On Portfolio Frictions, Asset Returns and Volatility
Technical Appendix

Aurélien Eyquem — University of Lausanne
Céline Poilly — AMSE
Anna Belianska — IMF

October 22, 2023

This technical appendix describes the full derivations of the model.

Contents

1 The Model

We consider a New-Keynesian model enriched with a financial accelerator mechanism in the spirit of Bernanke, Gertler, and Gilchrist (1999) and imperfect substitutability between corporate and government bonds through a quadratic cost. In this section, we describe in details the decisions rules and the model's steady state.

1.1 Households Sector

There is a continuum of identical households gathering a large number of workers/savers and financial intermediaries. In the spirit of Gertler and Karadi (2011), the agents acting on financial markets are thus incorporated within a large family. Following Christiano and Ikeda (2013) and Christiano, Motto, and Rostagno (2014), we perfect consumption insurance among family members. Before going to the full description of decisions of mutual funds and financial intermediairies, we start with the decisions of the workers/savers in the household, who consume, work, save and build raw capital, so as to maximize the discounted lifetime utility:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{\left(\widetilde{c}_t - \psi \ell_t^{\omega_w} X_t\right)^{1-\sigma} - 1}{1-\sigma} \right\}, \tag{1.1}$$

with $\widetilde{c}_t = c_t - hc_{t-1}$. The term X_t is defined by

$$X_t = \widetilde{c}_t^{\sigma_X} X_{t-1}^{1-\sigma_X}, \tag{1.2}$$

where $\beta \in (0,1)$ is the subjective discount factor, c_t denotes real consumption and ℓ_t denotes labor supply. Further, $\mathbb{E}_t$ is the expectation operator conditional on the information available in period t, σ_X drives the strength of the wealth effect on labor supply, ω_w^{-1} is the Frisch elasticity of labor supply, ψ is a normalizing constant (governs the relative disutility of labor effort) and σ is the inverse of the risk aversion coefficient.

As mentioned above, at the end of the period, after production, households purchase existing undepreciated capital $(1-\delta)\, k_{t-1}$ at nominal price Q_t, and combine it with investment i_t so as to produce end-of-period raw capital k_t following the technology:

$$k_t = (1-\delta)\, k_{t-1} + \left[1 - \frac{\varkappa}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2\right] i_t, \tag{1.3}$$

where $\varkappa$ captures the presence of investment adjustment costs. Households then sell the new stock of capital k_t to portfolio managers at the same price Q_t. Therefore, the representative household acts simply as a capital producer and since she is perfectly competitive, she takes the price of capital as given. The representative household faces the following budget constraint:

$$c_t + i_t + q_t\,(1-\delta)\, k_{t-1} + \mathcal{A}_t \leq w_t \ell_t + q_t k_t + \frac{R_{t-1}}{\pi_t}\mathcal{A}_{t-1} + \frac{\Pi_t}{P_t} + \text{div}_t - \frac{\text{T}_t}{P_t}, \tag{1.4}$$

where P_t is the price of final goods and $w_t \equiv W_t/P_t$ is the real wage rate. Let $\pi_t = (P_t/P_{t-1} - 1)$ denote the inflation rate and $\mathcal{A}_t$ denote real non-state contingent deposits issued to a representative mutual fund in period t, which pay back a risk-free gross nominal return, R_{t+1}. Finally, households receive div_t nominal profits from monopolistic intermediate good firms as well as a net lump-sum transfer sent to / received from financial intermediaries, Π_t. Let T_t denote lump-sum taxes (or transfers if negative) provided by the government. The Lagrangian is:

$$\mathcal{L}_t = \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{c} \frac{1}{1-\sigma}\left[\left(c_t - hc_{t-1} - \psi\,(\ell_t)^{\omega_w} X_t\right)^{1-\sigma} - 1\right] - v_t\left[X_t - (c_t - hc_{t-1})^{\sigma_X} X_{t-1}^{1-\sigma_X}\right] \\ -\lambda_t\left[c_t + i_t + \mathcal{A}_t - w_t\ell_t - q_t\left[1 - \Phi\left(\frac{i_t}{i_{t-1}}\right)\right] i_t - \frac{R_{t-1}}{\pi_t}\mathcal{A}_{t-1} - \frac{\Pi_t}{P_t} - \text{div}_t + \frac{\text{T}_t}{P_t}\right] \end{array} \right\}, \tag{1.5}$$

where λ_t and v_t are the Lagrangian multipliers associated with the budget constraint (1.4) and the definition (1.2), respectively. The FOCs wrt A_t, c_t, X_t, ℓ_t, i_t are

$$\beta \mathbb{E}_t \left\{ \lambda_{t+1} \frac{R_t}{\pi_{t+1}} \right\} = \lambda_t, \tag{1.6}$$

$$\left(\widetilde{c}_t - \psi X_t \ell_t^{\omega_w}\right)^{-\sigma} - \beta h \left(\widetilde{c}_{t+1} - \psi X_t \ell_{t+1}^{\omega_w}\right)^{-\sigma} + \sigma_X v_t \widetilde{c}_t^{\sigma_X - 1} X_{t-1}^{1-\sigma_X} - \beta h \sigma_X v_{t+1} \widetilde{c}_{t+1}^{\sigma_X - 1} X_t^{1-\sigma_X} = \lambda_t, \tag{1.7}$$

$$v_t + \psi \ell_t^{\omega_w} \left(\widetilde{c}_t - \psi \ell_t^{\omega_w} X_t\right)^{-\sigma} = \beta \left(1 - \sigma_X\right) \mathbb{E}_t \left\{ v_{t+1} \widetilde{c}_{t+1}^{\sigma_X} X_t^{-\sigma_X} \right\}, \tag{1.8}$$

$$\psi \omega_w \ell_t^{\omega_w - 1} X_t \left(\widetilde{c}_t - \psi X_t \ell_t^{\omega_w}\right)^{-\sigma} = \lambda_t w_t, \tag{1.9}$$

$$q_t^{-1} = 1 - \frac{\varkappa}{2} \left(\frac{i_t}{i_{t-1}} - 1\right)^2 - \varkappa \frac{i_t}{i_{t-1}} \left(\frac{i_t}{i_{t-1}} - 1\right) + \beta \varkappa \mathbb{E}_t \left\{ \frac{\lambda_{t+1} q_{t+1}}{\lambda_t q_t} \left(\frac{i_{t+1}}{i_t} - 1\right) \left(\frac{i_{t+1}}{i_t}\right)^2 \right\}. \tag{1.10}$$

with $\widetilde{c}_t = c_t - h c_{t-1}$.

