S_t P^*_{ft} C_{ft} = P_t C_t$. The demand functions of local and foreign goods are respectively

$$C_{ht} = \varepsilon \left(\frac{P_{ht}}{P_t}\right)^{-\lambda} C_t = \varepsilon \mathcal{P}_t^{\lambda} C_t \tag{B.96}$$

$$C_{ft} = (1-\varepsilon)\left(\frac{(1+\tau_t) S_t P^*_{ft}}{P_t}\right)^{-\lambda} C_t = (1-\varepsilon)\left(\frac{\mathcal{P}_t}{(1+\tau_t)\mathcal{S}_t}\right)^{\lambda} C_t \tag{B.97}$$

where $\mathcal{P}_t = P_t/P_{ht} = \left(\varepsilon + (1-\varepsilon)\left((1+\tau_t)\mathcal{S}_t\right)^{1-\lambda}\right)^{\frac{1}{1-\lambda}}$ represents the relative price of the home consumption good and $\mathcal{S}_t = S_t P^*_{ft}/P_{ht}$ denotes the Home terms-of-trade. The first-order conditions of the Home household imply

$$\beta \mathbb{E}_t \left\{ \frac{\mathcal{S}_{t+1} R^*_t \mathcal{P}_t C_t^{\sigma}}{\mathcal{S}_t \pi^*_{ft+1} \mathcal{P}_{t+1} C^{\sigma}_{t+1}\left(1+\nu\left(\frac{S_t B^*_t}{P_t} - \frac{SB^*}{P}\right)\right)} \right\} = 1 \tag{B.98}$$

$$\beta \mathbb{E}_t \left\{ \frac{R_t \mathcal{P}_t C_t^{\sigma}}{\pi_{ht+1} \mathcal{P}_{t+1} C^{\sigma}_{t+1}} \right\} = 1 \tag{B.99}$$

$$\chi H_t^{\psi} C_t^{\sigma} = \frac{\mathcal{W}_t}{\mathcal{P}_t} \tag{B.100}$$

where $\pi_{ht} = P_{ht}/P_{ht-1}$ and $\pi^*_{ft} = P^*_{ft}/P^*_{ft-1}$ are the gross rates of PPI inflation in the Home and Foreign country respectively, and $\mathcal{W}_t = W_t/P_{ht}$. Turning to the Foreign representative household, the consumption bundle and price index are respectively

$$C_t^* = \left(\varepsilon^{*1/\lambda} C_{ft}^{*1-1/\lambda} + (1-\varepsilon^*)^{1/\lambda} C_{ht}^{*1-1/\lambda}\right)^{\frac{1}{1-1/\lambda}} \tag{B.101}$$

$$P_t^* = \left(\varepsilon^* P_{ft}^{*1-\lambda} + (1-\varepsilon^*)\left((1+\tau_t^*)\frac{P_{ht}}{S_t}\right)^{1-\lambda}\right)^{\frac{1}{1-\lambda}} \tag{B.102}$$

and the corresponding demand function are

$$C^*_{ft} = \varepsilon^* \left(\frac{P_{ft}}{P_t^*}\right)^{-\lambda} = \varepsilon^* \mathcal{P}_t^{*\lambda} C_t^* \tag{B.103}$$

$$C^*_{ht} = (1-\varepsilon^*)\left(\frac{(1+\tau_t^*) P_{ht}}{S_t P_t^*}\right)^{-\lambda} = (1-\varepsilon^*)\left(\frac{\mathcal{S}_t \mathcal{P}_t^*}{(1+\tau_t^*)}\right)^{\lambda} C_t^* \tag{B.104}$$

where $\mathcal{P}_t^* = P_t^*/P^*_{ft} = \left(\varepsilon^* + (1-\varepsilon^*)\left((1+\tau_t^*)/\mathcal{S}_t\right)^{1-\lambda}\right)^{\frac{1}{1-\lambda}}$. The representative foreign household faces a different constraint, accessing only local bonds without paying adjustment costs. Its labor supply equation is

$$\chi C_t^{*\sigma} H_t^{*\psi} = \frac{W_t^*}{P_t^*} = \frac{\mathcal{W}_t^*}{\mathcal{P}_t^*} \tag{B.105}$$

where $\mathcal{W}_t^* = W_t^*/P_{ft}^*$ and the Euler equation associated with Foreign bonds gives

$$\beta \mathbb{E}_t \left\{ \frac{R_t^* \mathcal{P}_t^* C_t^{*\sigma}}{\pi_{ft+1}^* \mathcal{P}_{t+1}^* C_{t+1}^{*\sigma}} \right\} = 1 \tag{B.106}$$

