Debt, Sovereign Risk and Government Spending*

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Abstract

We investigate the link between the size of government indebtedness and the effectiveness of government spending shocks. We develop a New Keynesian model of sovereign risk in which the zero lower bound (ZLB) constraint on the nominal interest rate may be binding. In normal times, high levels of government debt to GDP lead to reduced output multipliers. They also lead to more severe economic downturns in the event of crises. However, when the ZLB binds, high debt levels produce larger output multipliers. Our results rely on the transmission of sovereign risk to the real economy through distorsionary taxation, and have interesting implications for the design of optimized spending policies at the ZLB.

Keywords: Sovereign Default Risk, Fiscal Policy.

JEL Classification: E62, E32.

1 Introduction

The massive rise in government debt levels and sovereign spreads that followed the 2008 Great Recession and the 2011 recession in countries of peripheral Europe raises the question of whether the level of public debt affects the effectiveness of government spending shocks. In other words, do high levels of sovereign debt undermine the ability of governments to make use of government spending to stabilize the economy? Conventional wisdom suggests that countries with high levels of public debt have less room for fiscal stimulation than countries with low levels of public debt in the event of an economic crisis, and would therefore advocate for low debt levels on average. For instance, Corsetti, Meier, and Müller (2012) show empirically that periods of fiscal stress are associated with lower spending output multipliers. The perceived reason would be that countries with high debt are closer to their natural debt limit, and that any fiscal stimulation might lead them to a sovereign debt crisis and/or to a risk of insolvency, or would require much more future

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tax efforts with adverse consequences. Building a model of sovereign risk, Corsetti, Kuester, Meier, and Müller (2013) further show that spending multipliers are much lower when the zero lower bound on the interest rate is binding, because central banks are unable to mitigate the effects of sovereign risk on the real economy that go through the rise in private spreads.

We shed new light on this question in a New-Keynesian model with sovereign risk à la Corsetti et al. (2013) and capital accumulation. Sovereign risk is a non-linear function of the debt ratio and the direct consequences of sovereign risk are ruled out since default risk is insured *ex-post*. The main impact of sovereign risk thus goes through the rise in the sovereign rate, that further increases the need to refinance debt and raises the level of public debt. In our model, as opposed to Corsetti et al. (2013), the transmission of sovereign risk does not rely on an *ad hoc* rise in private rates that further depresses private consumption, but on the assumption that government expenditure and the debt burden are financed through debt and a distorsionary tax on labor income that is set according to a feedback rule. In addition, we consider useful government spending and allow the public and private goods to be either substitutes or complements in the utility of households.

Within this framework, we find that the impact of the steady-state level of debt on government spending multipliers is negative during normal times, *i.e.* when shocks arise around the steady state. This result relies on the fact that a country with a higher steady-state level of public debt faces a larger increase in public debt after a spending shock, and that the corresponding increase in distorsionary taxes is larger, imposing more distortions on the economy, thus contributing negatively to the dynamics of GDP. This result is in perfect accordance with empirical estimates of spending multipliers conditional on a high level of debt (see Corsetti, Meier, and Müller (2012) or Ilzetzki, Mendoza, and Végh (2013)).

Further, when the economy is hit by a discount factor shock that is intended to mimic the effects of a financial crisis, we find that countries with a high initial level of debt are worse off in terms of output. The shock is known to produce large adverse effects on output, hours, real wages and inflation, pushing the economy to the lower bound of the nominal interest rate. The fall in output raises the debt-to-GDP ratio more for the economy with a relatively high initial debt-to-GDP ratio, which in turn requires a larger increase in the labor income tax rate and makes this economy experience a deeper recession and a slower recovery.

However, if a stimulus package—a public spending shock—is decided to alleviate the effects of the crisis, a novel result arises. Spending multipliers will be larger in the highly indebted economy. Of course, as already found in the literature, spending multipliers are larger than during normal times regardless of the initial debt-to-GDP ratio (see Christiano, Eichenbaum, and Rebelo (2011) and the abundant subsequent contributions). But conditional on hitting the ZLB, countries with a relatively high initial debt-to-GDP ratio have larger spending multipliers.

The chief reason is that highly indebted countries have larger steady-state labor income tax rates to sustain the larger debt-to-GDP ratios. At the ZLB, a spending shock raises goods demand very strongly since the real interest rate falls instead of rising during normal times. Private consumption is crowded in instead of being crowded out, and private investment falls less. As such, the aggregate demand for goods rises much more, which raises hours worked and the real wage more than during normal times. As a consequence public debt falls instead of rising during normal times. With higher steady-state levels of the labor income tax rate, economies that feature a relatively high initial debt-to-GDP ratio experience a much larger rise in fiscal receipts after the shock, and a larger fall in public debt, inducing a larger fall in the labor income tax rate. This contributes to stimulate GDP more in economies with a relatively high initial debt-to-GDP ratio, conditional on the ZLB being binding. Our experiments suggest that the chief driver of this result is the fact that government expenditure are financed using a distorsionary tax on labor income, as opposed to Corsetti et al. (2013) who assume lump-sum taxes. Sovereign risk and capital accumulation serve as amplification mechanisms. We also show that our results are magnified when the public and the private good are complements in the utility of households.

These results imply that an optimized spending rule at the ZLB should be less aggressive when public spending are more effective at stabilizing the economy, *i.e.* when public and private goods are complements and when the initial level of debt is high. In this last case, the potential welfare gains from an optimized spending rule are much larger—3 to 4 times—than when the initial level of debt is low. While fully optimal (Ramsey) policies are beyond the scope of this paper, our results have some potentially interesting policy implications, discussed in details in Section 6.