1.2 Financial Frictions

Each family is composed of a large number of workers/savers and a large number of financial intermediaries. The financial sector combines competitive mutual funds who collect deposits from savers and use them to provide financial funds to financial intermediaries. Following Christiano and Ikeda (2013), we classify financial intermediaries by their net worth, *i.e.* a financial intermediaries who possesses $N > 0$ units of net worth is a N-type financial intermediaries. Each N-type financial intermediary is sub-divided in two separated branches, namely the banking and the portfolio management branch, who make sequential and independent decisions.

1.2.1 Mutual Funds

There is a large number of identical and competitive mutual funds who collect deposits from savers and distribute loans to financial intermediaries. Each mutual fund has a large set of perfectly diversified loans across the N-type financial intermediaries. Therefore, even though the representative mutual fund is imperfectly informed about the ex-post return on a particular financial intermediary's project, perfect diversification of its loans ensures that it does not bear any risk at the aggregate level. The representative mutual fund receives deposits $\mathcal{A}_t$ from savers and distributes them at the beginning of period t to type-N bankers, who commit to return them at the end of the period with a return R_{t+1}^d. The competitive mutual fund then transfers deposits back to savers with a certain gross interest rate R_t that they take as given. The mutual fund has no other source of funds so the total amount of financial funds, transferred to the financial intermediaries, D_t, equals the level of deposits, $\mathcal{A}_t$. The mutual fund thus maximizes its equity level such that:

$$\max_{D_t} \quad \Pi_t^d = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{\lambda_{t+1}}{\lambda_t} \left\{ R_{t+1}^d \frac{D_t}{P_t} - R_t \mathcal{A}_t \right\}, \; s.t. \; d_t \equiv \frac{D_t}{P_t} = \mathcal{A}_t, \tag{1.11}$$

where the expression in brackets corresponds to the dividends of the mutual fund and $\beta^t \lambda_{t+1}/\lambda_t$ is the stochastic discount factor of the household. As the deposit market is competitive, the deposit market is characterized by a zero-profit condition implying $R_{t+1}^d = R_t$ in equilibrium, and the mutual fund acts as a passive entity that passes deposits from savers to financial intermediaries.

1.2.2 Portfolio Decisions

At the begining of period t, the first branch of financial intermediary, *i.e.* the type-N banker agrees on a debt contract with mutual funds so as to receive loans, $D_{N,t}$, which are combined with her own net worth to build a portfolio $\mathcal{P}_{N,t}$. At this stage, banker accumulates "deferred assets" which are supplied from a competitive market at a cost $1/R^p_{t+1}$. The portfolio experiences an idiosyncratic shock, ε, which might lead the financial intermediary to default. Taking $\mathcal{P}_{N,t}$ as given, the portfolio manager decides on the composition of the portfolio, *i.e.* the share used to purchase raw capital $k_{N,T}$ at real price q_t to the household, the remaining share being used to buy public bonds $b_{N,t}$ to the government at real price q^c_t. Once the composition of the portfolio is decided, the portfolio manager rents capital to intermediate firms at a rental rate z_t, receives the earnings from equity and bond holdings, repays the loans, and transfers part of the profits to households while also determining its net worth for the next period. Net worth is therefore pre-determined.

As explained by Christiano, Motto, and Rostagno (2014), the total net worth of all portfolio managers is given by

$$N_{t+1} = \int_0^\infty N f_t\,(N)\,\mathrm{d}N, \tag{1.12}$$

where $f_t\,(N)$ is the density of portfolio managers with net worth N. Bankers maximize their profits and benefit from perfect consumption insurance from the rest of the family.

The portfolio decisions are made in two sequential steps. First, bankers determine the total size of their portfolios given the agency problem described above. Second, portfolio managers take the size of portfolios as given and determine their composition by choosing weights of the two assets, namely equity and long-term government bonds, in the portfolio.

Bankers The N-type banker combines loans $D_{N,t}$ received in t and due in $t+1$ with net worth N to build a financial portfolio $\mathcal{P}_{N,t}$ such that

$$\mathcal{P}_{N,t} = D_{N,t} + N, \tag{1.13}$$

where the composition of $\mathcal{P}_{N,t}$ is described below. Once the portfolio has been built, it experiences an idiosyncratic shock, denoted by ε. As standard in the literature, we assume that ε is a random variable, which follows a unit-mean log-normal distribution and is i.i.d. across time and types, with a continuous and once-differentiable c.d.f., $F(\varepsilon)$, over a non-negative support such that $\int_0^\infty \mathrm{d}F(\varepsilon) = 1$ and $\mathrm{d}F(\bar{\omega}_t) = \varphi(\varepsilon)\mathrm{d}\varepsilon$. Before the realization of the shock, both the mutual fund and the banker share the same information about ε, meaning that they know its distribution given by $\mathbb{E}(\varepsilon) = 1$ and $\mathrm{V}(\varepsilon) = \sigma_{\varepsilon,t}$. Following Christiano, Motto, and Rostagno (2014), we assume that the variance of the idiosyncratic shock is time-varying, which can be seen as a "risk shock", or a "micro-uncertainty shock". This shock captures cross-sectional dispersion of idiosyncratic shocks to portfolio returns.

The banker drawing an idiosyncratic ε receives at the end of period t a rate of return εR^p_{t+1}, where R^p_{t+1} is the portfolio's return which, as we will see depends on the return of both assets and but not on N. The representative mutual fund provides loans $D_{N,t}$ at rate R^d_{t+1} to bankers through a standard debt contract. We denote $\bar{\varepsilon}_{t+1}$, the of ε from which a banker goes bankrupt such that:

$$\mathbb{E}_t \left\{ \bar{\varepsilon}_{t+1} R^p_{t+1} \mathcal{P}_{N,t} \right\} = \mathbb{E}_t \left\{ R^d_{t+1} D_{N,t} \right\}. \tag{1.14}$$

As shown in Equation (1.14), the threshold value of ε is such that the total return of project by a N-type portfolio and hit by a shock $\bar{\varepsilon}_{t+1}$ equals the total cost of its debt. Financial intermediaries with $\varepsilon \leq \bar{\varepsilon}_{t+1}$ declare bankruptcy as the total return on their portfolio is not large enough to repay their debt. To fully observe the portfolio's realized gross payoff, the mutual fund needs to monitor the banker at a cost proportional to this payoff and equals to $\mu \bar{\varepsilon}_{t+1} R^p_{t+1} \mathcal{P}_{N,t}$, where $\mu \in [0,1]$ is the monitoring cost parameter. As shown by Bernanke, Gertler, and Gilchrist (1999) or Christiano, Motto, and Rostagno (2014), the debt contract will be independent of N in equilibrium, so we drop the index for briefness.

