Debt, Sovereign Risk and Government Spending*

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Abstract

We investigate the relation between the size of government indebtedness and the effectiveness of government spending shocks. We develop New Keynesian model of sovereign default in which the zero lower bound (ZLB) constraint on the nominal interest rate may be binding. In normal times, high levels of debt to GDP lead to reduced output multipliers. High debt also leads to more severe economic downturns in the event of crises. Finally, when the ZLB binds, high debt levels produce larger output multipliers.

Keywords: Sovereign Default Risk, Fiscal Policy.

JEL Classification: E62, E32.

1 Introduction

Does the level of public debt affect the effectiveness of government spending shocks? Conventional wisdom suggests that countries with high levels of public debt have less room for fiscal stimulation than countries with low levels of public debt in the event of an economic crisis, and would therefore advocate for low debt levels. For instance, Corsetti, Meier, and Müller (2012) show that, empirically, periods of fiscal stress are associated with lower spending output multipliers. The perceived reason would be that countries with high debt are closer to their natural debt limit, and that any fiscal stimulation might lead them to a sovereign debt crisis and/or to insolvency, or would require much more future tax efforts with adverse consequences.

We investigate this question in a New-Keynesian model with sovereign risk à la Corsetti et al. (2013). Sovereign risk is a non-linear function of the debt ratio and the direct consequences of sovereign risk are ruled out since the risk is insured *ex-post*. The main impact of sovereign risk thus goes through the rise in the sovereign rate, that further increases need to refinance debt and raises the level of public debt. Sovereign crisis and insolvency are not considered however,

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as government expenditure (spending and the debt burden) are financed through debt and a distortionary tax on labor income, that follows a fiscal rule.

Within this framework, the impact of the level of debt in the steady state on government spending multipliers is negative during normal times, *i.e.* when shocks occur around the steady state. This result relies on the fact that a country with a higher steady-state level of public debt will face a larger increase in public debt after a spending shock due to sovereign risk, and that the corresponding long term increase in taxes will be larger, imposing more distortions on the economy, thus contributing negatively to GDP. This result is perfectly in accordance with empirical estimates of spending multipliers conditional on a high level of debt (see Corsetti, Meier, and Müller (2012) or Ilzetzki, Mendoza, and Végh (2013)).

Further, when the economy is hit by a discount factor shock that is intended to mimic the effects of a financial crises, countries with a high initial level of debt are worse off. The shock is known to produce large adverse effects on output, hours, real wages and inflation, pushing the economy to the ZLB of the nominal interest rate. The fall in output raises the debt ratio more for the highly indebted economy, which in turn requires a larger increase in the labor income tax rate and makes the high-debt economy experience a larger recession.

However, if a stimulus package – a public spending shock – is decided to alleviate the effects of the discount factor shock, a novel result arises, according to which spending multipliers will be larger in the highly indebted economy. Of course, as already found in the literature, spending multipliers are larger than during normal times regardless of the debt level (see Christiano, Eichenbaum, and Rebelo (2011) and the subsequent contributions). But conditional on hitting the ZLB, countries with high debt have larger spending multipliers.

The reason is that highly indebted countries have larger steady-state labor income tax rates. At the ZLB, a spending shock raises very strongly hours worked and the real wage, which enlarges the tax base a lot. With a larger steady state tax rate, high debt countries experience a rise in their fiscal receipts that is larger than the rise in spending, making public debt fall and dampening sovereign risk. Both contribute to stimulate GDP more than in low-debt economies. Indeed, because these economies have lower steady-state labor income tax rates, they experience a fall in their net fiscal receipts (receipts increase less than spending) and face a rise in public debt and sovereign risk, that contributes negatively to the dynamics of GDP. Spending multipliers are thus lower for the low debt countries, conditional on being at the ZLB. Nevertheless, we show that these larger multipliers will not make a country that faces joint discount factor and public spending shocks better off in terms of output gains/losses, unless interest rates remain very low (close to the ZLB) and public and private consumption are complementary.

Our paper relates to the literature on spending multipliers and how the economic environment may affect the latter. Empirically, one of the first paper to raise the question was Perotti (1999). More recently, the subject has been revived by Auerbach and Gorodnichenko (2012), investigating whether the business cycle position matters for the value of multipliers. Last, two recent papers respectively by Corsetti, Meier, and Müller (2012) and by Ilzetzki, Mendoza, and Végh (2013) more precisely question the impact of debt or fiscal stress and financial crises on spending multipliers. Their results converge in that fiscal stressed or highly indebted economies tend to be characterized by lower (and even negative) spending multipliers. According to Corsetti, Meier, and Müller (2012), when the economy experiences a financial crisis, spending multipliers are much larger than in normal times, a result that is consistent with more theoretical contributions like Christiano, Eichenbaum, and Rebelo (2011). None of the mentioned empirical papers does test the joint conditional impact of a financial crisis and the level of debt as we do. Our paper also belongs to a model-based literature that investigates the effects of the ZLB on the size of fiscal multipliers, skillfully summarized and referenced in Eggertsson (2011). To our knowledge, there are only very little papers questioning the effect of the initial debt level on the size of spending with sovereign risk. Closest to our paper is Corsetti, Kuester, Meier, and Müller (2013) but their analysis does not consider capital accumulation, and essentially focuses on the case of lump-sum taxes.[1] As will be clear, the use of distortionary taxes is crucial to our main result.

The paper is organized as follows. Section 2 presents the model. Section 3 details our calibration. Section 4 analyzes the Impulse Response Functions to spending shocks, discount factor shocks and combined shocks when the steady state level of debt is low or high. It also investigates the robustness of our main result to the assumption of complementarity (vs. substitutability) of public and private goods in the utility function of the households. Section 5 summarizes our results by presenting the value of spending multipliers at various horizons and under the different cases investigated. Section 6 compares the cumulative output gains/losses under various situations depending on the level of steady-state debt. Section 7 offers some concluding remarks.

2 Model

Our framework builds on a New Keynesian model of sovereign default. We rely on the specification of the probability of sovereign default developed by Corsetti et al. (2013) as a non-linear function of the public debt-to-GDP ratio. In this framework, default risk matters *ex-ante* but not *ex-post*. We also consider two additional features with respect to standard models. First, government spending and private consumption may be substitute or complementary to each other. Second, the zero lower bound (ZLB, henceforth) on the nominal interest is taken into account to study the implication of the sovereign risk and debt on the size of government spending multiplier when this constraint binds. The economy is populated by infinitely-lived and identical households, two types of firms: intermediate good firms and final good firms, the government and the central bank.

[1] In the working paper version, Corsetti et al. (2013) present some results when using distortionary taxes instead of lump-sum taxes in Section 4.3.2.