Our paper relates to the literature on spending multipliers that questions how the economic environment may affect the latter. Empirically, one of the first papers to raise the question was Perotti (1999). More recently, the subject has been revived by Auerbach and Gorodnichenko (2012), investigating whether the business cycle position matters for the value of multipliers. Two recent papers respectively by Corsetti, Meier, and Müller (2012) and by Ilzetzki, Mendoza, and Végh (2013) more precisely question the impact of debt or fiscal stress and financial crises on spending multipliers. Their results converge and conclude that fiscally stressed or highly indebted economies tend to be characterized by lower (and even negative) spending multipliers. According to Corsetti, Meier, and Müller (2012), when an economy experiences a financial crisis, spending multipliers are much larger than in normal times, a result that is consistent with more theoretical contributions like Christiano, Eichenbaum, and Rebelo (2011). Regrettably however, none of the mentioned empirical papers does test the joint conditional impact of a financial crisis and the level of debt as we do. Our paper also belongs to a model-based literature that investigates the effects of the ZLB on the size of fiscal multipliers, summarized and referenced in Eggertsson (2011). To our knowledge, there are only very few papers questioning the effect of the initial debt level on the size of spending with sovereign risk. Close to our paper is of course Corsetti et al. (2013) but their analysis does not consider capital accumulation, and essentially focuses on the case of lump-sum taxes. In addition, Corsetti et al. (2013) rely on an *ad hoc* relation between the sovereign and the private spread for the transmission of sovereign risk to the real economy, while we rely on distorsionary taxes. In this sense, our framework is closer to Nakata (2017), although we consider a richer model with capital accumulation and sovereign risk, and our main focus is not on Ramsey equilibria but on the size of government spending multipliers and optimized policies. Our paper also echoes the recent contribution of Bilbiie, Monacelli, and Perotti (2017), who derive optimal spending policies at the ZLB with an additional focus on spending multipliers. However, they disregard the impact of sovereign risk, consider lump-sum taxes and as such do not investigate the impact of the initial level of debt on optimal policies.

The paper is organized as follows. Section 2 presents the model. Section 3 details our calibration. Section 4 analyzes the Impulse Response Functions to spending shocks, discount factor shocks and combined shocks when the initial debt-to-GDP ratio is low or high. It also investigates the robustness of our main result to the various assumptions of the model. Section 5 summarizes our results by presenting the value of spending multipliers at various horizons and under the different cases considered. Section 6 investigates the design of optimized spending rules depending on the size of the initial debt-to-GDP ratio and on the elasticity of substitution between public and private goods. Section 7 concludes.

2 Model

Our framework builds on a New Keynesian model of sovereign risk. We rely on the specification of the probability of sovereign default proposed by Corsetti et al. (2013) where the latter is a non-linear function of the public debt-to-GDP ratio. In this framework, default risk matters *ex-ante* but not *ex-post*. The model also features capital accumulation, allows for the public good to provide utility to the households, and considers a monetary policy rule that is restricted by a (zero) lower bound on the nominal interest rate. The economy is populated by infinitely-lived and identical households, two types of firms, a government and a central bank.

2.1 Households

Households choose consumption, labor supply, deposits and government bonds maximizing lifetime welfare

$$E_t \left\{ \sum_{s=t}^{\infty} \left(e_s \beta^{s-t} \right) u\left(c_s, g_s, \ell_s\right) \right\}, \tag{1}$$

where $u_{\ell,t} \leq 0$, $u_{c,t} \geq 0$ and $u_{g,t} \geq 0$ are the first-order partial derivatives with respect to hours worked, consumption, and the amount of public spending. Parameter $\beta \in (0,1)$ denotes the discount factor and e_t is a discount factor shock that evolves according to

$$\log e_t = \rho_e \log e_{t-1} + \epsilon_t^e. \tag{2}$$

Households optimize subject to the following budget constraint

$$b_t^g + d_t + p_t\,(c_t + k_t) = (1-\chi_t)\,r_{t-1}^b b_{t-1}^g + r_{t-1}d_{t-1} + p_t\left((1-\tau_t)\,w_t\ell_t + R_t^k k_{t-1}\right) + \Pi_t + T_t^b. \quad (3)$$

In this equation, d_t denotes nominal deposits returning r_t between t and $t+1$ and b_t^g denotes the nominal amount of sovereign bonds that returns r_t^b. Sovereign returns are affected by default risk, χ_t, and variable T_t^b denotes the amount of *ex-post* insurance against sovereign default.[1] Further, c_t is the level of private consumption, w_t is the real wage, ℓ_t stands for hours worked and τ_t is a distorsionary tax on labor income. Variable k_t is the capital stock, and $R_{kt} = 1 + (1-\eta)\left(r_t^k - \delta\right)$ is the real gross return on capital where η is the (constant) capital income tax that comes with a deduction for capital depreciation δ.[2] Finally, Π_t comprises monopolistic profits from firms. An additional constraint to the optimization program is the law of capital accumulation

$$k_t - (1-\delta)\,k_{t-1} = i_t\left(1 - \left(\varphi^i/2\right)x_t^2\right), \quad (4)$$

where i_t is the amount of investment in physical capital, $x_t = i_t/i_{t-1} - 1$ is the growth rate of investment, and $\varphi_i > 0$ controls the size of the investment adjustment costs. First-order conditions with respect to deposits, sovereign bonds and labor supply imply

$$\begin{aligned} E_t\left\{\beta_{t+1} r_t/\pi_{t+1}\right\} &= 1, && (5)\\ E_t\left\{\beta_{t+1}\,(1-\chi_{t+1})\,r_t^b/\pi_{t+1}\right\} &= 1, && (6)\\ u_{\ell,t} + (1-\tau_t)\,u_{c,t}w_t &= 0, && (7)\end{aligned}$$

where $\beta_t = e_t\beta u_{c,t}/u_{c,t-1}$ and where $\pi_t = p_t/p_{t-1}$ is the inflation rate. The first two equations price deposits and sovereign bonds. The third equation relates the marginal disutility of working to the real wage, expressed in terms of the marginal utility of consumption. We define $q_t p_t u_{c,t}$ as the Lagrangian multiplier associated with the capital accumulation constraint, and derive the following first-order conditions with respect to the capital stock and investment:

$$\begin{aligned} E_t\left\{\beta_{t+1}\left(q_{t+1}\,(1-\delta) + (1-\eta)\,r_{t+1}^k + \eta\delta\right)\right\} &= q_t, && (8)\\ q_t\left(1 - \left(\varphi^i/2\right)x_t^2 - \varphi^i x_t\,(1+x_t)\right) + E_t\left\{\beta_{t+1}q_{t+1}\varphi^i x_{t+1}\,(1+x_{t+1})^2\right\} &= 1. && (9)\end{aligned}$$

[1] Both χ_t and T_t^b will be discussed in the government section.