B.2 Firms

A measure n of firms in the Home economy produce differentiated goods. The aggregate Home good is a composite of these differentiated goods, where the elasticity of substitution between individual goods is denoted $\epsilon > 1$. The production function for firm i in the Home country is

$$Y_t(i) = A_t H_t(i)^{1-\alpha} X_t(i)^{\alpha} \tag{B.107}$$

where A_t is an exogenous aggregate productivity term. Here, $X_t(i)$ represents the use of intermediate goods by the Home firm i and $L_t(i)$ the use of labor. We allow that intermediate good inputs are composed of Home and Foreign goods in a different composition than that of the consumption aggregator. Namely

$$X_t(i) = \left(\varepsilon_x^{\frac{1}{\lambda}} X_{ht}(i)^{\frac{\lambda-1}{\lambda}} + (1-\varepsilon_x)^{\frac{1}{\lambda}} X_{ft}(i)^{\frac{\lambda-1}{\lambda}} \right)^{\frac{\lambda}{\lambda-1}}$$

where $X_{jt}(i)$ is the Home firm i's use of inputs from country $j = h, f$. The profits of Home firm i are then represented as

$$\Pi_t(i) = ((1+s)P_{ht}(i) - MC_t)\, Y_t(i) \tag{B.108}$$

where $MC_t = A_t^{-1}(1-\alpha)^{\alpha-1}\alpha^{-\alpha}W_t^{1-\alpha}P_{xt}^{\alpha}$ denotes the firm's marginal cost, and

$$P_{xt} = \left(\varepsilon_x P_{ht}^{1-\lambda} + (1-\varepsilon_x)((1+\tau_t)S_t P_{ft}^*)^{1-\lambda} \right)^{\frac{1}{1-\lambda}} \tag{B.109}$$

is the price index relevant for the firm's use of intermediate inputs, $s_t(i)$ represents a subsidy that may be given to the firm to offset the monopoly distortion in pricing and where τ_t is an ad-valorem tarif on imports. Cost minimization by the firm implies:

$$(1-\alpha)\frac{Y_t(i)}{H_t(i)} = \frac{W_t}{MC_t} \text{ and } \alpha \frac{Y_t(i)}{X_t(i)} = \frac{P_{xt}}{MC_t} \tag{B.110}$$

with

$$X_{ht}(i) = \varepsilon_x \left(\frac{P_{ht}}{P_{xt}} \right)^{-\lambda} X_t(i) = \varepsilon_x \mathcal{P}_{xt}^{\lambda} X_t(i) \tag{B.111}$$

$$X_{ft}(i) = (1-\varepsilon_x) \left(\frac{(1+\tau_t)S_t P_{ft}^*}{P_{xt}} \right)^{-\lambda} X_t(i) = (1-\varepsilon_x) \left(\frac{\mathcal{P}_{xt}}{(1+\tau_t)\mathcal{S}_t} \right)^{\lambda} X_t(i) \tag{B.112}$$

where $\mathcal{P}_{xt}$ is the equivalent of $\mathcal{P}_t$ for intermediate goods.[25] The firm chooses its price to maximize its present value of expected profits, net of price adjustment costs

$$\mathbb{E}_t \left\{ \sum_{j=0}^{\infty} \omega_{t+j} \left(\Pi_{t+j}(i) - \frac{\phi}{2} \left(\frac{P_{ht+j}(i)}{P_{ht+j-1}(i)} - 1 \right)^2 P_{ht+j}(i) \, Y_{t+j}(i) \right) \right\} \tag{B.113}$$

where ω_t is the firm's nominal stochastic discount factor, and ϕ represents a price adjustment cost for the firm. Price adjustment costs are proportional to the nominal value of Home output, to be consistent with the nominal profit objective function of the firm. The first order condition for profit maximization for the Home firm i takes into account the individual demand of good i, *i.e* $Y_t^d(i) = (P_t(i)/P_t)^{-\epsilon} Y_t$ and is the same for all producers so that $P_{ht}(i) = P_{ht}$ and $Y_t(i) = Y_t$ and that the i index can be dropped. It implies

$$(1+s)(1-\epsilon) + \epsilon \mathcal{MC}_t - \phi \left(\pi_{ht} (\pi_{ht} - 1) - \mathbb{E}_t \left\{ \omega_{t+1} \pi_{ht+1} (\pi_{ht+1} - 1) \, Y_{t+1}/Y_t \right\} \right) = 0 \tag{B.114}$$

where

$$\mathcal{MC}_t = MC_t/P_{ht} = \mathcal{MC}_t = \frac{\mathcal{W}_t^{1-\alpha} \mathcal{P}_{xt}^{\alpha}}{A_t \alpha^{\alpha} (1-\alpha)^{1-\alpha}} \text{ and } \omega_t = \beta \frac{C_{t-1}^{\sigma} \mathcal{P}_{t-1}}{C_t^{\sigma} \mathcal{P}_t} \tag{B.115}$$

Using symmetry among producers, the factor demands can be rewritten as

$$(1-\alpha) \mathcal{MC}_t Y_t = \mathcal{W}_t H_t \text{ and } \alpha \mathcal{MC}_t Y_t = \mathcal{P}_{xt} X_t \tag{B.116}$$

where $\mathcal{P}_{xt} = P_{xt}/P_{ht}$.

B.3 Economic Policy

There are three separate levers of policy in this model. Fiscal policy may be used to subsidize monopoly firms. Trade policy may be used to levy tariffs on imports, and monetary policy may be used to either target inflation rates or exchange rates. In the case where firms are subsidized, we follow the literature in assuming that a fiscal authority chooses a subsidy to offset the steady-state monopoly markup. But we also allow for the possibility that the monopoly markup remains as a pre-existing distortion in the economy. As we see, this may have an important implication for both optimal monetary policy and trade policy.

[25] $\mathcal{P}_t$ and $\mathcal{P}_{xt}$ only differ by the presence of potentially different degrees of home biais.