2.1 Households

Households face a simple optimization problem as they choose consumption, labor supply, deposits and government bonds maximizing lifetime welfare

$$E_t\left(\sum_{s=t}^{\infty}\left(e_s\beta^{s-t}\right)u\left(c_s,g_s,\ell_s\right)\right), \tag{1}$$

where $u_{\ell,t}\leq 0$, $u_{c,t}\geq 0$ and $u_{g,t}\geq 0$ are the first-order partial derivatives with respect to hours worked, consumption, and public spending. $\beta\in(0,1)$, denotes the discount factor. e_t is a time t discount factor shock that evolves according to

$$\log e_t=\rho_e\log e_{t-1}+\epsilon_t^e. \tag{2}$$

Households optimize subject to the budget constraint

$$b_t^g+d_t+c_t+k_t=(1-\chi_t)\,r_{t-1}^b b_{t-1}^g+r_{t-1}^d d_{t-1}+(1-\tau_t)\,w_t\ell_t+R_t^k k_{t-1}+\Pi_t+T_t^b. \tag{3}$$

In this equation, d_t denote real deposits returning r_t^d between t and $t+1$ and b_t^g the real amount of sovereign bonds held that return r_t^b and that are affected by default risk, χ_t. Variable T_t^b denotes the amount of *ex-post* insurance against sovereign default.[2] Further, c_t is consumption, i_t is investment in physical capital, w_t denotes the real wage, ℓ_t hours worked and τ_t a distortionary tax on labor income. Variable k_t is the capital stock, and $R_{kt}=1+(1-\eta)\left(r_t^k-\delta\right)$ is the gross return on capital where η is the (constant) capital income tax that comes with a deduction for capital depreciation δ.[3] Finally, Π_t comprises monopolistic profits from firms. An additional constraint to the optimization program is the law of capital accumulation

$$k_t-(1-\delta)\,k_{t-1}=i_t\left(1-\left(\varphi^i/2\right)x_t^2\right), \tag{4}$$

where $x_t=i_t/i_{t-1}-1$ is the growth rate of investment, and $\varphi_i>0$ controls the size of the investment adjustment cost. First order conditions with respect to deposits, sovereign bonds and labor supply imply

$$\begin{aligned} E_t\left(\beta_{t+1}r_t^d\right) &= 1, && (5)\\ E_t\left(\beta_{t+1}\left(1-\chi_{t+1}\right)r_t^b\right) &= 1, && (6)\\ u_{\ell,t}+(1-\tau_t)\,u_{c,t}w_t &= 0, && (7)\end{aligned}$$

where $\beta_t=e_t\beta u_{c,t}/u_{c,t-1}$. The first two equations price deposits and sovereign bonds. The third equation relates the marginal disutility of working to the real wage, expressed in terms of the marginal utility of consumption. We define $q_t u_{c,t}$ as the Lagrangian multiplier associated with the capital accumulation constraint, and derive the following first order conditions with respect to the capital stock, and investment

$$\begin{aligned} E_t\left(\beta_{t+1}\left(q_{t+1}(1-\delta)+(1-\eta)\,r_{t+1}^k+\eta\delta\right)\right) &= q_t, && (8)\\ q_t\left(1-\left(\varphi^i/2\right)x_t^2-\varphi^i x_t\left(1+x_t\right)\right)+E_t\left(\beta_{t+1}q_{t+1}\varphi^i x_{t+1}\left(1+x_{t+1}\right)^2\right) &= 1. && (9)\end{aligned}$$

[2] Both χ_t and T_t^b will be discussed in the government section.

[3] A constant capital income tax η is introduced to obtain a realistic steady-state calibration for the labor income tax rate.

2.2 Firms

2.2.1 Final good firms

At time t, a perfectly competitive representative firm produces a final consumption good y_t. Final goods producers the final good by combining a continuum of intermediate goods indexed by $j \in [0,1]$, using the CES production function (Dixit-Stiglitz aggregator):

$$y_t = \left[\int_0^1 y_t\left(j\right)^{\frac{\theta-1}{\theta}}\,dj\right]^{\frac{\theta}{\theta-1}}, \tag{10}$$

where, θ denotes the elasticity of substitution across varieties and $y_t\left(j\right)$ denotes the time t input of intermediate good j. The firm takes the price of output p_t and the input price, $p_t\left(j\right)$, as given. Profit maximization leads to the following first order condition:

$$y_t\left(j\right) = \left(p_t\left(j\right)/p_t\right)^{-\theta} y_t. \tag{11}$$

Substituting (10) into (11) yields the following relationship between the aggregate price level and the price of intermediate goods:

$$p_t = \left[\int_0^1 p_t\left(j\right)^{1-\theta}\,dj\right]^{\frac{1}{1-\theta}}. \tag{12}$$

2.2.2 Intermediate good firms

Intermediate good $j \in (0,1)$ is produced by a monopolist using effective capital k_{t-1} in the production process and hiring labor in quantity ℓ_t, with the following production function

$$y_t\left(j\right) = k_{t-1}^{\iota}\left(j\right)\ell_t\left(j\right)^{1-\iota}, \tag{13}$$

where $\iota \in (0,1)$ is the share of effective capital. Intermediate good firms rent capital and labor in a perfectly competitive markets. Profits are distributed to households at the end of each period. Recall that r_t^k and w_t denote the net real rental rate on capital and the real wage, respectively. The firm's real marginal cost is given by

$$s_t = \left(\iota^{\iota}\left(1-\iota\right)^{1-\iota}\right)^{-1}\left(r_t^k\right)^{\iota}\left(w_t\right)^{1-\iota}. \tag{14}$$

The firm's time t profits are $\left[p_t\left(j\right)/p_t - s_t\right]p_t y_t\left(j\right)$, where $p_t\left(j\right)$ is firm j's price in period t. We assume there are price-setting frictions along the lines of Calvo (1983), where $1/\left(1-\gamma\right)$ and γ_p respectively denote the average length of contracts and an indexation parameter. The optimal pricing conditions are standard and therefore not reported.

2.3 Government

We adopt the approach of sovereign default from Corsetti et al. (2013). Actual *ex post* default is neutral while the *ex-ante* probability of default is crucial for the pricing of government debt and for real activity. Following Eaton and Gersovitz (1981), Arellano (2008) and others have modeled default as a strategic decision of the government. On the other hand, Bi (2012) consider default as the consequence of the government's inability to raise the funds necessary to honor its debt obligations. Under both approaches, the probability of sovereign default is closely and non-linearly related to the level of public debt to GDP. Focusing on the domestic economy, the *ex-ante* probability of default, pd_t, at a certain level of sovereign indebtedness, $by_t = b_t^g / (4y)$, will be given by the cumulative distribution function of the beta distribution:

$$pd_t = F_{beta} \left(by_t / by_{max}, \alpha_p, \beta_p \right), \tag{15}$$

where by_{max} denotes the upper end of the support for the debt to GDP ratio. Actual default occurs with probability pd_t so that

$$\chi_t = \Delta \text{ if } \mathcal{B} \left(pd_t \right) = 1, \tag{16}$$

$$\chi_t = 0 \text{ if } \mathcal{B} \left(pd_t \right) = 0, \tag{17}$$

where $\mathcal{B}(.)$ is a Bernoulli. Given these assumptions, the budget constraint of the government writes

$$b_t^g = r_t^b \left(1 - \chi_t\right) b_{t-1}^g + g_t - \tau_t w_t n_t - \eta \left(r_{kt} - \delta\right) k_{t-1} + T_t^b. \tag{18}$$