[2] A constant capital income tax η is introduced to obtain a realistic steady-state calibration for the labor income tax rate.

2.2 Firms

2.2.1 Final good producers

A perfectly competitive representative firm produces a final consumption good y_t using a continuum of intermediate goods indexed in $j \in [0,1]$, according to the following production function:

$$y_t = \left[\int_0^1 y_t\left(j\right)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}}, \tag{10}$$

where $y_t\left(j\right)$ denotes the time t input of intermediate good j and θ the elasticity of substitution across intermediate goods. The firm takes the price of output p_t and the input price $p_t\left(j\right)$ as given. Profit maximization leads to the following first-order condition:

$$y_t\left(j\right) = \left(p_t\left(j\right)/p_t\right)^{-\theta} y_t. \tag{11}$$

Substituting (10) into (11) yields the following relationship between the aggregate price level and the price of intermediate goods:

$$p_t = \left[\int_0^1 p_t\left(j\right)^{1-\theta} dj \right]^{\frac{1}{1-\theta}}. \tag{12}$$

2.2.2 Intermediate good producers

Intermediate good $j \in (0,1)$ is produced under monopolistic competition using capital k_{t-1} and labor ℓ_t with the following production function:

$$y_t\left(j\right) = k_{t-1}\left(j\right)^{\iota} \ell_t\left(j\right)^{1-\iota}, \tag{13}$$

where $\iota \in (0,1)$ is the share of capital in value-added. Intermediate good producers rent the capital stock and hire workers in perfectly competitive markets. Profits are distributed to the households at the end of each period. Recalling that r_t^k and w_t respectively denote the real gross rental rate on capital and the real wage, the real marginal cost of intermediate good producers is:

$$s_t = \left(\iota^{\iota}\left(1-\iota\right)^{1-\iota}\right)^{-1} \left(r_t^k\right)^{\iota} \left(w_t\right)^{1-\iota}. \tag{14}$$

Profits are $\left[p_t\left(j\right)/p_t - s_t\right] p_t y_t\left(j\right)$, where $p_t\left(j\right)$ is the price of the good produced by firm j in period t. We assume there are Calvo price-setting contracts, where $1/\left(1-\gamma\right)$ and γ_p respectively represent the average length of contracts and an indexation parameter. The optimal pricing conditions are standard and therefore not reported.

2.3 Government

We adopt the approach of sovereign risk of Corsetti et al. (2013). Actual *ex post* default is neutral while the *ex-ante* probability of default is crucial for the pricing of government debt. Following Eaton and Gersovitz (1981), Arellano (2008) and others have modeled default as a strategic decision of the government. On the other hand, Bi (2012) considers default as the consequence of the government's inability to raise enough fiscal revenues to refinance its debt. Under both approaches, the probability of sovereign default is closely and non-linearly related to the level of public debt to GDP. Thus, we assume that the *ex-ante* probability of default pd_t at a given level of real sovereign indebtedness $by_t = b_t^{rg}/\left(4y\right)$ is given by the cumulative distribution function of the following beta distribution:

$$pd_t = F_{beta}\left(by_t/by_{max}, \alpha_p, \beta_p\right), \tag{15}$$

where by_{max} denotes the upper end of the support for the debt-to-GDP ratio. Actual default occurs with probability pd_t so that:

$$\chi_t = \Delta \text{ if } \mathcal{B}\left(pd_t\right) = 1, \tag{16}$$

$$\chi_t = 0 \text{ if } \mathcal{B}\left(pd_t\right) = 0, \tag{17}$$

where $\mathcal{B}\left(.\right)$ is a Bernoulli and Δ is the size of the hair-cut. Given these assumptions, the budget constraint of the government writes

$$b_t^g = r_t^b\left(1-\chi_t\right)b_{t-1}^g + p_t\left(g_t - \tau_t w_t n_t - \eta\left(r_{kt}-\delta\right)k_{t-1}\right) + T_t^b. \tag{18}$$

Once again, potential losses from default are fully compensated, so that only *ex-ante* default risk matters. As a consequence:

$$T_t^b = r_t^b \chi_t b_{t-1}^g, \tag{19}$$

and the consolidated budget constraint writes:

$$b_t^g = r_t^b b_{t-1}^g + p_t\left(g_t - \tau_t w_t n_t - \eta\left(r_{kt}-\delta\right)k_{t-1}\right), \tag{20}$$

or, in real terms

$$b_t^{rg} = r_t^b b_{t-1}^{rg}/\pi_t + g_t - \tau_t w_t n_t - \eta\left(r_{kt}-\delta\right)k_{t-1}. \tag{21}$$

The stability of public debt in the long run is granted by the following tax rule:

$$\tau_t - \tau = d_\tau\left(b_{t-1}^{rg} - b^{rg}\right), \tag{22}$$

and government spending evolve according to:

$$\log(g_t/g) = \rho_g \log(g_{t-1}/g) + \varepsilon_t^g. \tag{23}$$

Although actual default is not considered in our set-up, sovereign default Although actual default is not considered in our set-up, sovereign default risk has major real consequences. A rise in default risk raises the real sovereign rate r_t^b, leads to a rise in public debt that subsequently triggers a rise in the distortionary tax rate. As the latter goes up, hours worked, output, investment, asset prices and inflation collapse. So, even in the absence of actual default, sovereign default risk can be a key driver of the dynamics of the economy.

2.4 Central Bank

We assume that the gross nominal interest rate r_t is set according to

$$r_t = \max(1, rn_t), \tag{24}$$

where rn_t is the desired gross nominal interest rate chosen by the central bank, according to:

$$\log(rn_t/r) = d_\pi \log(\pi_t/\pi) + d_y \log(y_t/\widetilde{y}_t), \tag{25}$$

where π_t is the inflation rate and $\widetilde{y}_t$ is the natural level of output.[3] Parameter d_π and d_y determine the response of rn_t respectively to inflation and the output gap, respectively. The central bank sets r_t equal to rn_t if and only if its policy rule implies a non-negative level for the nominal interest rate. Otherwise, the ZLB binds and r_t equals one.