C The Competitive Equilibrium

We assume that governments rebate the proceeds from tariffs – net from the production subsidy s – to the household using lump-sum transfers. Given that Rotemberg costs are paid in units of local goods and using the demand functions for intermediate and final goods, the goods market clearing conditions are given by:

$$Y_t \left(1 - \frac{\phi}{2} \left(\pi_{ht} - 1\right)^2\right) = D_t + D^*_{xt} \tag{C.1}$$

$$Y^*_t \left(1 - \frac{\phi}{2} \left(\pi^*_{ft} - 1\right)^2\right) = D^*_t + D_{xt} \tag{C.2}$$

where

$$D_t = \varepsilon \mathcal{P}_t^{\lambda} \left(C_t + \Lambda_t\right) + \varepsilon_x \mathcal{P}_{xt}^{\lambda} X_t \; ; \; D_{xt} = \frac{n}{1-n} \left(\frac{\mathcal{S}_t^{-1}}{1+\tau_t}\right)^{\lambda} \left(\left(1-\varepsilon\right) \mathcal{P}_t^{\lambda} \left(C_t + \Lambda_t\right) + \left(1-\varepsilon_x\right) \mathcal{P}_{xt}^{\lambda} X_t\right) \tag{C.3}$$

$$D^*_t = \varepsilon^* \mathcal{P}_t^{*\lambda} C^*_t + \varepsilon^*_x \mathcal{P}_{xt}^{*\lambda} X^*_t \; ; \; D^*_{xt} = \frac{1-n}{n} \left(\frac{\mathcal{S}_t}{1+\tau^*_t}\right)^{\lambda} \left(\left(1-\varepsilon^*\right) \mathcal{P}_t^{*\lambda} C^*_t + \left(1-\varepsilon^*_x\right) \mathcal{P}_{xt}^{*\lambda} X^*_t\right) \tag{C.4}$$

The labor market clearing conditions are:

$$(1-\alpha) \mathcal{MC}_t A_t L_t^{-\alpha} X_t^{\alpha} = \chi \mathcal{P}_t C_t^{\sigma} L_t^{\psi} \tag{C.5}$$

$$(1-\alpha) \mathcal{MC}^*_t A^*_t L_t^{*-\alpha} X_t^{*\alpha} = \chi \mathcal{P}^*_t C_t^{*\sigma} L_t^{*\psi} \tag{C.6}$$

Finally, Home bonds are in zero net supply so that $B_t = 0$ and the clearing condition on the market for Foreign bonds writes

$$nB^*_t + \left(1-n\right) B^{**}_t = 0 \tag{C.7}$$

Defining $b_t = \frac{S_t B^*_t}{P_t}$ and $b^*_t = \frac{B^{**}_t}{P^*_t}$ as the real per-capita net foreign asset positions, the latter condition implies

$$nb_t + \left(1-n\right) \frac{\mathcal{S}_t \mathcal{P}^*_t}{\mathcal{P}_t} b^*_t = 0 \tag{C.8}$$

Further, the modified IUP condition stemming from the combination of Home and Foreign Euler Equations writes

$$\mathbb{E}_t \left\{ \frac{\mathcal{S}_{t+1} \omega_{t+1}}{\mathcal{S}_t \omega^*_{t+1} \left(1 + \nu \left(b_t - b\right)\right)} - 1 \right\} = 0 \tag{C.9}$$

where $\omega_t = \beta \frac{C_{t-1}^{\sigma} \mathcal{P}_{t-1}}{C_t^{\sigma} \mathcal{P}_t}$ and $\omega^*_t = \beta \frac{C_{t-1}^{*\sigma} \mathcal{P}^*_{t-1}}{C_t^{*\sigma} \mathcal{P}^*_t}$. Last, the consolidation of the Home budget constraint with other equilibrium and market clearing conditions gives

$$b_t = \frac{\mathcal{S}_t \mathcal{P}_{t-1}}{\mathcal{S}_{t-1} \mathcal{P}_t \omega^*_t} b_{t-1} + \mathcal{P}_t^{-1} \left(D^*_{xt} - \frac{1-n}{n} \mathcal{S}_t D_{xt}\right) \tag{C.10}$$

Using appropriate substitutions, the above equations can be reduced to a system of two NK Phillips Curves (Equations (C.11) and (C.12) below), two goods market clearing conditions (Equations (C.13) and (C.14) below) and Equations (C.15)-(C.17) that describe the external equilibrium – the terms of trade and two net foreign asset positions. Conditional on a given set of tariffs $\{\tau_t, \tau_t^*\}$ and inflation rates implemented through monetary policy $\left\{\pi_{ht}, \pi_{ft}^*\right\}$, these equations determine $\{C_t, C_t^*, Y_t, Y_t^*, b_t, b_t^*, \mathcal{S}_t\}$. Nominal interest rates are then deduced from the Home and Foreign equation on local bonds.