Once again, potential losses from default are fully compensated, so that only *ex-ante* default risk matters. As a consequence

$$T_t^b = r_t^b \chi_t b_{t-1}^g, \tag{19}$$

and the consolidated budget constraint writes

$$b_t^g = r_t^b b_{t-1}^g + g_t - \tau_t w_t n_t - \eta \left(r_{kt} - \delta\right) k_{t-1}. \tag{20}$$

The stability of public debt in the long run is granted by the following tax rule:

$$\tau_t - \tau = d_\tau \left(b_{t-1}^g - b^g \right). \tag{21}$$

Finally, public spending evolve according to

$$\log \left(g_t / g \right) = \rho_g \log \left(g_{t-1} / g \right) + \varepsilon_t^g. \tag{22}$$

Although actual default is not considered in our set-up, sovereign default risk has major real consequences. A rise in default risk raises the real sovereign rate r_t^b, leads to a rise in public debt that subsequently triggers a rise in the distortionary tax rate. As the latter goes up, hours worked, output, investment, asset prices and inflation collapse. So even in the absence of actual default, sovereign default risk can be a major driver of the dynamics of the economy.

2.4 Central Bank

We assume that the gross nominal interest rate r_t is set according to

$$r_t = \max \left(1, rn_t\right), \tag{23}$$

where rn_t is the desired gross nominal interest rate chosen by the central bank, such that

$$\log(rn_t/r) = d_\pi \log(\pi_t/\pi) + d_y \log(y_t/\widetilde{y}_t), \tag{24}$$

where π_t is the inflation rate and $\widetilde{y}_t$ is the natural level of output.[4] d_π and d_y determine the response of rn_t to inflation and the output gap, respectively. The central bank sets r_t equal to rn_t if and only if its policy rule implies a non-negative level for the nominal interest rate. Otherwise, the ZLB binds and r_t equals one. The relation between the gross nominal interest rate and the real deposits rate is given by the following Fisher equation:

$$r_t = r_t^d E_t(\pi_{t+1}). \tag{25}$$

2.5 Aggregation

The clearing condition on the intermediate goods market is

$$k_{t-1}^{\iota}\ell_t^{1-\iota} = \int_0^1 y_t(j)\, dj = y_t dp_t, \tag{26}$$

where $dp_t = \int_0^1 (p_t(j)/p_t)^{-\theta} dj \geq 1$ measures price dispersion, and where we have used the equilibrium condition on the labor market $\int_0^1 \ell_t(j)\, dj = \ell_t$. On the final goods market, the clearing condition simply writes

$$y_t = c_t + i_t + g_t. \tag{27}$$

The clearing condition for government bonds is just

$$b_t^g = b_t. \tag{28}$$

Since deposits are in zero net supply, the clearing condition is

$$d_t = 0. \tag{29}$$

3 Calibration

Table 1 presents the model's parameters which are calibrated to peripheral countries of the Euro Area and to fit the quarterly frequency. We adopt a formulation of the utility that can give rise to complementarity or substitutability between private and public consumption

$$u(c_t, g_t, n_t) = \log(\widetilde{c}_t) - \omega\ell_t^{1+\psi}/1+\psi, \tag{30}$$

where

$$\widetilde{c}_t = \left(\kappa c_t^{\frac{\nu-1}{v}} + (1-\kappa)\, g_t^{\frac{\nu-1}{v}}\right)^{\frac{\nu}{\nu-1}},\ \nu > 0. \tag{31}$$

In Equation (31), κ denotes the weight of private consumption in the effective consumption index, and ν is the elasticity of substitution between private and public consumption spending. When

[4] As in Gertler and Karadi (2011), variations in the mark-up will serve as a proxy for variations in the output gap.

$\nu = 0$, public and private consumption expenditure are complements. As ν increases, private and public consumption become more and more substitutable, and $\nu \to \infty$ capture perfect substitutability. This specification is in the spirit of Leeper, Traum, and Walker (2015), but our choice of a CES specification is justified by the need to capture the diminishing marginal returns to public spending in order to achieve a given level of effective consumption, *ceteris paribus*. Following Bouakez and Rebei (2007), we choose different scenarios depending on the value of ν. In the first scenario $\nu = 1$, public and private spending are substitutable. In the second scenario $\nu = 0.45$, public and private spending are mildly complementary.[5]

The discount factor is $\beta = 0.99$ implying an annual real interest rate of roughly 4 percent. The inverse of the Frisch elasticity of labor supply is typically a controversial and important parameter. We impose $\psi = 3$ to capture relatively sluggish labor markets, a value that lies in-between the calibration of Corsetti et al. (2013) and that of Galí, López-Salido, and Vallés (2007). On the production side, the share of effective capital is $\iota = 0.33$, and the steady state depreciation rate is $\delta = 0.018$ (7% annually). The investment adjustment cost parameter, φ_i, is set to 1.8. The steady-state mark-up is 30%, implying $\theta = 4.33$, the Calvo parameter on price contracts is $\gamma = 0.75$ a standard value in quarterly models of price adjustments, and the indexation parameter is $\gamma_P = 0.5$. Using OECD data for 2008, we build a measure of hours worked and set $\ell = 0.3049$.[6] The value of ω is adjusted to match this value. Proceeding similarly, the share of public expenditure in GDP and the level of public debt to GDP are respectively $s_g = 0.1924$. The capital income tax rate is $\eta = 0.45$. We investigate two cases for the debt level, $b^g/(4y) = 0.6$ referred to as the case of low debt, and $b^g/(4y) = 1.15$, referred to as the case of high debt. The steady-state labor income tax rate is adjusted to match the debt-to-GDP ratio target. We get $\tau = 0.3117$ in the case of low debt and $\tau = 0.4532$ when the steady-state level of debt is high. Both numbers are reasonable in comparison of observed labor income tax rates in peripheral countries of Europe. We also impose the value of κ so that marginal utilities from private and public consumption are equal in the steady state.[7] Default parameters are calibrated after Corsetti et al. (2013): the size of default is $\Delta = 0.55$ and parameters of the default distribution are $\alpha_p = 3.7$, $\beta_p = 0.54$ and $by_{\max} = 2.56$. Finally, the feedback parameter of the fiscal rule is $d_\tau = 0.25$ to make the model stationary. Parameters of the Taylor rule are $d_\pi = 1.5$ and $d_y = 0.125$ and the persistence of shocks is $\rho_e = \rho_g = 0.9$.

[5] This value is also roughly consistent with the point estimates found by Auray and Eyquem (2017), between 0.5 and 0.6, and with the estimates of Leeper, Traum, and Walker (2015), according to which public and private consumption are complementary.

[6] The country sample includes Greece, Ireland, Italy, Portugal and Spain. Average statistics are computed as weighted averages.