2.5 Aggregation

The clearing condition on the intermediate goods market is

$$k_{t-1}^{\iota} \ell_t^{1-\iota} = \int_0^1 y_t(j)\, dj = y_t dp_t, \tag{26}$$

where $dp_t = \int_0^1 (p_t(j)/p_t)^{-\theta} dj \geq 1$ measures the dispersion of prices. On the final goods market, the clearing condition simply writes:

$$y_t = c_t + i_t + g_t. \tag{27}$$

Further, the clearing condition for government bonds is $b_t^g = b_t$, and since deposits are in zero net supply, the clearing condition is $d_t = 0$.

[3] As in Gertler and Karadi (2011), variations in the mark-up will serve as a proxy for variations in the output gap.

3 Calibration

We assign values to the parameters of the model to match features of peripheral countries of the Euro Area at a quarterly frequency. Table 1 summarizes these parameter values. We adopt a formulation of the utility where private and public goods provide direct utility:

$$u\left(c_t, g_t, n_t\right) = \log\left(\widetilde{c}_t\right) - \omega \ell_t^{1+\psi} / 1 + \psi, \tag{28}$$

where

$$\widetilde{c}_t = \left(\kappa c_t^{\frac{\nu-1}{v}} + (1-\kappa)\, g_t^{\frac{\nu-1}{v}}\right)^{\frac{\nu}{\nu-1}},\ \nu > 0. \tag{29}$$

In Equation (29), κ denotes the weight of private consumption in the effective consumption index, and ν is the elasticity of substitution between the private and the public good. When $\nu = 0$, public and private goods are pure complements. As ν increases, private and public goods become more and more substitutable, and pure substitutability arises when $\nu \to \infty$. This specification is in the spirit of Leeper, Traum, and Walker (2015), but our choice of a CES specification is justified by the need to capture the diminishing marginal returns to public spending in order to achieve a given level of effective consumption, *ceteris paribus*. Following Bouakez and Rebei (2007), we consider alternative values of ν. In the first case we impose $\nu = 1$ so that public and private goods are substitutable. In the second case, we set $\nu = 0.45$ so that public and private goods are mild complements.[4] In each case, we impose the value of κ so that the marginal utilities of private and public goods are equal in the steady state.[5]

The discount factor is $\beta = 0.99$ implying an annual real interest rate of 4.1 percents. The inverse of the Frisch elasticity of labor supply is typically a controversial and important parameter. We impose $\psi = 3$ to capture relatively sluggish labor markets, a value that lies in-between the calibration of Corsetti et al. (2013) and that of Galí, López-Salido, and Vallés (2007). On the production side, the share of capital is $\iota = 0.33$, and the steady-state depreciation rate is $\delta = 0.018$ (7% annually). The investment adjustment cost parameter, φ_i, is set to 1.8. The steady-state mark-up is 30%, implying $\theta = 4.33$, the Calvo parameter on price contracts is $\gamma = 0.75$ a standard value in quarterly models of price adjustments, and the indexation parameter is $\gamma_P = 0.5$. Default parameters are calibrated after Corsetti et al. (2013): the size of default is $\Delta = 0.55$ and parameters of the default distribution are $\alpha_p = 3.7$, $\beta_p = 0.54$ and $by_{\max} = 2.56$. The feedback parameter of the fiscal rule is $d_\tau = 0.25$ to ensure medium-run fiscal solvency.

[4]This second value is also roughly consistent with the point estimates found by Auray and Eyquem (2017), between 0.5 and 0.6, and with the estimates of Leeper, Traum, and Walker (2015), according to which public and private goods are complements.

[5]The value of κ that is consistent with the optimal provision of public good in the steady-state is indeed $u_c = u_g$ and implies:

$$\kappa = g^{-\frac{1}{\nu}} / \left(c^{-\frac{1}{\nu}} + g^{-\frac{1}{\nu}}\right).$$

Parameters of the Taylor rule are $d_\pi = 1.5$ and $d_y = 0.125$ and the persistence of shocks is $\rho_e = \rho_g = 0.9$. In the baseline calibration, we impose $b^g/(4y) = 0.8$. Using OECD data for 2008, we build a weighted-average measure of hours worked and adjust the value of ω to get $\ell = 0.3049$.[6] Proceeding similarly, the share of public consumption in GDP is $s_g = 0.1924$. The capital income tax rate is $\eta = 0.45$ and the labor income tax rate is adjusted to match the debt-to-GDP ratio target, implying $\tau = 0.3402$.

Table 1: Baseline parameter values

Discount factor	$\beta = 0.99$
Edgeworth preference parameter	$\nu = \{1, 0.45\}$
Inverse of the Frisch elasticity of labor supply	$\psi = 3$
Steady state depreciation rate of capital	$\delta = 0.018$
Production function, capital parameter	$\iota = 0.33$
Investment adjustment cost parameter	$\varphi_i = 1.8$
Steady-state mark-up	$\theta/(\theta - 1) = 1.3$
Calvo parameter	$\gamma = 0.75$
Indexation parameter	$\gamma^p = 0.5$
Fraction of time spent working	$\ell = 0.3049$
Labor disutility parameter	$\omega =$ adjusted
Default size	$\Delta = 0.55$
Default parameter	$\alpha_p = 3.7$
Default parameter	$\beta_p = 0.54$
Default parameter	$by_{\max} = 2.56$
Government debt to annual GDP	$b^g/(4y) = 0.8$
Government spending to GDP	$s_g = 0.1924$
Labor income tax rate	$\tau = 0.3402$
Capital income tax rate	$\eta = 0.45$
Tax rule parameter	$d_\tau = 0.25$
Taylor rule, response to inflation	$d_\pi = 1.5$
Taylor rule, response to output gap	$d_y = 0.125$

4 Impulse response functions

In the following experiments, we investigate the responses of our model economy under two polar cases for the debt level, $b^g/(4y) = 0.6$ referred to as the case of low debt, and $b^g/(4y) = 1.15$, referred to as the case of high debt. In each case, the calibration remains the same except for the steady-state level of debt and for the labor income tax rate.[7] The latter adjusts to match the targeted debt-to-GDP ratio. We get $\tau = 0.3118$ in the case of low debt and $\tau = 0.4380$ when the steady-state level of debt is high. Both numbers are reasonable in comparison of observed labor income tax rates in peripheral countries of Europe.