$$(1+s)(1-\epsilon) + \epsilon \mathcal{MC}_t - \phi\left(\pi_{ht}(\pi_{ht}-1) - \mathbb{E}_t\left\{\omega_{t+1}\pi_{ht+1}(\pi_{ht+1}-1)\frac{Y_{t+1}}{Y_t}\right\}\right) = \tag{C.11}$$

$$(1+s)(1-\epsilon) + \epsilon \mathcal{MC}_t^* - \phi\left(\pi_{ft}^*\left(\pi_{ft}^*-1\right) - \mathbb{E}_t\left\{\omega_{t+1}^*\pi_{ft+1}^*\left(\pi_{ft+1}^*-1\right)\frac{Y_{t+1}^*}{Y_t^*}\right\}\right) = \tag{C.12}$$

$$Y_t\left(1-\frac{\phi}{2}(\pi_{ht}-1)^2\right) - D_t - D_{xt}^* = \tag{C.13}$$

$$Y_t^*\left(1-\frac{\phi}{2}\left(\pi_{ft}^*-1\right)^2\right) - D_t^* - D_{xt} = \tag{C.14}$$

$$nb_t + (1-n)\frac{\mathcal{S}_t\mathcal{P}_t^*}{\mathcal{P}_t}b_t^* = \tag{C.15}$$

$$\mathbb{E}_t\left\{\frac{\mathcal{S}_{t+1}\omega_{t+1}}{\mathcal{S}_t\omega_{t+1}^*(1+\nu(b_t-b))} - 1\right\} = \tag{C.16}$$

$$b_t - \frac{\mathcal{S}_t\mathcal{P}_{t-1}}{\mathcal{S}_{t-1}\mathcal{P}_t\omega_t^*}b_{t-1} - \mathcal{P}_t^{-1}\left(D_{xt}^* - \frac{1-n}{n}\mathcal{S}_tD_{xt}\right) = \tag{C.17}$$

where

$$D_t = \varepsilon\mathcal{P}_t^{\lambda}(C_t+\Lambda_t) + \varepsilon_x\mathcal{P}_{xt}^{\lambda}X_t\ ;\ D_{xt} = \frac{n}{1-n}\left(\frac{\mathcal{S}_t^{-1}}{1+\tau_t}\right)^{\lambda}\left((1-\varepsilon)\mathcal{P}_t^{\lambda}(C_t+\Lambda_t) + (1-\varepsilon_x)\mathcal{P}_{xt}^{\lambda}X_t\right) \tag{C.18}$$

$$D_t^* = \varepsilon^*\mathcal{P}_t^{*\lambda}C_t^* + \varepsilon_x^*\mathcal{P}_{xt}^{*\lambda}X_t^*\ ;\ D_{xt}^* = \frac{1-n}{n}\left(\frac{\mathcal{S}_t}{1+\tau_t^*}\right)^{\lambda}\left((1-\varepsilon^*)\mathcal{P}_t^{*\lambda}C_t^* + (1-\varepsilon_x^*)\mathcal{P}_{xt}^{*\lambda}X_t^*\right) \tag{C.19}$$

and where $\omega_t = \beta\frac{C_{t-1}^{\sigma}\mathcal{P}_{t-1}}{C_t^{\sigma}\mathcal{P}_t}$; $\omega_t^* = \beta\frac{C_{t-1}^{*\sigma}\mathcal{P}_{t-1}^*}{C_t^{*\sigma}\mathcal{P}_t^*}$, $\mathcal{MC}_t = \frac{\left(\mathcal{P}_t\chi H_t^{\psi}C_t^{\sigma}\right)^{1-\alpha}\mathcal{P}_{xt}^{\alpha}}{A_t\alpha^{\alpha}(1-\alpha)^{1-\alpha}}$ and $\mathcal{MC}_t^* = \frac{\left(\mathcal{P}_t^*\chi H_t^{*\psi}C_t^{*\sigma}\right)^{1-\alpha}\mathcal{P}_{xt}^{*\alpha}}{A_t\alpha^{\alpha}(1-\alpha)^{1-\alpha}}$ with

$$H_t = \left(\frac{(1-\alpha)(\mathcal{P}_t\chi C_t^{\sigma})^{-\alpha}\mathcal{P}_{xt}^{\alpha}Y_t}{A_t\alpha^{\alpha}(1-\alpha)^{1-\alpha}}\right)^{\frac{1}{1+\alpha\psi}},\ H_t^* = \left(\frac{(1-\alpha)(\mathcal{P}_t^*\chi C_t^{*\sigma})^{-\alpha}\mathcal{P}_{xt}^{*\alpha}Y_t^*}{A_t^*\alpha^{\alpha}(1-\alpha)^{1-\alpha}}\right)^{\frac{1}{1+\psi\alpha}} \tag{C.20}$$

$$X_t = \frac{\alpha\left(\mathcal{P}_t\chi H_t^{\psi}C_t^{\sigma}\right)^{1-\alpha}\mathcal{P}_{xt}^{\alpha-1}Y_t}{A_t\alpha^{\alpha}(1-\alpha)^{1-\alpha}},\ X_t^* = \frac{\alpha\left(\mathcal{P}_t^*\chi H_t^{*\psi}C_t^{*\sigma}\right)^{1-\alpha}\mathcal{P}_{xt}^{*\alpha-1}Y_t^*}{A_t^*\alpha^{\alpha}(1-\alpha)^{1-\alpha}} \tag{C.21}$$

D Appendix: Alternative parameter values

Table 7 describes the results of the currency and trade war under alternative parameter values. For a larger trade elasticity, assuming $\lambda = 6$, equilibrium tariffs in the trade war are substantially lower. Tariffs are higher than the baseline when the monopoly markup is lower ($\epsilon = 11$, implying a 10 percent markup), and lower in the case of greater home bias in preferences and production. In addition, a smaller weight of intermediate goods, and a lower elasticity of intertemporal substitution also leads to higher Nash equilibrium tariff rates. A lower Frisch elasticity of substitution in labor supply has minimal effects on equilibrium tariff rates, but leads to a 1 percentage point rise in the equilibrium inflation rate.