[7] The value of κ that is consistent with the optimal provision of public good in the steady-state is indeed $u_c = u_g$ and implies:

$$\kappa = g^{-\frac{1}{\nu}} / \left(c^{-\frac{1}{\nu}} + g^{-\frac{1}{\nu}}\right).$$

Table 1: Calibrated parameter values

Discount factor	$\beta = 0.99$
Edgeworth preference parameter	$\nu = \{1, 0.45\}$
Inverse of the Frisch elasticity of labor supply	$\psi = 3$
Steady state depreciation rate of capital	$\delta = 0.018$
Production function, capital parameter	$\iota = 0.33$
Investment adjustment cost parameter	$\varphi_i = 1.8$
Steady-state mark-up	$\theta/(\theta-1) = 1.3$
Calvo parameter	$\gamma = 0.75$
Indexation parameter	$\gamma^p = 0.5$
Fraction of time spent working	$\ell = 0.3049$
Labor disutility parameter	$\omega =$ adjusted
Default size	$\Delta = 0.55$
Default parameter	$\alpha_p = 3.7$
Default parameter	$\beta_p = 0.54$
Default parameter	$by_{\max} = 2.56$
Government debt to annual GDP	$b^g/(4y) = \{0.6, 1.15\}$
Government spending to GDP	$s_g = 0.1924$
Labor income tax rate	$\tau =$ adjusted
Capital income tax rate	$\eta = 0.45$
Tax rule parameter	$d_\tau = 0.25$
Taylor rule, response to inflation	$d_\pi = 1.5$
Taylor rule, response to output gap	$d_y = 0.125$

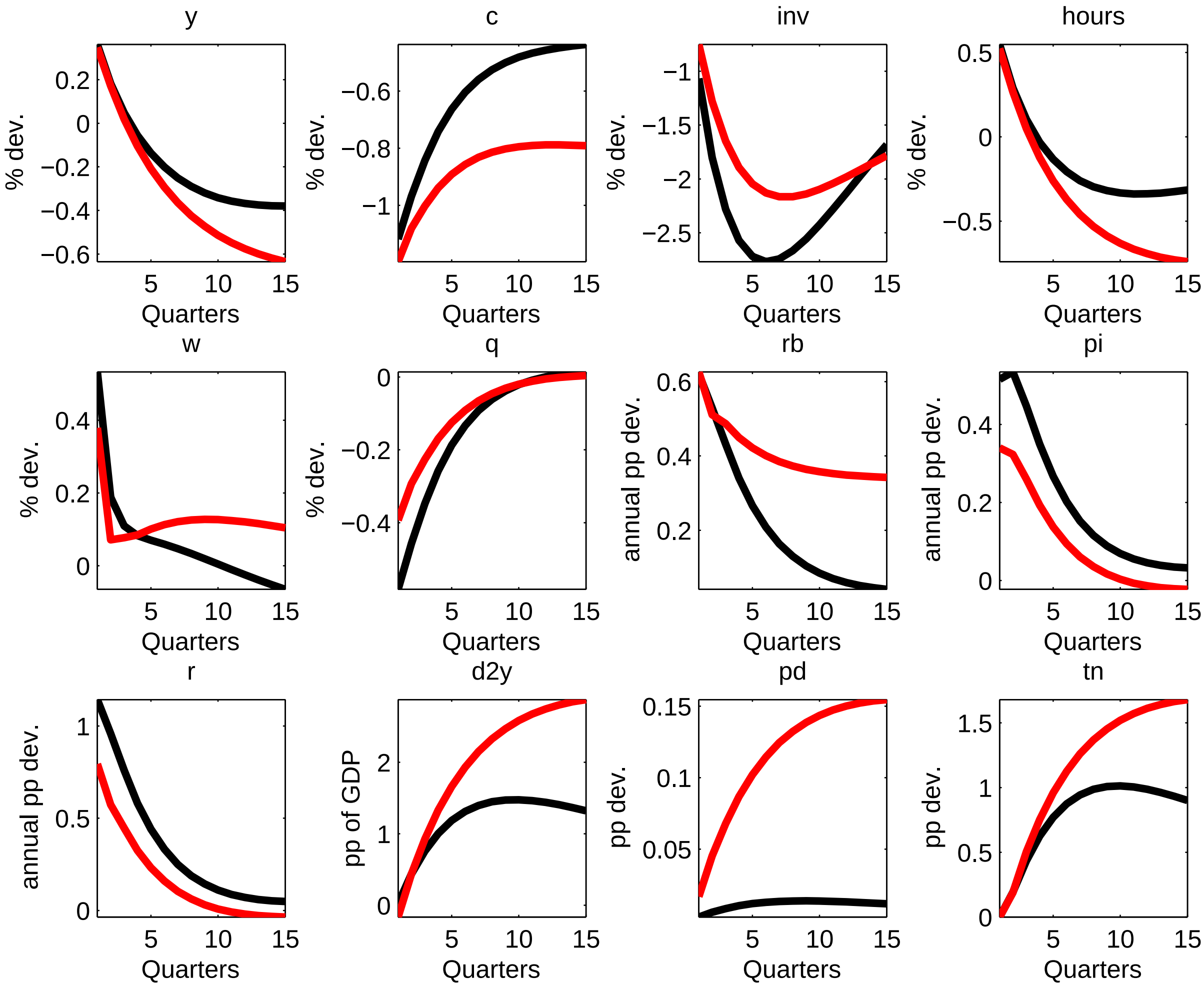
4 Impulse response functions

4.1 Public spending shock around the steady state

Figure 1 reports the Impulse Response Functions (IRFs hereafter) of the economy to a one percent public spending shock with substitutable public and private goods ($\nu = 1$). The model is solved using a non-linear two boundaries Newton-type algorithm to capture accurately the potential non-linearities induced by the model. The increase in public spending is partly financed through distortionary taxes while the remaining of the rise in public spending is absorbed by public debt. This is slightly different from usual analyzes that focus on multipliers when public spending is financed by lump-sum taxes. The black solid line show the response of the economy with a low steady-state level of public debt to GDP (60%) while the red line shows the response with a high steady-state public debt to GDP ratio (115%).

Figure 1 shows that a shock has positive effects on impact on output, crowds out private consumption (remember that public and private goods are substitutes) as well as investment. Wages and hours increase since final good demand rises, as illustrated by the rise in the inflation rate. In terms of public finance, the labor income tax and public debt both increase, and the sovereign rate increases as well. The most interesting feature of this Figure lies in the differences between a low or a high steady-state debt-to-GDP ratio. With a higher steady-state ratio, a public spending shock leads the debt ratio to rise much more than with a low steady-state indebtedness. The shock induces the probability of default and the labor income tax rate to rise much more as well. As a consequence, the overall distortion imposed on hours worked and hence GDP are magnified.

Figure 1: IRFs after a 1% public spending shock around the steady state with $\nu = 1$.



Black: low steady-state debt to GDP ratio (60%). Red: high steady-state debt to GDP ratio (115%).

The dynamics of wages is thus less positive in this case, which translates into a moderate inflation dynamics and a more moderate associated dynamics of the nominal interest rate.