If $\varepsilon \leq \bar{\varepsilon}$, the earnings of a N-type banker are zero. If $\varepsilon > \bar{\varepsilon}$, the expected profit of a N-type banker writes:

$$\Pi_{t+1} = \mathbb{E}_t \left\{ \int_{\bar{\varepsilon}_{t+1}}^{\infty} \left[\varepsilon R^p_{t+1} \mathcal{P}_{N,t} - R^d_{t+1} D_{N,t} \right] \mathrm{d}F(\varepsilon) \right\}, \tag{1.15}$$

$$= \mathbb{E}_t \left\{ R^p_{t+1} \mathcal{P}_{N,t} \int_{\bar{\varepsilon}_{t+1}}^{\infty} \left[\varepsilon - \bar{\varepsilon}_{t+1} \right] \mathrm{d}F(\varepsilon) \right\}. \tag{1.16}$$

Let $[1 - F_t(\bar{\varepsilon}_{t+1})]$ be the probability that a banker is hit by an idiosyncratic shock larger than the threshold, $\bar{\varepsilon}_{t+1}$ such that

$$1 - F(\bar{\varepsilon}_{t+1}) = 1 - \mathrm{prob}(\varepsilon \leq \bar{\varepsilon}_{t+1}) = \int_{\bar{\varepsilon}_{t+1}}^{\infty} \varphi(\varepsilon) \mathrm{d}\varepsilon, \tag{1.17}$$

and $G(\bar{\varepsilon}_{t+1})$, the expected value of the shock for defaulting bankers

$$G(\bar{\varepsilon}_{t+1}) = \int_0^{\bar{\varepsilon}_{t+1}} \varepsilon \varphi(\varepsilon) \mathrm{d}\varepsilon. \tag{1.18}$$

As standard in the literature, we define $\Gamma(\bar{\varepsilon}_{t+1})$, the share of average earnings $R^p_{t+1} \mathcal{P}_{N,t}$ paid to the mutual fund, such that:

$$\Gamma(\bar{\varepsilon}_{t+1}) = \bar{\varepsilon}_{t+1} \int_{\bar{\varepsilon}_{t+1}}^{\infty} \varphi(\varepsilon) d\varepsilon + \int_0^{\bar{\varepsilon}_{t+1}} \varepsilon \varphi(\varepsilon) \mathrm{d}\varepsilon, \tag{1.19}$$

$$= \bar{\varepsilon}_{t+1} \left[1 - F(\bar{\varepsilon}_{t+1}) \right] + G(\bar{\varepsilon}_{t+1}).$$

Using definitions of $\Gamma(\bar{\varepsilon}_{t+1})$ in Equation (1.19), we can rewrite the profit's expression (1.16) as:

$$\Pi_{t+1} = \mathbb{E}_t \left\{ R^p_{t+1} \mathcal{P}_{N,t} \left[1 - \Gamma(\bar{\varepsilon}_{t+1}) \right] \right\}. \tag{1.20}$$

The participation constraint of the mutual funds is:

$$\underbrace{\mathbb{E}_t \left\{ \left[\Gamma(\bar{\varepsilon}_{t+1}) - \mu G(\bar{\varepsilon}_{t+1}) \right] R^p_{t+1} \mathcal{P}_{N,t} \right\}}_{\text{mutual fund's earnings}} \geq \underbrace{R_t \left(\mathcal{P}_{N,t} - N \right)}_{\text{funds paid to savers}}. \tag{1.21}$$

We denote $x_{N,t} \equiv \mathcal{P}_{N,t}/N$ as the individual leverage ratio and $\check{r}_t \equiv R^p_{t+1}/R_t$. The optimal contract consists in choosing x and $\bar{\varepsilon}$ in order to maximize type$-N$ banker's expected returns with respect to the participation constraint of the private intermediary. For briefness, we drop type sub-indexes. The banker maximizes its expected profit (1.20) subject to the mutual fund's participation constraint and gives back all profits to the rest of the family in exchange for perfect consumption insurance. The optimization problem is:

$$\max_{x_t, \bar{\varepsilon}_{t+1}} \; \mathbb{E}_t \; \left\{ \left[1 - \Gamma(\bar{\varepsilon}_{t+1}) \right] \check{r}_t x_{N,t} \right\}, \tag{1.22}$$

subject to:

$$\mathbb{E}_t \left\{ \left[\Gamma(\bar{\varepsilon}_{t+1}) - \mu G(\bar{\varepsilon}_{t+1}) \right] \check{r}_t x_{N,t} \right\} \geq \left(x_{N,t} - 1 \right). \tag{1.23}$$

The FOCs are with respect to x_t, $\bar{\varepsilon}_{t+1}$ and Λ_t are respectively:

$$\begin{aligned}
\mathbb{E}_t \left\{ \left[1 - \Gamma(\bar{\varepsilon}_{t+1}) + \Lambda_t \left[\Gamma(\bar{\varepsilon}_{t+1}) - \mu G(\bar{\varepsilon}_{t+1}) \right] \right] \check{r}_t \right\} &= \mathbb{E}_t \left\{ \Lambda_t \right\}, &(1.24)\\
\mathbb{E}_t \left\{ \check{r}_t x_{N,t} \left[-\Gamma_\varepsilon(\bar{\varepsilon}_{t+1}) + \Lambda_t \left(\Gamma_\varepsilon(\bar{\varepsilon}_{t+1}) - \mu G_\varepsilon(\bar{\varepsilon}_{t+1}) \right) \right] \right\} &= 0, &(1.25)\\
\mathbb{E}_t \left\{ \left[\Gamma(\bar{\varepsilon}_{t+1}) - \mu G(\bar{\varepsilon}_{t+1}) \right] \check{r}_t x_{N,t} \right\} &= \left(x_{N,t} - 1 \right). &(1.26)
\end{aligned}$$

where Λ_t is the Lagrangian multiplier and $\Gamma_\varepsilon(\cdot)$ and $G_\varepsilon(\cdot)$ denote the derivative of $\Gamma(\cdot)$ and $G(\cdot)$ w.r.t. $\bar{\varepsilon}$. Combining the FOCs (1.24) and (1.25) yields:

$$\mathbb{E}_t \left\{ \check{r}_t \left[1 - \Gamma(\bar{\varepsilon}_{t+1}) \right] + \frac{\Gamma_\varepsilon(\bar{\varepsilon}_{t+1})}{\left[\Gamma_\varepsilon(\bar{\varepsilon}_{t+1}) - \mu G_\varepsilon(\bar{\varepsilon}_{t+1}) \right]} \left[\check{r}_t \left[\Gamma(\bar{\varepsilon}_{t+1}) - \mu G(\bar{\varepsilon}_{t+1}) \right] - 1 \right] \right\} = 0.$$