Table 7: Trade and Currency Wars under alternative parameter values

	Trade and currency war - no subsidy ($s = 0$)						
	Base.	$\lambda = 6$	$\epsilon = 11$	$\varepsilon = \varepsilon_x = 0.75$	$\alpha = 0.2$	$\sigma = 2$	$\psi = 0$
π_h	1.0474	1.0479	1.0170	1.0465	1.0278	1.0433	1.0580
π_f^*	1.0474	1.0479	1.0170	1.0465	1.0278	1.0433	1.0580
τ	0.4053	0.1644	0.4352	0.4164	0.4612	0.4314	0.3912
τ^*	0.4053	0.1644	0.4352	0.4164	0.4612	0.4314	0.3912
S	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
C	0.1900	0.1960	0.2130	0.1826	0.4572	0.3310	0.1674
C^*	0.1900	0.1960	0.2130	0.1826	0.4572	0.3310	0.1674
H	0.8733	0.8845	0.9000	0.8575	0.8946	1.5055	0.8012
H^*	0.8733	0.8845	0.9000	0.8575	0.8946	1.5055	0.8012
Home welf. loss (%)	14.4376	12.5938	6.2893	16.6131	5.5570	86.2889	18.3069
Foreign welf. loss (%)	14.4376	12.5938	6.2893	16.6131	5.5570	86.2889	18.3069

E Appendix: Model with DCP

The model under DCP differs in only a few features. The nominal exchange rate is still flexible, but the impact of exchange rate changes on the Home terms of trade is muted since both its exports and imports are priced in its own currency. As we show below, this has significant implications for the equilibrium of the policy game. The true price index for the Home consumer under DCP now becomes

$$P_t = \left(\varepsilon P_{ht}^{1-\lambda} + (1-\varepsilon)((1+\tau_t)P_{ft})^{1-\lambda}\right)^{\frac{1}{1-\lambda}} \tag{E.22}$$

where P_{ft} (instead of P_{ft}^* previously) is the price of the Foreign good set in Home currency, which implies

$$\mathcal{P}_t = \frac{P_t}{P_{ht}} = \left(\varepsilon + (1-\varepsilon)((1+\tau_t)\mathcal{S}_t)^{1-\lambda}\right)^{\frac{1}{1-\lambda}} \tag{E.23}$$

where $\mathcal{S}_t = P_{ft}/P_{ht}$. By contrast, the price index for the Foreign economy is unchanged since the Home country firm sets all prices in Home currency, which implies

$$\mathcal{P}_t^* = P_t^*/P_{ft}^* = \left(\varepsilon^* + (1-\varepsilon^*)\left((1+\tau_t)/\mathcal{S}_t^*\right)^{1-\lambda}\right)^{\frac{1}{1-\lambda}} \tag{E.24}$$

where $\mathcal{S}_t^* = S_t P_{ft}^*/P_{ht}$ is the equivalent of the terms of trade in the PCP model. Relative price indices for intermediate goods $\mathcal{P}_t$ and $\mathcal{P}_{xt}$ are modified in the exact same way. The optimal pricing condition of the Home firm is as before, the firm chooses one price which is then converted to the Foreign currency when exported. But the Foreign firm charges separate prices to the local firms and households (in Foreign currency) and to the Home firms and households (in Home currency). The profits of the Foreign firm i are then represented as

$$\Pi_t^*(i) = (1+s^*(i))\left(P_{ft}^*(i)Y_{ft}^*(i) + S_t^{-1}P_{ft}(i)Y_{ft}(i)\right) - MC_t^*\left(Y_{ft}^*(i) + Y_{ft}(i)\right) \tag{E.25}$$

where $MC_t^* = A_t^{*-1}(1-\alpha)^{\alpha-1}\alpha^{-\alpha}W_t^{*1-\alpha}P_{xt}^{*\alpha}$ and where $Y_{ft}(i) = D_{xt}(i)$ and $Y_{ft}^*(i) = D_t^*(i)$ in equilibrium. Foreign firm i maximizes

$$\mathbb{E}_t\left\{\sum_{j=0}^{\infty}\omega_{t+j}^*\left(\begin{array}{c}\Pi_{t+j}^*(i) - \frac{\phi}{2}\left(\frac{P_{ft+j}^*(i)}{P_{ft+j-1}^*(i)} - 1\right)^2 P_{ft+j}^*(i)Y_{ft+j}^*(i) \\ -\frac{\phi}{2}\left(\frac{P_{ft+j}(i)}{P_{ft+j-1}(i)} - 1\right)^2 S_{t+j}^{-1}P_{ft+j}(i)Y_{ft+j}(i)\end{array}\right)\right\} \tag{E.26}$$

Note that the Foreign firm incurs costs of price adjustment for sales to the Home country that are separate from those pertaining to sales to the domestic consumers and firms. The first order conditions for profit maximization for the Foreign firm i selling to the Home country is

$$(1+s^*)(1-\epsilon)+\epsilon\left(\mathcal{S}_t^*/\mathcal{S}_t\right)\mathcal{MC}_t^*-\phi\pi_{ft}\left(\pi_{ft}-1\right)+\mathbb{E}_t\left\{\omega_{t+1}^*\frac{D_{xt+1}}{D_{xt}}\frac{\mathcal{S}_t^*/\mathcal{S}_t}{\mathcal{S}_{t+1}^*/\mathcal{S}_{t+1}}\phi\pi_{ft+1}\left(\pi_{ft+1}-1\right)\right\}=0 \tag{E.27}$$

where $\mathcal{MC}_t^*=MC_t^*/P_{ft}^*$ while the condition for the Foreign firm i selling to the Foreign country is

$$(1+s^*)(1-\epsilon)+\epsilon\mathcal{MC}_t^*-\phi\pi_{ft}^*\left(\pi_{ft}^*-1\right)+\mathbb{E}_t\left\{\omega_{t+1}^*\frac{D_{t+1}^*}{D_t^*}\phi\pi_{ft+1}^*\left(\pi_{ft+1}^*-1\right)\right\}=0 \tag{E.28}$$