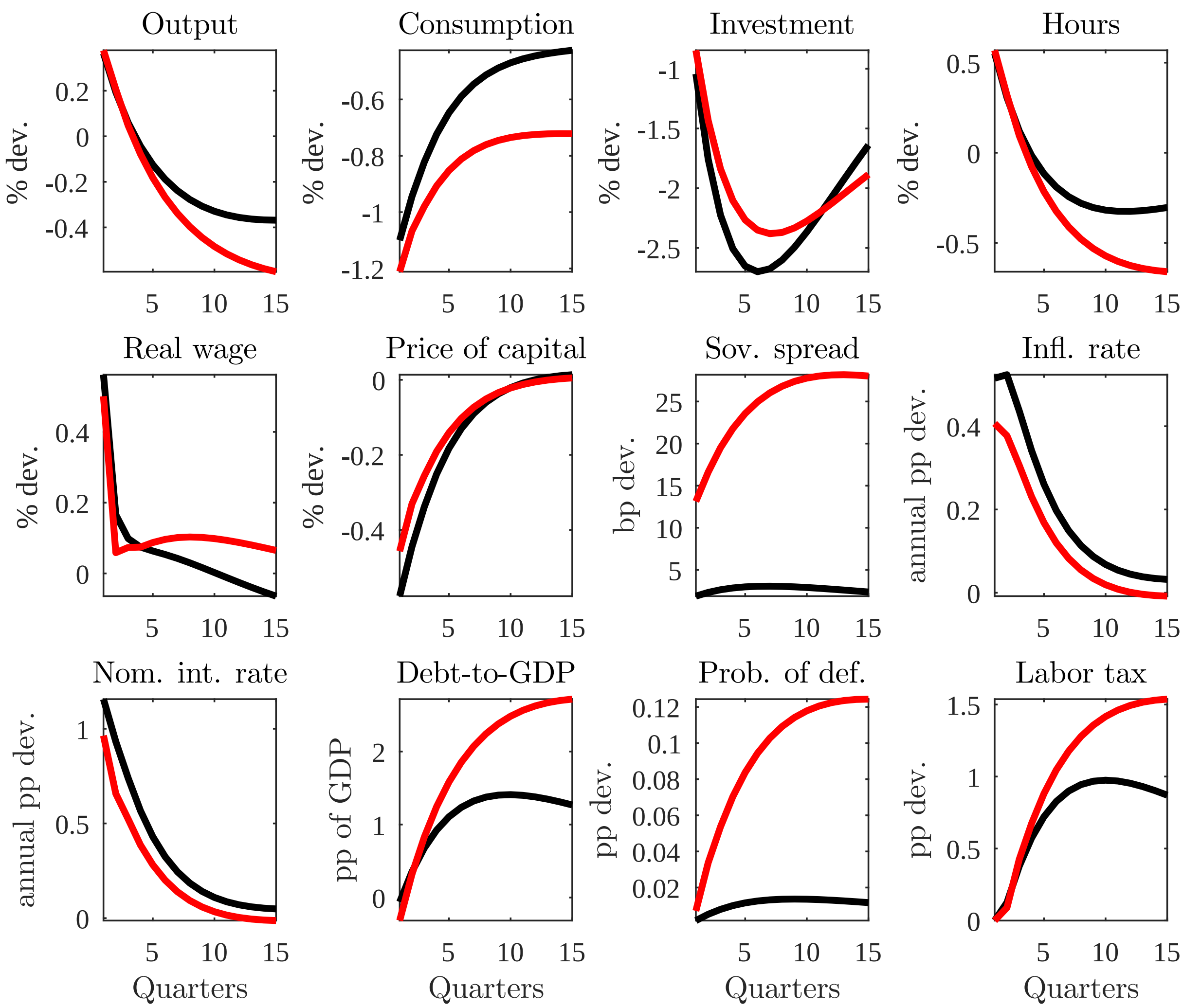
[6] The country sample includes Greece, Ireland, Italy, Portugal and Spain.

[7] In particular, the labor disutility parameter ω remains unchanged, so when the labor income tax rate changes the steady-state level of hours worked changes as well.

4.1 Public spending shock around the steady state

Figure 1 reports the Impulse Response Functions (IRFs hereafter) of the economy to a one percent public spending shock with substitutable public and private goods ($\nu = 1$). The model is solved using a non-linear two boundaries Newton-type algorithm to capture accurately the potential non-linearities induced by the model. The black solid line shows the response of the economy with a low steady-state level of public debt to GDP (60%) while the red line shows the response with a high steady-state public debt to GDP ratio (115%).

Figure 1: IRFs after a 1% public spending shock around the steady state with $\nu = 1$.



Black: $b^g/(4y) = 0.6$ and $\tau = 0.3118$. Red: $b^g/(4y) = 1.15$ and $\tau = 0.4380$.

Figure 1 shows that a spending shock has positive effects on output, crowds out private consumption (remember that public and private goods are substitutes), as well as investment. The real wage and hours increase since the demand for final goods rises, as illustrated by the rise in the inflation rate. In terms of public finance, the labor income tax and public debt both increase, and the sovereign rate increases as well. The most interesting feature of this Figure lies in the differences between a low or a high steady-state debt-to-GDP ratio. With a higher steady-state ratio, a public spending shock leads the debt ratio to rise much more than with a low steady-state indebtedness. The shock induces the probability of default and the labor income tax rate to rise much more as well. As a consequence, the overall distorsions imposed on hours worked and hence GDP are magnified. The dynamics of the real wage is thus less positive in this case, which translates into smoother inflation and nominal interest rate dynamics.

Those differences result from the combination of distorsionary taxes and sovereign default risk. In the absence of sovereign default risk, the dynamics of the real sovereign rate would be very similar under both calibrations, implying a similar dynamics of the labor income tax rate and thus a closer dynamics in all relevant macroeconomic aggregates and prices. With sovereign default risk and lump-sum taxation, the differences in debt and sovereign rate dynamics would not feed back to other variables of the economy, producing an identical path for output, consumption or investment.[8]

From the above experiment, we conclude that countries with high steady-state levels of public debt to GDP face higher sovereign default risk and suffer from potentially less effective public spending policies when the latter are partly financed through distorsionary taxes. After a spending shock with high initial debt-to-output ratio, taxes increase more in the medium run than if the initial ratio was low. As such, our first result confirms existing studies according to which countries with high levels of public debt face lower public spending output multipliers (see Ilzetzki, Mendoza, and Végh (2013) or Corsetti, Meier, and Müller (2012)).