Plugging the FOC (1.26) in the previous equation yields

$$\check{r}_t = \frac{1}{x_{N,t}} \left[\left[1 - \Gamma(\bar{\varepsilon}_{t+1}) \right] \left[1 - \mu \frac{G_\varepsilon(\bar{\varepsilon}_{t+1})}{\Gamma_\varepsilon(\bar{\varepsilon}_{t+1})} \right] \right]^{-1}. \tag{1.27}$$

Portfolio composition Once the size of the portfolio has been determined by the debt contract described above, the second branch of financial intermediaries, namely the portfolio manager, takes $\mathcal{P}_{N,t}$ as given and decides on the amount of securities from households and from the government she wants to purchase. A type-N portfolio $\mathcal{P}_{N,t}$ is made of raw capital $k_{N,t}$ purchased from the household and government bonds $b_{N,t}$:

$$\mathcal{P}_{N,t} = Q_t k_{N,t} + Q_t^c b_{N,t}, \tag{1.28}$$

where Q_t and Q_t^c are the nominal market prices of one unit of capital and one bond, respectively. In real terms, we have that

$$\mathfrak{p}_t = q_t k_{N,t} + q_t^c b_{N,t}, \tag{1.29}$$

where $\mathfrak{p}_{N,t} \equiv \mathcal{P}_{N,t}/P_t$, $q_t = Q_t/P_t$, and $q_t^c = Q_t^c/P_t$. Let $\omega_{N,t}$ denote the share of capital in the portfolio and $1 - \omega_{N,t}$ the share of government bonds, so that:

$$\omega_{N,t} = \frac{Q_t k_{N,t}}{\mathcal{P}_{N,t}}, \tag{1.30}$$

$$1 - \omega_{N,t} = \frac{Q_t^c b_{N,t}}{\mathcal{P}_{N,t}}. \tag{1.31}$$

The total gross return on the portfolio bought in period t and redeemed in period $t+1$ is denoted by $R_{t+1}^p \mathcal{P}_{N,t}$, and is defined as:

$$\frac{R_{t+1}^p \mathcal{P}_{N,t}}{\pi_{t+1}} \equiv \frac{R_{t+1}^k}{\pi_{t+1}} Q_t k_{N,t} + \frac{R_{t+1}^b}{\pi_{t+1}} Q_t^c b_{N,t} - \frac{\varpi}{2} \left(\omega_{N,t} - \omega\right)^2 \frac{\mathcal{P}_{N,t}}{\pi_{t+1}}, \tag{1.32}$$

where $\varpi \geq 0$ is a portfolio adjustment cost. Let denote $r_t^k \equiv R_t^k/\pi_t$ and $r_t^b \equiv R_t^b/\pi_t$ denote the real returns on equity and government bonds, respectively. Dividing the above Equation (1.32) by $\mathcal{P}_{N,t}/\pi_{t+1}$, we obtain the definition of the returns on the portfolio:

$$r_{t+1}^p \equiv r_{t+1}^k \frac{Q_t k_{N,t}}{\mathcal{P}_{N,t}} + r_{t+1}^b \frac{Q_t^c b_{N,t}}{\mathcal{P}_{N,t}} - \frac{\varpi}{2} \left(\omega_{N,t} - \omega\right)^2, \tag{1.33}$$

$$= \omega_{N,t} r_{t+1}^k + \left(1 - \omega_{N,t}\right) r_{t+1}^b - \frac{\varpi}{2} \left(\omega_{N,t} - \omega\right)^2. \tag{1.34}$$

An N-type portfolio manager chooses the weight $\omega_{N,t}$ to maximize the expected flow of portfolio returns that are kept after reimbursement of the loans:

$$\max_{\omega_{N,t}} \mathbb{E}_t \left\{ \beta \frac{\lambda_{t+1}}{\lambda_t} \left[1 - \Gamma(\bar{\varepsilon}_{t+1})\right] \left[r_{t+1}^k \omega_{N,t} + r_{t+1}^b (1 - \omega_{N,t}) - \frac{\varpi}{2} \left(\omega_{N,t} - \omega\right)^2 \right] \mathcal{P}_{N,t} \right\}. \tag{1.35}$$

The FOC is:

$$\mathbb{E}_t \left\{ \beta \frac{\lambda_{t+1}}{\lambda_t} \left[r_{t+1}^k - r_{t+1}^b - \varpi \left(\omega_t - \omega\right) \right] \left[1 - \Gamma(\bar{\varepsilon}_{t+1})\right] \mathcal{P}_t \right\} = 0. \tag{1.36}$$

1.2.3 Net Worth

The aggregate portfolio is given by

$$\mathcal{P}_t = \int_0^\infty \mathcal{P}_{N,t} f_t\left(N\right) \mathrm{d}N = \mathcal{P}_{N,t}. \tag{1.37}$$

In addition, aggregate expected profits are obtained by integration of (1.20) over N, which gives

$$\mathbb{E}_t\left\{[1-\Gamma(\bar{\varepsilon}_{t+1})]\,\check{r}_t\int_0^\infty \frac{\mathcal{P}_{N,t}}{N} f_t\left(N\right)\mathrm{d}N\right\} = \mathbb{E}_t\left\{[1-\Gamma(\bar{\varepsilon}_{t+1})]\,\check{r}_t\frac{\mathcal{P}_{N,t}}{N_{t+1}}\right\}. \tag{1.38}$$

After the banker has chosen the size of the portfolio, after the portfolio manager has chosen its composition and after the mutual fund has been reimbursed at the end of date $t+1$, an exogenous fraction $1-\gamma$ of the financial intermediary's assets are transferred to workers/savers. Therefore, the higher the net worth of financial intermediaries, the higher the wealth of the family. The complementary fraction γ remains in the hands of the financial intermediary to build portfolios along with deposits in the next period. In addition, each financial intermediary receives a transfer $W_t^e = \chi w_t \ell_t$ from the savers/workers that is proportional to their labor income.

The nominal aggregate net worth at the end of period t, N_t, is given by:

$$N_t = \gamma V_t + W_t^e, \tag{1.39}$$

and V_t is aggregate equity from portfolio holdings in period t, such that:

$$V_t = [1-\Gamma(\bar{\varepsilon}_t)]\,R_t^p \mathcal{P}_{t-1}. \tag{1.40}$$

1.2.4 Returns on capital and government bonds

The gross real rate of the returns on capital equals to:

$$r_t^k \equiv \frac{z_t + (1-\delta)q_t}{q_{t-1}}, \tag{1.41}$$

where δ is the capital depreciation rate. The gross real one-period return on a bond is defined as follows;

$$r_t^b = \frac{1+\rho q_t^c}{q_{t-1}^c}. \tag{1.42}$$

1.3 Goods Sector

We now describe the production sector, composed of final goods producers and intermediate goods producers.