Finally, Home exports and imports become

$$D_{xt} = \frac{n}{1-n}\mathcal{S}_t^{-\lambda}(1+\tau_t)^{-\lambda}\left((1-\varepsilon)\mathcal{P}_t^{\lambda}C_t+(1-\varepsilon_x)\mathcal{P}_{xt}^{\lambda}X_t\right) \tag{E.29}$$

$$D_{xt}^* = \frac{1-n}{n}\mathcal{S}_t^{*\lambda}(1+\tau_t^*)^{-\lambda}\left((1-\varepsilon^*)\mathcal{P}_t^{*\lambda}C_t^*+(1-\varepsilon_x^*)\mathcal{P}_{xt}^{*\lambda}X_t^*\right) \tag{E.30}$$

and the market clearing conditions

$$Y_t\left(1-\frac{\phi}{2}(\pi_{ht}-1)^2\right) = D_t+D_{xt}^* \tag{E.31}$$

$$Y_t^*\left(1-\frac{\phi}{2}(\pi_{ft}-1)^2-\frac{\phi}{2}\left(\pi_{ft}^*-1\right)^2\right) = D_t^*+D_{xt} \tag{E.32}$$

The modified UIP condition becomes

$$\beta\mathbb{E}_t\left\{\frac{\mathcal{S}_{t+1}^*\omega_{t+1}}{\mathcal{S}_t^*\omega_{t+1}^*(1+\nu(b_t-b))}-1\right\}=0 \tag{E.33}$$

and net foreign assets

$$b_t-b_{t-1}\frac{\mathcal{S}_t^*\mathcal{P}_{t-1}}{\mathcal{S}_{t-1}^*\mathcal{P}_t\omega_t^*}-\mathcal{P}_t^{-1}\left(D_{xt}^*-\frac{1-n}{n}\mathcal{S}_tD_{xt}\right) = 0 \tag{E.34}$$

$$nb_t+(1-n)\frac{\mathcal{S}_t^*\mathcal{P}_t^*}{\mathcal{P}_t}b_t^* = 0 \tag{E.35}$$

F Appendix: Bonds denominated in Home currency

When internationally traded bonds are denominated in the Home currency, the Home budget constraint writes

$$B_t + P_{ht}C_{ht} + (1+\tau_t)\, S_t P^*_{ft} C_{ft} = B_{t-1}R_{t-1} + W_t H_t + \Pi_t + TR_t \tag{F.36}$$

where B_t is the amount of Home currency-denominated bonds bought by Home households, paying return R_t between t and $t+1$, which implies the following FOCs

$$\beta \mathbb{E}_t \left\{ \frac{R_t \mathcal{P}_t C_t^{\sigma}}{\pi_{ht+1} \mathcal{P}_{t+1} C_{t+1}^{\sigma}} \right\} = 1 \tag{F.37}$$

$$\chi H_t^{\psi} C_t^{\sigma} = \frac{\mathcal{W}_t}{\mathcal{P}_t} \tag{F.38}$$

where $\pi_{ht} = P_{ht}/P_{ht-1}$. The representative Foreign household faces a modified constraint, accessing both local and Home bonds:

$$S_t^{-1} B_t^* + B_t^{**} + P_{ht}C_{ht} + (1+\tau_t)\, S_t P^*_{ft} C_{ft} + P_t^* \Lambda_t^* = S_t^{-1} B^*_{t-1} R_{t-1} + B^{**}_{t-1} R^*_{t-1} + W_t H_t + \Pi_t + TR_t \tag{F.39}$$

where B_t^* and B_t^{**} are respectively the amounts of Home and Foreign currency-denominated bonds bought by Foreign households, paying returns R_t and R_t^* between t and $t+1$. Buying Home bonds incurs the payment of a small adjustment cost $\Lambda_t^* = \frac{\nu}{2}\left(\frac{B_t^*}{S_t P_t^*} - \frac{B^*}{SP^*}\right)^2$, proportional to the deviation of real Foreign bonds to their steady-state value. The bundle structure of adjustment costs mimics that of final goods. The Euler equations associated with Home and Foreign bonds give

$$\beta \mathbb{E}_t \left\{ \frac{R_t S_t P_t^* C_t^{*\sigma}}{S_{t+1} P^*_{t+1} C^{*\sigma}_{t+1} \left(1 + \nu \left(\frac{B_t^*}{S_t P_t^*} - \frac{B^*}{SP^*}\right)\right)} \right\} = 1 \tag{F.40}$$

$$\beta \mathbb{E}_t \left\{ \frac{R_t^* \mathcal{P}_t^* C_t^{*\sigma}}{\pi^*_{ft+1} \mathcal{P}^*_{t+1} C^{*\sigma}_{t+1}} \right\} = 1 \tag{F.41}$$