Those differences require the combination of distortionary taxes and sovereign default risk. In the absence of sovereign default risk, the dynamics of the real sovereign rate would be very similar under both calibration, implying a similar dynamics of the labor income tax rate and thus a closer dynamics in all relevant macroeconomic aggregates and prices. With sovereign default risk and lump-sum taxation, the differences in debt and sovereign rate dynamics would not feed back to other variables of the economy, producing an identical path for output, consumption or investment.[8]

From the above experiment, we learn that countries with a high steady-state levels of public debt to GDP face higher sovereign default risk and suffer from potentially less effective public spending policies when the latter are partly financed through increases in taxes. This is due to the fact that taxes increase more in the medium run than if they have low steady-state debt-to-GDP ratios. As such, our first result confirms existing studies according to which countries with high levels of public debt face lower public spending output multipliers (see Ilzetzki, Mendoza, and Végh (2013) or Corsetti, Meier, and Müller (2012)).

4.2 Discount factor shock

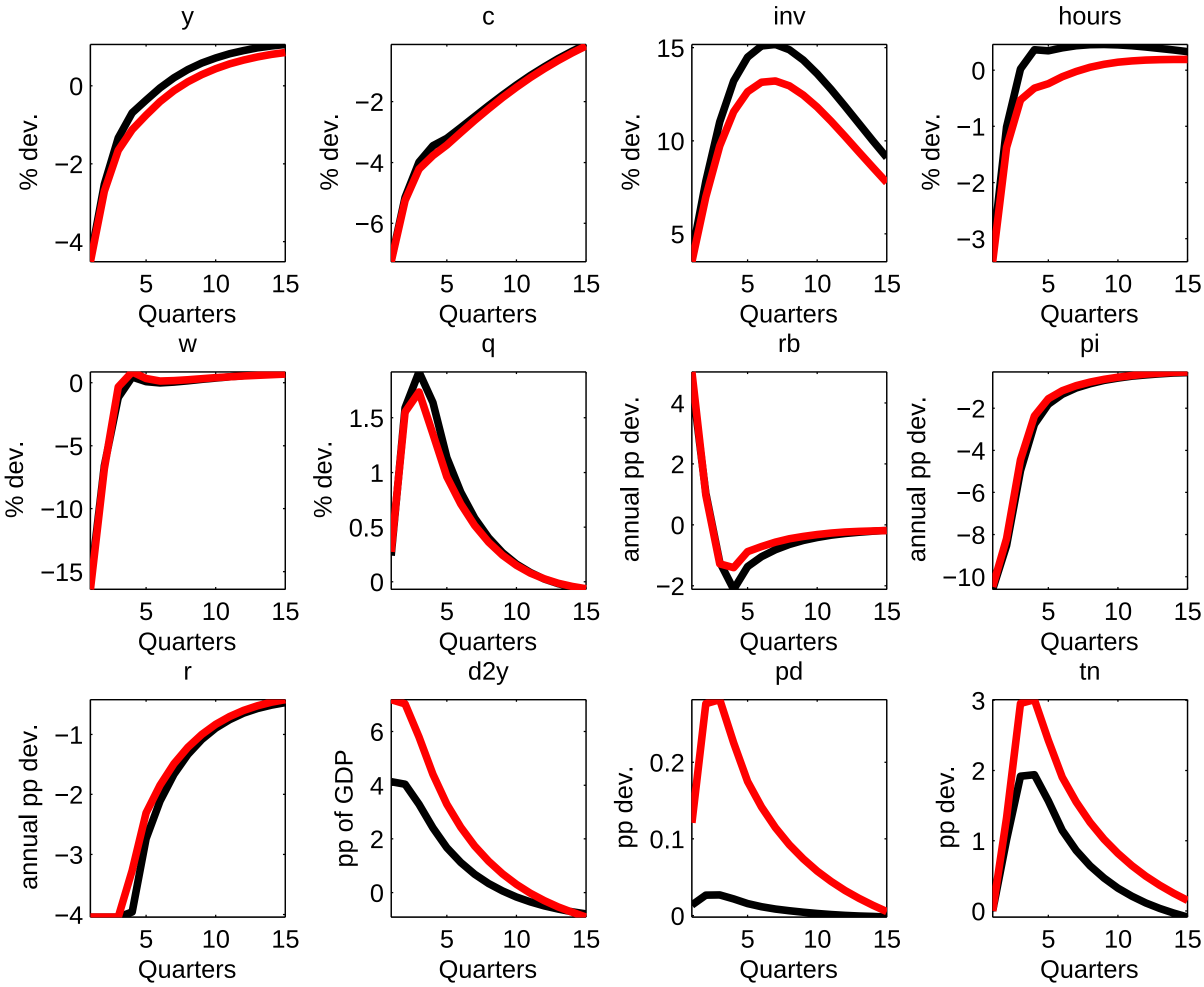
Figure 2 reports the dynamics of the economy after a discount factor shock, once again for substitutable private and public goods ($\nu = 1$). The size of the shock is 1.2 percent, which leads to a roughly 4 percent drop in output.

While this type of shock does not mimic perfectly the effects of the 2008 financial crisis because investment rises, it still produces a large drop in consumption, hours worked, output, wages, inflation and the nominal interest rate. The latter hits the ZLB immediately. The shock also raises the debt-to-GDP ratio because output falls and because hours and wages go down, which in turn lowers fiscal receipts. A consequence of the increase in the debt-to-GDP ratio is the rise in sovereign default risk, that strengthens the increase in public debt. Overall, a large increase in the labor income tax rate is required to make this rise in indebtedness sustainable in the long run, with additional adverse effects on hours worked and output.

With a high steady-state level of public debt to GDP ratio, the economy is more exposed to the rising sovereign default risk, taxes rise more and hours fall more and recover more slowly. High steady-state levels of public debt thus produce a larger recession and a more sluggish recovery. However, high steady-state debt to GDP ratio has a positive effect on the period over which the ZLB is binding, which is shorter. The reason is the increase in taxes, which is greater in the case of high steady-state public debt levels. As hours worked and real wages fall more, the level of government debt is driven up, leading to a rise in the labor income tax rate. In fact,

[8] Remember that sovereign risk is priced but insured, so if the consequences of sovereign risk are not channeled to the real economy (in the form of distortionary taxes in our case), then its consequences are null.

Figure 2: IRFs after a discount factor shock with $\nu = 1$.