1.3.1 Final Good Producers

The final good y_t, used for consumption and investment, is produced in a competitive market by combining a continuum of intermediate goods indexed by $j \in [0,1]$, *via* the CES production function:

$$y_t = \left(\int_0^1 y_{j,t}^{\frac{\theta_p - 1}{\theta_p}}\,\mathrm{d}j\right)^{\frac{\theta_p}{\theta_p - 1}}, \tag{1.43}$$

where $y_{j,t}$ denotes the overall demand for an intermediate good j and θ_p is the elasticity of demand for an intermediate good. The maximization of profits yields typical demand functions:

$$y_{j,t} = \left(\frac{P_{j,t}}{P_t}\right)^{-\theta_p} y_t, \tag{1.44}$$

with

$$P_t = \left(\int_0^1 P_{j,t}^{1-\theta_p} \mathrm{d}j\right)^{\frac{1}{1-\theta_p}}, \tag{1.45}$$

where $P_{j,t}$ denotes the price of an intermediate good produced by firm j.

1.3.2 Intermediate Good Producers

Production Function Type-j intermediate good is produced with the following constant returns to scale technology:

$$y_{j,t} = a_t \ell_{j,t}^{1-\alpha} k_{j,t-1}^{\alpha}, \tag{1.46}$$

where a_t is a measure of TFP, defined below. Let $\mathbb{S}\left(y_{j,t}\right)$ denote the total real cost of producing $y_{j,t}$ units of good j:

$$\mathbb{S}\left(y_{j,t}\right) = w_t \ell_{j,t} + z_t k_{j,t-1}, \tag{1.47}$$

Each monopolistic firm chooses capital and labor services to minimize $\mathbb{S}\left(y_{j,t}\right)$ subject to the production function (1.46), taking w_t and z_t as given. Accordingly, labor and capital demands are:

$$w_t = s_t(1-\alpha)\frac{y_t}{\ell_{j,t}^h}, \tag{1.48}$$

$$z_t = s_t \alpha \frac{y_t}{k_{j,t-1}}. \tag{1.49}$$

where $s_t \equiv \partial \mathbb{S}\left(\cdot\right) / \partial y_{j,t}$ is the real marginal cost.

Price Setting At each point in time, type-j monopolistic firm maximizes its profit taking into account a quadratic adjustment cost it faces:

$$\max_{p_{j,t}} \quad \mathbb{E}_t \sum_{k=0}^{\infty} \beta^k \frac{\lambda_{t+k}}{\lambda_{t+k-1}} \left\{ \frac{P_{j,t+k}}{P_{t+k}} y_{j,t+k} - \mathbb{S}\left(y_{j,t+k}\right) - \frac{\kappa_p}{2}\left(\frac{P_{j,t+k}}{P_{j,t+k-1}} - 1\right)^2 y_{j,t+k}, \right\}$$

subject to $y_{j,t} = \left(\frac{P_{j,t}}{P_t}\right)^{-\theta_p} y_t$. Paramater $\kappa_p > 0$ measures the degree of price rigidity. The FOC gives the New Keynesian Phillips Curve in the symmetric equilibrium

$$1 - \kappa_p\left(\pi_t - 1\right)\pi_t + \kappa_p \beta \mathbb{E}_t \left\{ \frac{\lambda_{t+1}}{\lambda_t}\left(\pi_{t+1} - 1\right)\pi_{t+1}\frac{y_{t+1}}{y_t} \right\} = \theta_p\left(1 - s_t\right). \tag{1.50}$$

1.4 Government and Central Bank

1.4.1 Fiscal Policy

The budget constraint of the government is given by:

$$q_t^c b_t + \mathrm{T}_t = \left(1 + \rho q_t^c\right) b_{t-1} + g_t, \tag{1.51}$$

where $t_t \equiv \mathrm{T_t} \ / P_t$ is the real amount of lump-sum taxes. Since from (1.42), we have $r_t^b q_{t-1}^c = 1 + \rho q_t^c$, the constraint can be expressed as:

$$q_t^c b_t + \mathrm{T}_t = r_t^b q_{t-1}^c b_{t-1} + g_t, \tag{1.52}$$

where g_t denotes public spending which follows an exogenous process to be defined below. Further, lump-sum taxes evolve according to the following feedback rule:

$$\log\left(t_t / t\right) = \phi_b \log\left(b_{t-1} / b\right) + \phi_y \log\left(y_{t-1} / y\right) \tag{1.53}$$

1.4.2 Monetary Policy

We assume that the central bank follows a standard Taylor rule:

$$\log\left(\frac{R_t}{R}\right) = \rho_R \log\left(\frac{R_{t-1}}{R}\right) + (1 - \rho_R)\left[\phi_\pi \log\left(\frac{\pi_t}{\pi}\right) + \phi_y \left(\frac{y_t}{y_{t-1}}\right)\right], \tag{1.54}$$

where $\rho_R \in (0, 1)$ is a smoothing parameter, ϕ_π is the elasticity of R_t with respect to inflation deviations and ϕ_y the elasticity with respect to output growth.

1.5 Resource Constraint

We derive the resource constraint starting with the household's budget constraint (1.4):

$$c_t + i_t + q_t\left(1 - \delta\right) k_{t-1} + \mathcal{A}_t = w_t \ell_t + q_t k_t + r_{t-1} \mathcal{A}_{t-1} + \frac{\Pi_t}{P_t} + \mathrm{div}_t - \frac{\mathrm{T}_t}{P_t}, \tag{1.55}$$

where lump-sum transfers to households and dividends are defined by:

$$\frac{\Pi_t}{P_t} = (1 - \gamma) v_t - \chi w_t \ell_t \quad \text{and} \quad \mathrm{div}_t = y_t - w_t \ell_t - z_t k_{t-1} - \frac{\kappa_p}{2}\left(\pi_t - 1\right)^2 y_t. \tag{1.56}$$

Plugging (1.56) into (1.55) yields:

$$c_t + i_t + q_t\left(1 - \delta\right) k_{t-1} + \mathcal{A}_t = w_t \ell_t + q_t k_t + r_{t-1} \mathcal{A}_{t-1} + (1 - \gamma) v_t - \chi w_t \ell_t + y_t - w_t \ell_t - z_t k_{t-1} - \frac{\kappa_p}{2}\left(\pi_t - 1\right)^2 y_t - \frac{\mathrm{T}_t}{P_t} \tag{1.57}$$

Using the net worth expression in real terms (1.39) $n_t - \chi w_t \ell_t = \gamma v_t$, yields:

$$c_t + i_t + q_t (1-\delta) k_{t-1} + \mathcal{A}_t = q_t k_t + \frac{R_{t-1}}{\pi_t} \mathcal{A}_{t-1} + v_t - n_t + y_t - z_t k_{t-1} - \frac{\kappa_p}{2} (\pi_t - 1)^2 y_t - \frac{\mathrm{T}_t}{P_t}. \tag{1.58}$$