The other equilibrium conditions are unchanged. Foreign bonds are in zero net supply so that $B_t^{**} = 0$ and the clearing condition on the market for Home bonds writes

$$nB_t + (1-n)\, B_t^* = 0 \tag{F.42}$$

Defining $b_t = \frac{B_t}{P_t}$ and $b_t^* = \frac{B_t^*}{S_t P_t^*}$ as the real per-capita net foreign asset positions, the latter condition implies (as before)

$$nb_t + (1-n)\, \frac{\mathcal{S}_t \mathcal{P}_t^*}{\mathcal{P}_t} b_t^* = 0 \tag{F.43}$$

Further, the modified IUP condition stemming from the combination of Home and Foreign Euler Equations writes[26]

$$\mathbb{E}_t \left\{ \frac{\mathcal{S}_{t+1}\omega_{t+1}\left(1+\nu\left(b_t^* - b^*\right)\right)}{\mathcal{S}_t \omega_{t+1}^*} - 1 \right\} = 0 \tag{F.44}$$

where $\omega_t = \beta \frac{C_{t-1}^{\sigma}\mathcal{P}_{t-1}}{C_t^{\sigma}\mathcal{P}_t}$ and $\omega_t^* = \beta \frac{C_{t-1}^{*\sigma}\mathcal{P}_{t-1}^*}{C_t^{*\sigma}\mathcal{P}_t^*}$. Last, the consolidation of the Home budget constraint with other equilibrium and market clearing conditions gives

$$b_t = \frac{\mathcal{P}_{t-1}}{\mathcal{P}_t \omega_t} b_{t-1} + \mathcal{P}_t^{-1}\left(D_{xt}^* - \frac{1-n}{n}\mathcal{S}_t D_{xt}\right) \tag{F.45}$$

Notice that, combining this equation with the modified UIP delivers a condition that is identical to that of the baseline model, up to the adjustment costs.

[26] With bonds denominated in Foreign currency, the modified UIP condition was

$$\mathbb{E}_t \left\{ \frac{\mathcal{S}_{t+1}\omega_{t+1}}{\mathcal{S}_t \omega_{t+1}^*\left(1+\nu\left(b_t - b\right)\right)} - 1 \right\} = 0$$

G Appendix: Index linked Bonds

The text draws a key distinction between the outcome of the trade and currency war in an environment where bonds are denominated in Foreign currency relative to that of Home currency. Here we briefly outline a scenario where internationally traded bonds are indexed to *ex-post* nominal price changes. In this case we show that the outcome of the non-cooperative policy game in tariffs and inflation is fully symmetric.

First, we write Home budget constraint writes

$$B_t + D_t\mathcal{I}_t + P_{ht}C_{ht} + (1+\tau_t)\, S_t P^*_{ft} C_{ft} = B_{t-1}R_{t-1} + D_{t-1}\mathcal{I}_t R_{I,t-1} + W_t H_t + \Pi_t + TR_t \tag{G.46}$$

where again B_t is the amount of Home currency-denominated bonds bought by Home households, paying return R_t between t and $t+1$. However, these bonds are no longer traded internationally. Now however, D_t represents the real internationally traded bond whose price is $\mathcal{I}_t = P_{h,t} + S_t P_{f,t}$. That is, the effective price of the real bond is the sum of domestic and foreign prices, evaluated in Home currency. This assumption implies that the real bond is essentially indexed to changes in the Home currency nominal value of goods, coming from either the Home or Foreign country. Then $R_{I,t-1}$ is the predetermined rate of return on the real internationally traded bond.

The FOC for the Home currency bond is the same as before. The FOC for internationally traded bond is expressed as

$$\beta\mathbb{E}_t\left\{\frac{R_{I,t}P_t C_t^{\sigma}\mathcal{I}_{t+1}}{P_{t+1}C_{t+1}^{\sigma}\mathcal{I}_t}\right\} = 1 \tag{G.47}$$

This can be rewritten as

$$\beta\mathbb{E}_t\left\{\frac{R_{I,t}\tilde{\mathcal{P}}_t C_t^{\sigma}}{\tilde{\mathcal{P}}_{t+1}C_{t+1}^{\sigma}}\right\} = 1 \tag{G.48}$$

where

$$\tilde{\mathcal{P}}_t = \left(\varepsilon s_{ht}^{1-\lambda} + (1-\varepsilon)\left((1+\tau_t)\, s^*_{f,t}\right)^{1-\lambda}\right)^{\frac{1}{1-\lambda}}$$

and $s_{h,t} = \frac{P_{h,t}}{P_{h,t}+S_t P_{f,t}} = 1 - s_{f,t}$ is a normalization of the domestic goods price, and $s_{h,t} + s_{f,t} = 1$ by construction, so that the terms of trade are now expressed as $\frac{1-s_{f,t}}{s_{h,t}}$.