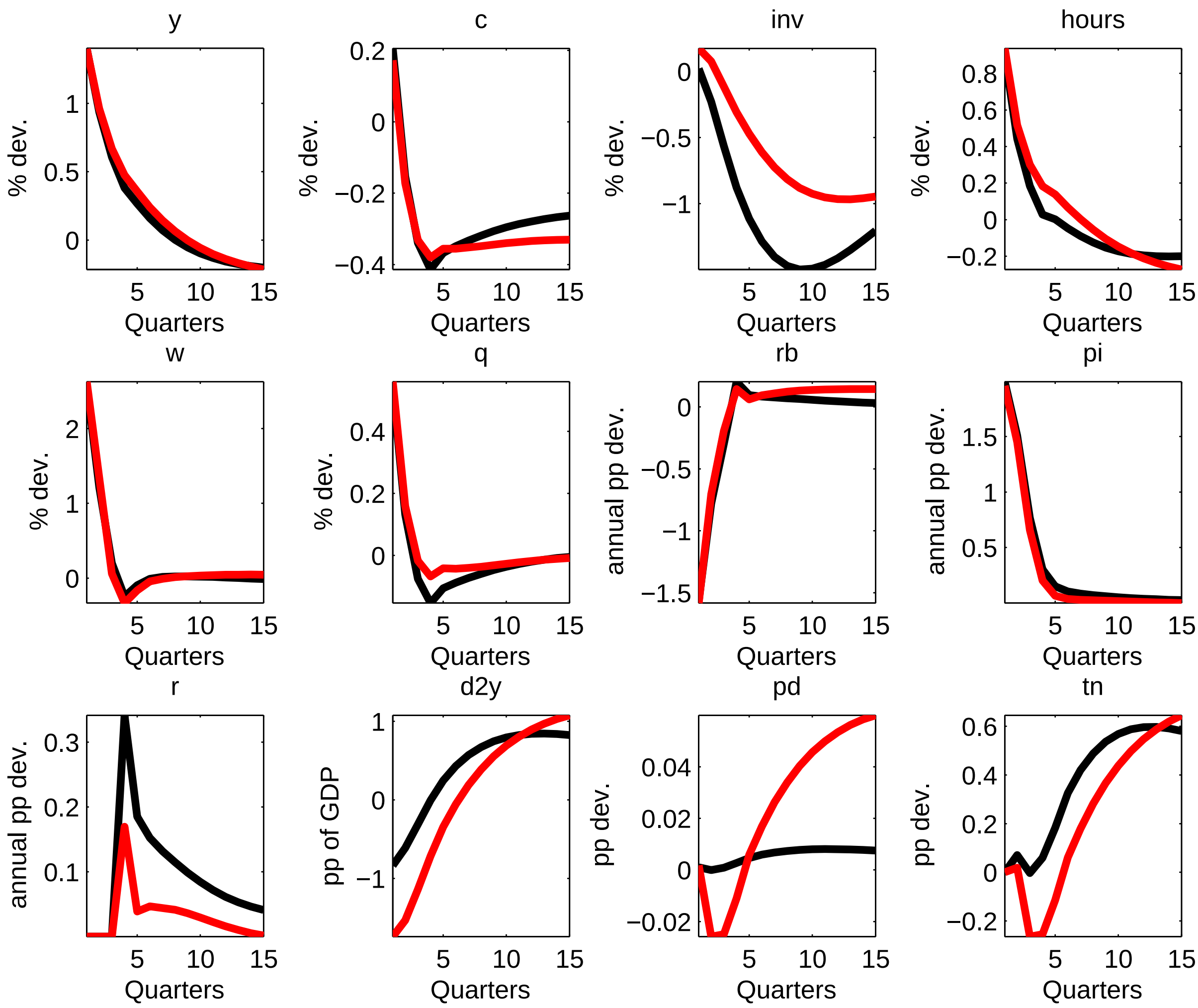
Black: low steady-state debt to GDP ratio (60%). Red: high steady-state debt to GDP ratio (115%).

Eggertsson (2011) shows that, at the ZLB, a wage tax increase produces higher marginal costs for intermediate goods producers, translating in inflation pressures that lower the real interest rate. This leads to a quicker exit from the ZLB. However, this shorter period of ZLB and its positive spillovers does not lead to a less severe economic downturn: the recession induced by a discount factor shock remains deeper under a high steady-state level of debt to GDP.

4.3 Public spending shock during a crisis

In this section we discuss the effect of a one percent increase in public spending conditional on a 1.2 percent discount factor shock. Figure 3 shows that the responses are different and depend on whether the economy is in a low or high steady-state level of public debt to GDP.

Figure 3: IRFs after a 1% public spending shock conditional on a discount factor shock with $\nu = 1$.



Black: low steady-state debt to GDP ratio (60%). Red: high steady-state debt to GDP ratio (115%).

On impact, when the zero bound is strictly binding, an increase in government spending leads to a rise in output, marginal cost, and inflation, both under low and high initial levels of public debt to GDP. The increase in expected inflation lowers the real interest rate, which drives up private consumption and investment. This rise in private consumption expenditure and investment leads to a further rise in output, marginal cost, and expected inflation and a further decline in the real interest rate. The debt to GDP ratio declines due to the increase in output and prices. This result is in line with Christiano, Eichenbaum, and Rebelo (2011) findings. Interestingly, with a high steady-state debt-to-GDP, output increases more than with low steady-state ratio, leading to a larger value of the government spending multiplier. In the case of high debt-to-GDP, we observe a negative impact response on the probability of sovereign default, the debt-to-GDP ratio and thus on the labor income tax while the impact response is positive with a low steady-state ratio.

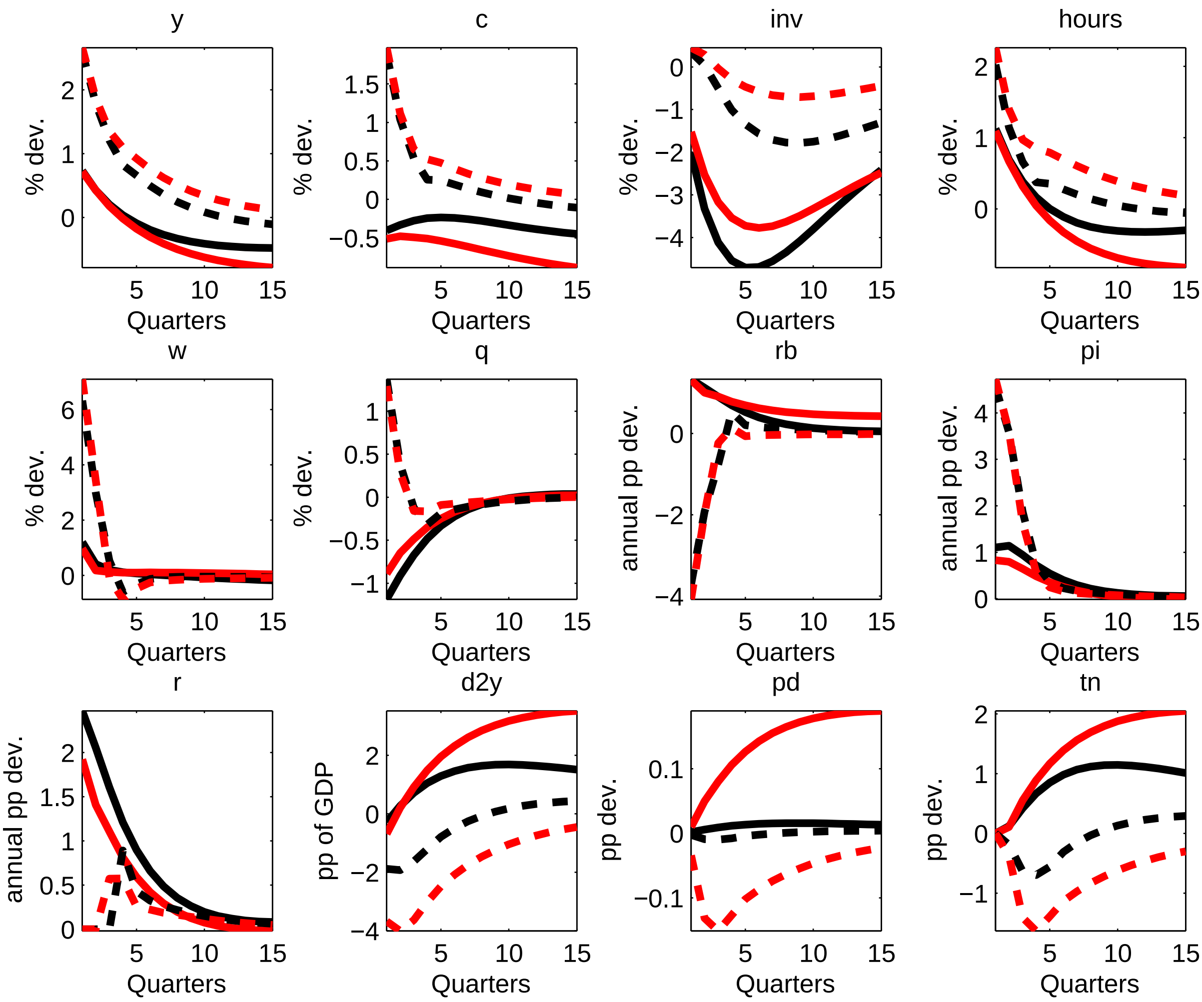
In our model, why does the increase in public spending lead to larger effects on output and favorable effect on public finance during a crisis? Recall that the probability of sovereign default and the labor income tax rate are positive functions of the level of public debt. Output multiplier is larger with a high initial level of debt when the ZLB is binding because the tax revenues rise more than with a low initial level of debt. A "positive" spillover from in the high debt case is that the steady-state tax rate is larger, which raises fiscal revenues more than under low debt after a public spending shocks that raises wages and hours worked. Hence, tax receipts *rise* with a high steady-state level of debt while they *fall* with a low steady-state debt level. As a consequence, the dynamics of public debt is negative (respectively positive) implying a fall (resp. rise) of the labor income tax with positive (resp. negative) effects on economic activity.