The value of a firm (1.40) yields:

$$v_t = [1 - \Gamma(\bar{\varepsilon}_t)] r_t^p \mathfrak{p}_{t-1}. \tag{1.59}$$

Plugging (1.11) and (1.59) into (1.57):

$$c_t + i_t + [q_t (1-\delta) + z_t] k_{t-1} + \underbrace{(d_t + n_t)}_{\mathfrak{p}_t} = q_t k_t + r_{t-1} d_{t-1} + [1 - \Gamma(\bar{\varepsilon}_t)] r_t^p \mathfrak{p}_{t-1} + y_t - \frac{\kappa_p}{2} (\pi_t - 1)^2 y_t - \frac{\mathrm{T}_t}{P_t}. \tag{1.60}$$

Using the portfolio decomposition (1.29), $\mathfrak{p}_t = q_t k_t + q_t^c b_t$:

$$c_t + i_t + [q_t (1-\delta) + z_t] k_{t-1} + q_t^c b_t = r_{t-1} d_{t-1} + [1 - \Gamma(\bar{\varepsilon}_t)] r_t^p \mathfrak{p}_{t-1} + y_t - \frac{\kappa_p}{2} (\pi_t - 1)^2 y_t - \frac{\mathrm{T}_t}{P_t}. \tag{1.61}$$

Using the expression for the real return of capital (1.41):

$$c_t + i_t + q_t^c b_t = y_t + r_{t-1} d_{t-1} + [1 - \Gamma(\bar{\varepsilon}_t)] r_t^p \mathfrak{p}_{t-1} - r_t^k q_{t-1} k_{t-1} - \frac{\kappa_p}{2} (\pi_t - 1)^2 y_t - \frac{\mathrm{T}_t}{P_t}. \tag{1.62}$$

Finally, plugging the portfolio return (1.34) yields:

$$\begin{aligned} c_t + i_t + q_t^c b_t = y_t + r_{t-1} d_{t-1} + [1 - \Gamma(\bar{\varepsilon}_t)] \left[r_t^k q_{t-1} k_{t-1} + r_t^b q_{t-1}^c b_{t-1} - \frac{\varpi}{2} (\omega_{t-1} - \omega)^2 \frac{\mathfrak{p}_{t-1}}{\pi_t} \right] \\ - r_t^k q_{t-1} k_{t-1} - \frac{\kappa_p}{2} (\pi_t - 1)^2 y_t - \frac{\mathrm{T}_t}{P_t}, \end{aligned}$$

which simplifies to

$$c_t + i_t + q_t^c b_t - r_t^b q_{t-1}^c b_{t-1} = y_t + r_{t-1} d_{t-1} - \frac{\varpi}{2} (\omega_{t-1} - \omega)^2 \frac{\mathfrak{p}_{t-1}}{\pi_t} - \Gamma(\bar{\varepsilon}_t) r_t^p \mathfrak{p}_{t-1} - \frac{\kappa_p}{2} (\pi_t - 1)^2 y_t - \frac{\mathrm{T}_t}{P_t}. \tag{1.63}$$

Using the government budget constraint (1.51) that we plug into the resource constraint:

$$y_t = c_t + i_t + (1 + \rho q_t^c) b_{t-1} + g_t - r_t^b q_{t-1}^c b_{t-1} + \frac{\varpi}{2} (\omega_{t-1} - \omega)^2 \frac{\mathfrak{p}_{t-1}}{\pi_t} - r_{t-1} d_{t-1} + \Gamma(\bar{\varepsilon}_t) r_t^p \mathfrak{p}_{t-1} + \frac{\kappa_p}{2} (\pi_t - 1)^2 y_t. \tag{1.64}$$

Since from (1.42) $r_t^b q_{t-1}^c = 1 + \rho q_t^c$, and plugging (1.13) with (1.21), we finally get the resource constraint:

$$y_t = c_t + i_t + g_t + \frac{\varpi}{2} (\omega_{t-1} - \omega)^2 \frac{\mathfrak{p}_{t-1}}{\pi_t} + \mu G(\bar{\varepsilon}_t) r_t^p \mathfrak{p}_{t-1} + \frac{\kappa_p}{2} (\pi_t - 1)^2 y_t. \tag{1.65}$$

1.6 Shocks

The public spending and the TFP shocks follow an AR(1) process enriched with time-varying volatility such that:

$$\log(x_t) = (1-\rho_x)\log(x) + \rho_x x_{t-1} + \exp(\sigma_t^x)\varepsilon_t^x, \tag{1.66}$$

$$\sigma_t^x = (1-\rho_{\sigma^x})\,\sigma^x + \rho_{\sigma^x}\sigma_{t-1}^x + \eta_{\sigma^x}\varepsilon_t^{\sigma x}, \tag{1.67}$$

where $x_t = \{g_t, a_t\}$, and $\varepsilon_t^x \sim N(0,1)$, $\varepsilon_t^{\sigma^x} \sim N(0,1)$. We also consider a risk shock *à la* Christiano et al. (2014) by allowing the variance of idiosyncratic shocks, $\sigma_{\varepsilon,t}$, to vary over time. ε_t is log-normally distributed such that $\varepsilon \sim \log N(0, \sigma_{\varepsilon,t}^2)$ and therefore, $\log(\varepsilon)$ is normally distributed so that we have $\log(\varepsilon) \sim N(\mu_\varepsilon, \xi_t^2)$ where $\xi_t^2 = \log(1+\sigma_{\varepsilon,t}^2)$. We assume that:

$$\xi_t = \rho_\xi \xi_{t-1} + \exp(\sigma^\xi)\varepsilon_t^\xi, \tag{1.68}$$

where $\varepsilon_t^x \sim N(0,1)$.

1.7 Model's Steady State

Normalization We assume a zero steady-state inflation rate, $\pi = 1$ and we normalize a so that $y = 1$.

Interest Rates From Equation (1.6), we the real risk-free interest rate is given by $r = \beta^{-1}$, while using the definition of $\check{r}_t$, we get the portfolio interest rate $R^p = \check{r}\beta^{-1}$ as soon as $\pi = 1$. Finally, plugging Equations (1.42)-(1.36) into (1.34) yields:

$$q^c = (R^b - \rho)^{-1},\ R^k = R^b \text{ and } R^p = R^b. \tag{1.69}$$

Production Sector From Equation (1.50), $s = \left(\theta_p - 1\right)/\theta_p$. Further, $q = 1$. From Equation (1.41), we get:

$$z = q[r^k - (1-\delta)]. \tag{1.70}$$

Equations (1.3) and (1.49) implies that $i/k = \delta$ and $y/k = z/s\alpha$. Also, from Equations (1.48), we deduce:

$$\frac{w}{y} = s(1-\alpha)\frac{1}{\ell}, \tag{1.71}$$

where the value of ℓ is set exogenously, imposed by our calibration.