From equation (G.48) it is apparent that the Home country cannot affect the return on the internationally traded bond through *ex-post* inflation. But given the symmetry of the model, the same applies to the Foreign country. The analogous FOC for the Foreign country holdings of the internationally traded bond is now

$$\beta\mathbb{E}_t\left\{\frac{R_{I,t}P_t^* S_t C_t^{*\sigma}\mathcal{I}_{t+1}}{P^*_{t+1}C^{*\sigma}_{t+1}S_{t+1}\mathcal{I}_t\left(1+\nu\left(\frac{D_t^*\mathcal{I}_t}{S_t P_t^*} - \frac{D^*_{t-1}\mathcal{I}_{t-1}}{S_{t-1}P^*_{t-1}}\right)\right)}\right\} = 1 \tag{G.49}$$

which may be rewritten as

$$\beta \mathbb{E}_t \left\{ \frac{R_{I,t} \tilde{\mathcal{P}}_t^* C_t^{*\sigma}}{\tilde{\mathcal{P}}_{t+1}^* C_{t+1}^{*\sigma} \left(1 + \nu \left(\frac{D_t^*}{\tilde{\mathcal{P}}_t^*} - \frac{D_{t-1}^*}{\tilde{\mathcal{P}}_{t-1}^*}\right)\right)} \right\} = 1 \qquad \text{(G.50)}$$

where

$$\tilde{\mathcal{P}}_t^* = \left(\varepsilon s_{ft}^{1-\lambda} + (1-\varepsilon)\left((1+\tau_t^*)\, s_{h,t}^*\right)^{1-\lambda}\right)^{\frac{1}{1-\lambda}}$$

Again, it is apparent that the Foreign country cannot affect the real return on internationally traded bonds through *ex-post* inflation.

Figure (3) illustrate the counterparts of Figure (1) for this example of real index linked bonds. The models is more stylized and abstracts from trade in intermediate goods so numbers should not be compared directly.

Figure 3: Trade and currency war with trade imbalances and index linked bonds

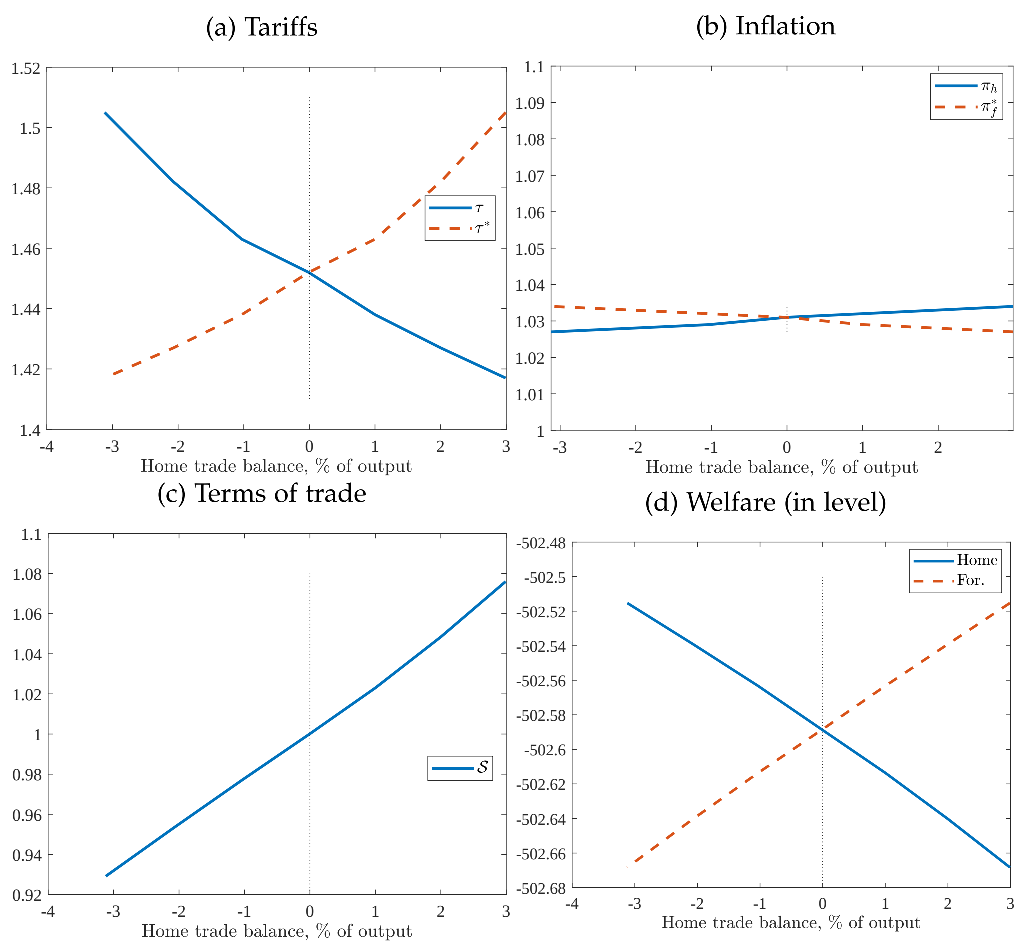


Figure 3 shows that when index linked bonds are considered, the outcome of the trade and currency war is fully symmetric – in contrast with Figure 1 – and depends only on the NFA position of the countries. When the Home country is an external creditor, so that it runs a trade balance deficit, it raises tariffs above the baseline zero-NFA case, and vice-versa for the Foreign country. At the same time, the inflation outcomes for each country reflect the incentive to use inflation to manipulate the terms of trade. Thus, when Home is a creditor country again, it follows an inflation rate lower than the baseline zero-NFA case, in order to achieve a terms of trade strategic advantage.

The bottom two panels of Figure 3 show the impact of steady-state NFA position on the terms of trade and welfare. The Home terms of trade are appreciated when the Home country is a creditor, as in the example of the text, and attains a higher welfare (lower welfare losses) in the same situation.