4.4 Complementarity between public and private goods

In the previous case we have considered that public and private expenditure on goods were substitutable, with an elasticity $\nu = 1$. However, empirical evidence favor estimates pointing to a mild complementarity, at least for Canada (see Bouakez and Rebei (2007)). Therefore, we investigate whether considering complementary public and private expenditure ($\nu = 0.45$) changes anything to our results. Figure 4 below reports the IRFs to a 1 percent public spending shock around the steady state (solid lines) and conditional on a discount factor shock (dashed lines) under low- or high-public debt to GDP ratios when public and private goods are Edgeworth complements.

Figure 4 indicates that the pattern observed with substitutable private and public expenditure is robust. In "normal" times, output is more stimulated after a public spending shock when the initial level of debt is low rather than high, while an opposite result is observed when the zero lower bound is binding. The major difference between complementarity and substitutability lies in the response of private consumption, that is more strongly crowded-in (or less crowded-out) when private and public expenditure are complementary, therefore producing much larger government spending output multipliers than in the same situation with substitutability.

Figure 4: IRFs after a 1% public spending shock with $\nu = 0.45$.



Solid: around the steady state. Dashed: conditional on a discount factor shock. Black: low steady-state debt to GDP ratio (60%). Red: high steady-state debt to GDP ratio (115%).

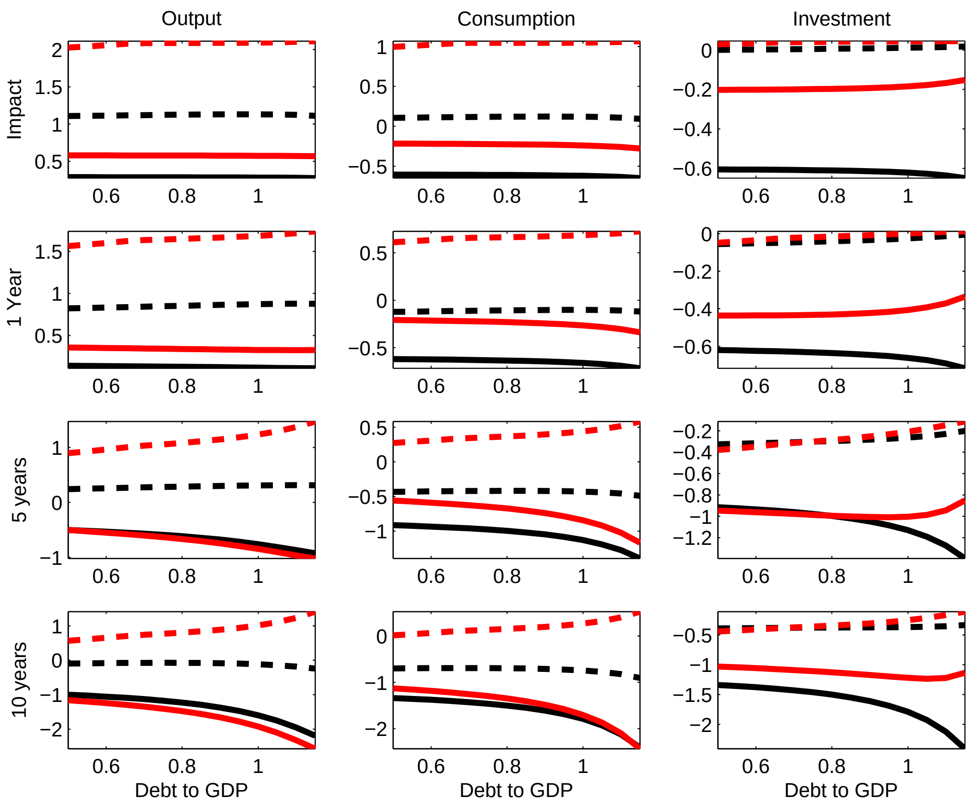
5 Government spending multipliers

We now compute the present value government spending output multipliers under all the cases presented above for different values of the initial level of public debt. Present value multipliers are defined as the discounted euro increase in output, consumption or investment that results from a euro increase in public spending:

$$PVM_{x,T} = \frac{\sum_{j=1}^{T} \beta^j \left(x_{t+j} - x_t\right)}{\sum_{j=1}^{T} \beta^j \left(g_{t+j} - g_t\right)}, \text{ for } x = y, c, i \tag{32}$$

Figure 5 reports the present value multipliers of output, consumption and investment on impact ($T = 1$), one year after the shock ($T = 4$), 5 years after the shock ($T = 20$) and 10 years after the shock ($T = 40$).