Household Sector The dynamics of X_t from (1.2) gives

$$X = (1-h)c, \tag{1.72}$$

Further, defining $A = \left[(1-h)c\left(1-\psi\ell_t^{\omega_w}\right)\right]^{-\sigma}$, we get from (1.7) and (1.8):

$$\lambda = (A + \sigma_X v)\,, \tag{1.73}$$

$$v = \frac{\psi\ell^{\omega_w} A}{\beta(1-\sigma_X)-1}. \tag{1.74}$$

Plugging (1.74) into (1.73) yields:

$$\lambda = A\left(1 + \frac{\sigma_X \psi \ell^{\omega_w}}{\beta(1-\sigma_X)-1}\right). \tag{1.75}$$

Plugging into (1.9) finally yields:

$$\psi = \frac{(1-\beta h)w}{\ell^{\omega_w}\left[\frac{\omega_w X}{\ell} + \frac{\sigma_X(1-\beta h)w}{1-\beta(1-\sigma_X)}\right]}. \tag{1.76}$$

Portofolio Composition We deduce the portfolio composition from Equations (1.29)-(1.34):

$$\frac{\mathfrak{p}}{y} = q\frac{k}{y} + q^c\frac{b}{y}. \tag{1.77}$$

$$\omega = 1 - \frac{q^c b}{\mathfrak{p}} \tag{1.78}$$

$$\frac{k}{\mathfrak{p}} = \omega \tag{1.79}$$

Financial Frictions The steady-state of optimal contract equations yields:

$$\check{r}\left(1-\Gamma(\bar{\varepsilon}) + \Lambda\left[\Gamma(\bar{\varepsilon}) - \mu G(\bar{\varepsilon})\right]\right) = \Lambda, \tag{1.80}$$

$$\frac{\Gamma_\omega(\bar{\varepsilon})}{\Gamma_\omega(\bar{\varepsilon}) - \mu G_\omega(\bar{\varepsilon})} = \Lambda, \tag{1.81}$$

$$\Gamma(\bar{\varepsilon}) - \mu G(\bar{\varepsilon}) = \frac{x-1}{\check{r}x}. \tag{1.82}$$

We choose $\bar{\varepsilon}$, σ_ε, γ, and μ in order to match the following moments: (*i*) an annually rate of business failure of 3 per cent annually, or $F(\bar{\varepsilon}) = 0.03/4$ in quarterly terms; (*ii*) a annual spread of 150 basis points or $\check{r} = 1.015^0.25$, and (*iii*) a leverage ratio of $x \equiv \mathfrak{p}/n = 6$. In "funx3.m", we use the Matlab command fsolve to pick up these values.

Rest of the Steady State The value of a firm is given by Equation (1.40), $v = \left[1-\Gamma(\bar{\varepsilon})\right] R^p \mathfrak{p}$, which implies, using Equation (1.65):

$$\frac{c}{y} = 1 - \frac{g}{y} - \frac{i}{y} - \frac{\mathfrak{p}}{y}\left[R^p \mu G(\bar{\varepsilon})\right]. \tag{1.83}$$

The government budget constraint writes:

$$\frac{\mathrm{T}}{y} = \left(R^b - 1\right) q^c \frac{b}{y} + \frac{g}{y}, \tag{1.84}$$

with $\frac{w\ell^h}{y} = \frac{\Pi(1-\alpha)}{\theta_p/\left(\theta_p-1\right)}$, $\frac{w^e}{y} = \frac{(1-\Pi)(1-\alpha)}{\theta_p/\left(\theta_p-1\right)}$, $\frac{zk}{y} = s\alpha$. Finally, the definitions of the leverage ratio and the financial intermediaries' balance sheet imply $x = \mathfrak{p}/n$ and $d/n = x - 1$.

2 A Simplified Two-Asset Model

Model Description We now develop a two-asset model in which we abstract from the financial accelerator mechanism to show that the wedge between the returns on the two assets is independent from this assumption. Absent financial frictions, the portfolio decisions are made directly by the representative household, while the mutual fund and the portfolio managers are removed from the structure of the model. We simplify the model as much as possible and thus also abstract from investment adjustment costs. There is a continuum of identical households who consume, work and save so as to maximize their flow of expected utility given by Expression (1.1). With zero investment adjustment cost, the law of motion of capital (1.3) becomes:

$$k_t = (1-\delta)\, k_{t-1} + i_t, \tag{2.1}$$

and the budget constraint (1.4) becomes:

$$c_t + \underbrace{q_t k_t + q_t^c b_t}_{\mathfrak{p}_t} \leq w_t \ell_t^h + \underbrace{z_t k_{t-1} + q_t (1-\delta)\, k_{t-1} + (1+\rho q_t^c)\, b_{t-1} - \frac{\varpi}{2}\left(\omega_{t-1} - \bar{\omega}\right)^2 \mathfrak{p}_{t-1}}_{r_t^p \mathfrak{p}_{t-1}} + \mathrm{div}_t - \frac{\mathrm{T}_t}{P_t}. \tag{2.2}$$

Savings of the representative household are invested in two assets which are stored in a "portfolio". Precisely, at time t, the representative household buys firm's equity k_t at real price q_t and public bonds b_t at real price q_t^c. Thus, her portfolio of assets at time t corresponds to the sum of the newly acquired assets $\mathfrak{p}_t \equiv q_t k_t + q_t^c b_t$. In the same period, the representative household earns the return from renting the pas capital stock at a real rental rate z_t. She also receives the depreciated equity claims $(1-\delta)\, k_{t-1}$, valued at a price q_t. Finally, public bonds purchased in $t-1$ at price q_{t-1}^c provide a return R_t^b. The representative household pays a quadratic adjustment cost whenever the capital share in her portfolio in $t-1$, $\omega_t \equiv \frac{q_t k_t}{q_t k_t + q_t^c b_t}$, deviates from its steady-state value.