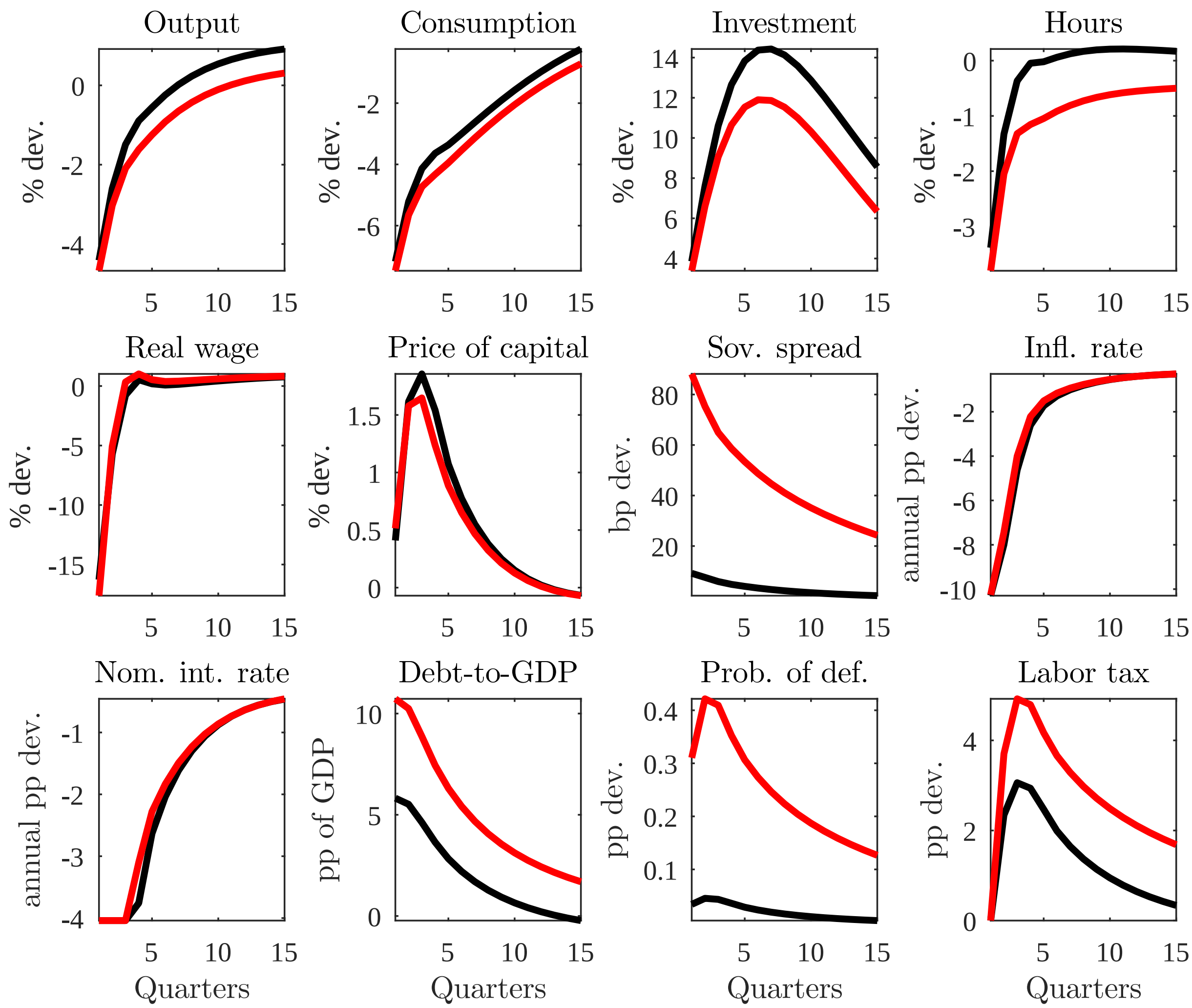
4.2 Discount factor shock

Figure 2 reports the dynamics of the economy after a discount factor shock, once again in the case of substitutable private and public goods ($\nu = 1$). The size of the shock is 1.2 percent, which leads to a 4 percent drop in output.

This type of shock does not mimic perfectly the effects of the 2008 financial crisis, essentially because private investment rises. Still, it produces a large drop in consumption, hours worked, output, the real wage, inflation and the nominal interest rate. The latter hits the ZLB immediately and remains constrained for 4 quarters. The shock also raises the debt-to-GDP ratio because output falls and because the tax base (hours times the real wage) falls, which in turn lowers fiscal revenues. A consequence of the increase in the debt-to-GDP ratio is the rise in sovereign default risk illustrated by the positive sovereign spread. The latter further strengthens the increase in public debt. Overall, a large increase in the labor income tax rate is required to

[8] Remember that sovereign risk is priced but insured, so if the consequences of sovereign risk are not channeled to the real economy (in the form of distorsionary taxes in our case), then its consequences are null.

Figure 2: IRFs after a discount factor shock with $\nu = 1$.



Black: $b^g/(4y) = 0.6$ and $\tau = 0.3118$. Red: $b^g/(4y) = 1.15$ and $\tau = 0.4380$.

make this rise in public debt sustainable in the long run, with additional adverse effects on hours worked and output.

With a high initial debt-to-GDP ratio, the economy is more exposed to the rising sovereign default risk: the spread rises by more than 80 basis points (as opposed to roughly 10 basis points with a low initial debt ratio), taxes rise more, hours worked fall more and the recovery is much slower. When the economy features a high initial level of debt, the recession is thus deeper and the recovery more sluggish.