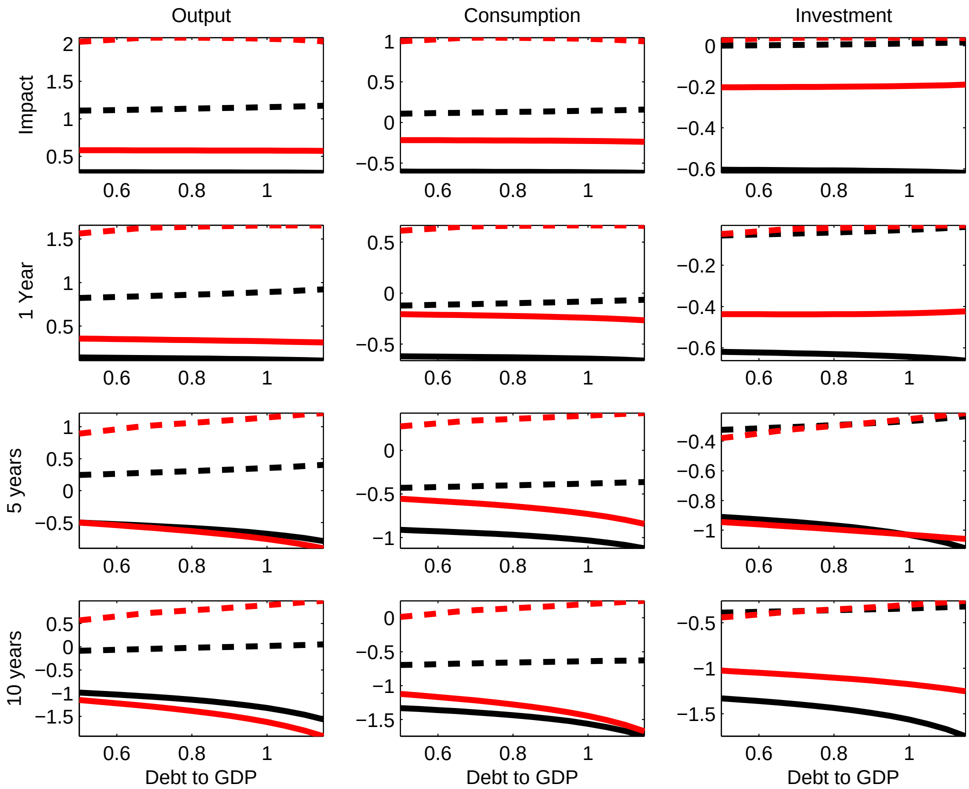
Figure 5: Present value multipliers.



Solid: normal times. Dashed: conditional on a discount factor shock. Black: $\nu = 1$. Red: $\nu = 0.45$.

First, Figure 5 confirms the previous results in the sense that multipliers are clearly larger when the ZLB is binding than when shocks occur around the steady state. The latter are 0.5 or less on impact in normal times while they reach 1.2 to 2 in ZLB episodes. Second, complementarity between public and private goods does produce much higher multipliers compared to the substitutable case. Third, the level of debt has a negative impact on multipliers during normal times and a positive or weakly negative impact on multipliers during ZLB episodes. In addition, the impact is magnified when the horizon is longer. How do these results compared to similar multipliers computed without sovereign risk? To answer this question, the same exercise is conducted imposing a constant default parameter, *i.e.* $\chi_t = \chi$ and the results are depicted in Figure 6.

Figure 6: Present value multipliers without sovereign risk.



Solid: normal times. Dashed: conditional on a discount factor shock. Black: $\nu = 1$. Red: $\nu = 0.45$.

This exercise reveals that sovereign risk is not the qualitative determinant of the effect uncovered by our previous analysis: the global pattern of multipliers and the impact of steady-state indebtedness is preserved when sovereign risk is not considered. This confirms that it is the steady-state level of the labor income tax rate that is at the heart of our main effect. However, sovereign risk acts as a quantitative amplifier, especially at high debt levels: the impact of the level of debt in the steady state is more strongly negative when shocks are computed around the steady state and more positive when computed during ZLB episodes.

6 Cumulative output losses

A discount factor shock unambiguously depresses output more with a high steady-state debt level, while the public spending shock stimulates output more with a high debt level conditional on a discount factor shock. Hence, are there configurations in which a country with a high steady-state level of debt is better off – after the combined shocks – than a country with a low steady-state level of debt? In other words, are there situations where the output gains from a public spending shock under high debt overturn the output losses from the discount factor shock? To answer this question, we compute the per period discounted output losses or gains – over 20 quarters – after a combined shock under alternative configurations. The results are depicted in Table 2.

Table 2: Per period discounted output gains over 20 quarters, in %

	$\nu = 1$		$\nu = 0.45$	
ε_t^g	$by = 0.6$	$by = 1.15$	$by = 0.6$	$by = 1.15$
1%	**0.1247**	**-0.0633**	**0.5489**	**0.7057**
2%	**0.2485**	**0.1659**	0.5768	0.6220
3%	**0.3317**	**0.2118**	0.4133	0.3057
4%	0.2675	0.1164	0.2378	-0.0094
5%	0.0961	-0.1538	0.0515	-0.3129

Note: Bold numbers correspond to cases where the ZLB is binding for at least one period. $\nu = 1$ is the case where public and private consumption are substitutable and $\nu = 0.45$ where they are complementary. $by = 0.6$ is the case of low steady-state debt and $by = 1.15$ is the case of high steady-state debt.

Table 2 shows that countries with low steady-state debt levels are better off in many situations after a combined shock. First, when public and private consumption are substitutable ($\nu = 1$), a country with a low steady-state level of debt is always better off (in terms of cumulative output gains) than a country with a high debt level, whatever the size of the public spending shock. The former always generates cumulative output gains over the short run (due to the positive effects of the public spending shock) while the latter may generate output losses, either because the multiplier is small (when the size of the shock is small) or because of the negative effects of enhanced sovereign risk (when the size of the shock is large).

In the case of complementarity between public and private consumption, countries with high steady-state levels of debt can be better off only if the size of the public spending shock remains relatively small. In this case, the positive spillover from low interest rates identified in the previous sections dominates the output losses from the discount factor shock. Whenever the public spending shock becomes too large, the gains associated with a very low interest rate are more than compensated by the depressing effects of rising sovereign risk in the medium run. Hence, in most situations, a country with a low level of debt is better armed to face a crisis along with an expansionary public spending policy. Only if the crisis implies very low interest rates, and public and private consumption are complementary can a country be better off (in terms of output) with a high steady-state level of debt.

7 Conclusion

This paper investigated the impact of the level of debt in a New-Keynesian model with sovereign risk, capital accumulation and a ZLB constraint on the nominal interest rate. We found that countries with high debt were more fragile in the event of a financial crisis, experiencing larger economic downturns. We also found that the spending multipliers during normal times were lower with high debt, and that spending multipliers were large at the ZLB. These results are in line with the literature.

The novel result of the paper was that economies with high debt had larger spending multipliers at the ZLB. The result was driven by a positive spillover on net fiscal receipts from having a larger steady-state level of tax rates. This effect was magnified by the presence of sovereign risk, that makes the economy more sensitive to fluctuations of the debt to GDP ratio. This result did not necessarily imply that countries with high initial debt were better off after a crisis and a stimulus package compared to countries with low initial debt. Only when interest rates were very low (close to the ZLB) and public and private consumption were complementary could countries with high steady-state levels of public debt be better off in terms of output.

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