The Lagrangian writes:

$$\mathcal{L}_t = \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{c} \frac{1}{1-\sigma}\left[\left(\widetilde{c} - \psi\left(\ell_t\right)^{\omega_w} \mathrm{X}_t\right)^{1-\sigma} - 1\right] - v_t\left[\mathrm{X}_t - \widetilde{c}^{\sigma_X} \mathrm{X}_{t-1}^{1-\sigma_X}\right] \\ -\lambda_t \left[\begin{array}{c} c_t + q_t^c b_t + q_t k_t - (1-\delta)\, q_t k_{t-1} - w_t \ell_t \\ -z_t k_{t-1} - (1+\rho q_t^c)\, b_{t-1} + \frac{\varpi}{2}\left(\frac{q_{t-1}k_{t-1}}{q_{t-1}k_{t-1}+q_{t-1}^c b_{t-1}} - \bar{\omega}\right)^2 \left(q_{t-1}k_{t-1} + q_{t-1}^c b_{t-1}\right) \\ -\frac{\Pi_t}{P_t} - \mathrm{div}_t + \frac{\mathrm{T}_t}{P_t} \end{array} \right] \end{array} \right\}, \tag{2.3}$$

and the portfolio composition is determined by the FOCs wrt to k_t and b_t, which are:

$$\lambda_t = \beta \mathbb{E}_t \lambda_{t+1} \left\{ \frac{z_{t+1} + (1-\delta)\, q_{t+1}}{q_t} - \varpi\left(\omega_t - \omega\right) - \varpi\left(\omega_t - \omega\right)\left[\frac{(\omega_t - \omega)}{2} - \omega_t\right] \right\}, \tag{2.4}$$

$$\lambda_t = \beta \mathbb{E}_t \lambda_{t+1} \left\{ \frac{1 + \rho q_{t+1}^c}{q_t^c} - \varpi\left(\omega_t - \omega\right)\left[\frac{(\omega_t - \omega)}{2} - \omega_t\right] \right\}. \tag{2.5}$$

Let us define $r_{t+1}^k \equiv \frac{[z_{t+1}+(1-\delta)q_{t+1}]}{q_t}$ as the real return on capital, which corresponds to the real rental rate of capital z_t, augmented by the depreciation rate of capital, and $r_{t+1}^b = \frac{1+\rho q_t^c}{q_{t-1}^c}$ as the real return on bonds.

Pricing Equation Notice that we can rewrite Equation (2.4) and (2.5)

$$\mathrm{E}_t \left\{ \beta \frac{\lambda_{t+1}}{\lambda_t}\left[r_{t+1}^k - \varpi\left(\omega_t - \omega\right) + \frac{\varpi}{2}\left(\omega_t^2 - \omega^2\right)\right] \right\} = 1, \tag{2.6}$$

$$\mathrm{E}_t \left\{ \beta \frac{\lambda_{t+1}}{\lambda_t}\left[r_{t+1}^b + \frac{\varpi}{2}\left(\omega_t^2 - \omega^2\right)\right] \right\} = 1, \tag{2.7}$$

where the expression $M_{t,t+1} \equiv \beta\lambda_{t+1}/\lambda_t$ corresponds to the stochastic discount factor.

In order to show the intuitions, let's apply the risk-adjusted log-linearization of the model, in the spirit of Bianchi et al. (2022). Precisely, assuming that variables in levels follow a log-normal distribution and applying a first-order approximation yields that Equation (2.6) becomes

$$\log\left(\mathrm{E}_t \left\{ M_{t,t+1}\left[r_{t+1}^k - \varpi\left(\omega_t - \omega\right) + \frac{\varpi}{2}\left(\omega_t^2 - \omega^2\right)\right] \right\}\right) = 0. \tag{2.8}$$

Remark. *If X_t is conditionally lognormally distributed, it has the convenient property*

$$\log\left(\mathrm{E}_t\left\{X_t\right\}\right) = \mathrm{E}_t\left\{\log\left(X_t\right)\right\} + \frac{1}{2}\mathrm{V}\left\{\log\left(X_t\right)\right\}, \tag{2.9}$$

where $\mathrm{V}\left\{\log\left(X_t\right)\right\}$ *is the variance of* $\log\left(X_t\right)$.

■

In the absence of portfolio frictions $\varpi = 0$, Equation (2.8) becomes

$$\mathrm{E}_t\left\{\log\left(M_{t,t+1} r_{t+1}^k\right)\right\} + \frac{1}{2}\mathrm{V}\left\{\log(M_{t,t+1} r_{t+1}^k)\right\} = 0. \tag{2.10}$$

Let denote $\tilde{x} = \log(\tilde{x})$, such that

$$\mathrm{E}_t\left\{\tilde{M}_{t,t+1}\right\} + \mathrm{E}_t\left\{\tilde{r}^k_{t+1}\right\} + \frac{1}{2}\mathrm{V}\left\{\tilde{M}_{t,t+1} + \tilde{r}^k_{t+1}\right\} = 0. \tag{2.11}$$

Using the property $\mathrm{V}\{X_t + Y_t\} = \mathrm{V}\{X_t\} + \mathrm{V}\{Y_t\} + 2\mathrm{cov}\{X_t, Y_t\}$, where $\mathrm{cov}\{X_t, Y_t\}$ is the covariance between X_t and Y_t, this gives

$$\mathrm{E}_t\left\{\tilde{r}^k_{t+1}\right\} = -\mathrm{E}_t\left\{\tilde{M}_{t,t+1}\right\} - \frac{1}{2}\mathrm{V}\left\{\tilde{M}_{t,t+1}\right\} - \frac{1}{2}\mathrm{V}\left\{\tilde{r}^k_{t+1}\right\} - \mathrm{cov}\left\{\tilde{M}_{t,t+1}, r^k_{t+1}\right\}. \tag{2.12}$$

Notice that $\mathrm{E}_t\left\{\tilde{M}_{t,t+1}\right\}$ captures the consumption smoothing motive and $\mathrm{V}\left\{\tilde{M}_{t,t+1}\right\}$ captures the precautionary saving motive.

We now applies the same formula in presence of portfolio frictions $\varpi > 0$. Equation (2.8) is

$$\mathrm{E}_t\{M_{t,t+1}\underbrace{\left[r^k_{t+1} - \varpi\left(\omega_t - \omega\right) + \frac{\varpi}{2}\left(\omega_t^2 - \omega^2\right)\right]}_{r^k_{t+1}(\omega_t)}\} = 1, \tag{2.13}$$

where $r^k_{t+1}(\omega_t)$ is the return of equity which internalizes the portfolio frictions. Therefore, Equation (2.12) become

$$\mathrm{E}_t\left\{\tilde{r}^k_{t+1}(\omega_t)\right\} = -\mathrm{E}_t\left\{\tilde{M}_{t,t+1}\right\} - \frac{1}{2}\mathrm{V}\left\{\tilde{M}_{t,t+1}\right\} - \frac{1}{2}\mathrm{V}\left\{\tilde{r}^k_{t+1}(\omega_t)\right\} - \mathrm{cov}\left\{\tilde{M}_{t,t+1}, r^k_{t+1}(\omega_t)\right\}. \tag{2.14}$$

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