4.3 Public spending shock during a crisis

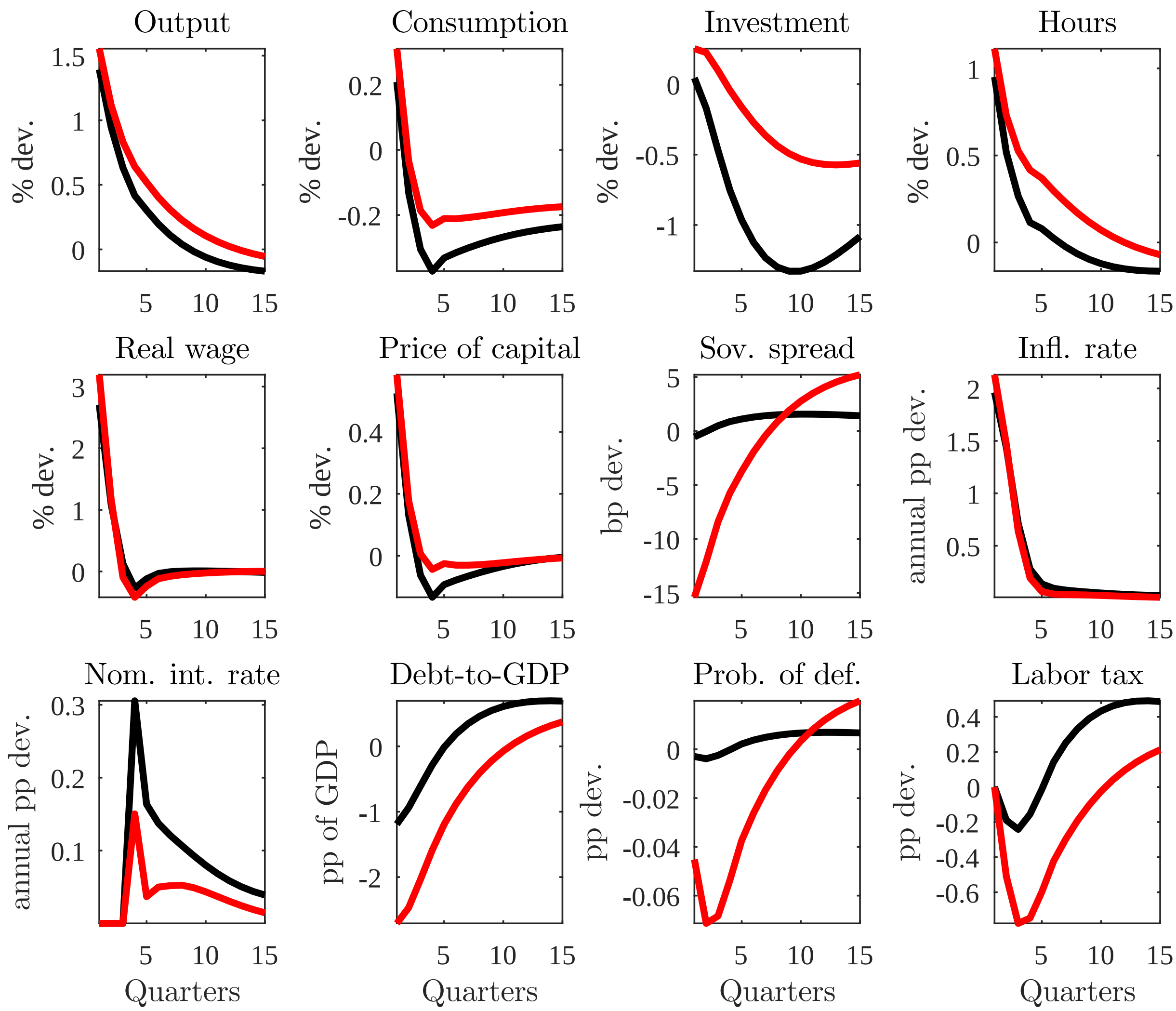
What are the effects of a 1 percent increase in public spending *conditional* on a 1.2 percent discount factor shock. Figure 3 reports the net effects of a spending shock in a recession induced by a discount factor shock, that is the difference between the response to a joint shock and the response to a discount factor shock only. Figure 3 shows that the responses are very different from the case of a spending shock around the steady state. They also vary significantly whether the economy features a high or low initial debt-to-GDP ratio.

On impact, when the zero bound is strictly binding, an increase in government spending leads to a rise in output, marginal cost, and inflation in both cases. The increase in expected inflation lowers the real interest rate, which drives up private consumption. This rise in private consumption expenditure leads to a further rise in output, marginal cost, and expected inflation and a further decline in the real interest rate. These results line up very well with the findings of Christiano, Eichenbaum, and Rebelo (2011). Interestingly enough, output increases more in the case of a high initial debt-to-GDP ratio, leading to a larger value of the government spending multiplier. We explain these results in the following way.

When the ZLB is binding, a spending shock lowers the real interest rate and raises the price of capital instead of raising the real interest rate and lowering the price of capital. As a consequence investment is less crowded out than when the ZLB is not binding, and private consumption is crowded in instead of being crowded out. Aggregate demand is thus boosted by much more, which results in a massive rise in labor demand: hours worked and the real wage both rise more. Hours increase by more than 1% (instead of 0.5% out of the ZLB) and the real wage increases by more than 3% (instead of 0.6%). These effects are clearly not new and explained extensively in several papers (see Christiano, Eichenbaum, and Rebelo (2011) and the subsequent contributions).

Our contribution lies in the fact that this massive rise in the tax base leads to a *fall* in the debt-to-GDP ratio, a fall in sovereign spreads and a fall in the tax rate on labor income, and that these movements are larger for economies featuring a large initial debt-to-GDP ratio. In addition, economies with a relatively low initial ratio face a relatively flat default probability schedule.

Figure 3: IRFs after a 1% public spending shock conditional on a discount factor shock with $\nu = 1$.



Black: $b^g/(4y) = 0.6$ and $\tau = 0.3118$. Red: $b^g/(4y) = 1.15$ and $\tau = 0.4380$.

Hence, when these economies face a shock that raises (respectively lowers) the level of public debt, they experience a moderate increase (resp. fall) in sovereign risk and in the subsequent spread. Economies featuring a relatively large initial debt-to-GDP ratio face a much steeper default probability schedule, inducing larger movements in sovereign risk and the subsequent spread. Therefore when the public spending shock occurs at the ZLB and produces a fall in the debt-to-GDP ratio, sovereign risk falls more, which further lowers the level of debt to be financed and leads labor taxes to fall more, producing a larger output multiplier.

Appendix A presents the IRFs produced by alternative versions of our model to assess the robustness of our results to the various assumptions considered. For instance, Figure 7 shows that our results do not strongly rely on the presence of capital accumulation. If anything, the absence of capital accumulation ($\iota = 0$) attenuates the differences between present-value public spending multipliers under alternative initial debt-to-GDP ratios but our main mechanism remains: the massive increase in the tax base leads public debt and the associated sovereign risk and spread to fall more when the initial level of public debt is high.

Further, Figure 8 suggests that sovereign risk alone is not the primary driver of our results. Indeed, IRFs without sovereign risk (Figure 8) exhibit a similar (though attenuated) pattern: both economies experience a rise in the tax base and a fall in debt-to-GDP that favors larger multipliers, and the economy with a relatively high initial debt-to-GDP ratio experiences a larger boom. This is because the economy with a high initial level of debt features a larger value of the tax rate on labor: for a given increase in the tax base, the high-debt economy experiences a larger increase in fiscal receipts and a larger fall in the debt-to-GDP ratio with positive feedback effects on output through a larger fall in the tax rate. Quantitatively however, sovereign risk acts as an amplifier of this mechanism and is therefore an additional transmission mechanism. So neither capital accumulation or sovereign risk are crucial in generating the result but both matter quantitatively.

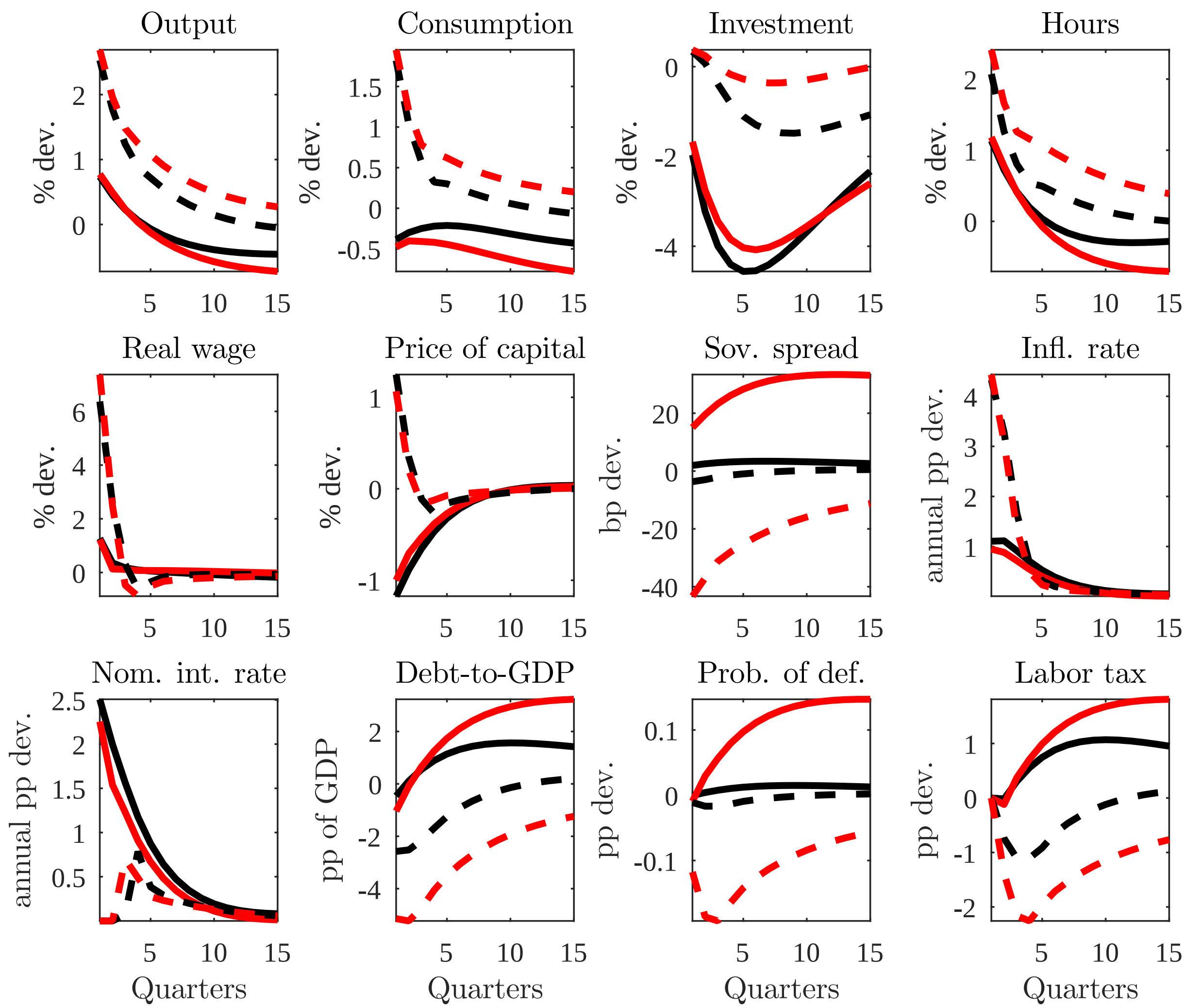
Finally, Figure 9 shows that distorsionary taxation is crucial to generate our results. With lump-sum taxes, differences in initial levels of debt matter for the pricing of sovereign risk, the probability of default and the dynamics of public debt, but these differences do not affect real variables. This clearly relates to the fact that default risk matters *ex-ante* but not *ex-post* in our model, and that an additional transmission mechanism is required for sovereign risk to matter, distorsionary taxes our baseline model.

4.4 Complementarity between public and private goods

In the previous case, we considered that public and private expenditure on goods are substitutable, with an elasticity $\nu = 1$. However, empirical evidence favor estimates pointing to a mild complementarity (see Bouakez and Rebei (2007)). We now investigate whether considering complementary public and private goods ($\nu = 0.45$) alters our results. Figure 4 below reports the

IRFs to a 1 percent public spending shock around the steady state (solid lines) and conditional on a discount factor shock (dashed lines) under relatively low or high initial debt-to-GDP ratios when public and private goods are complements in utility.

Figure 4: IRFs after a 1% public spending shock with $\nu = 0.45$.



Solid: around the steady state. Dashed: conditional on a discount factor shock. Black: $b^g/(4y) = 0.6$ and $\tau = 0.3118$. Red: $b^g/(4y) = 1.15$ and $\tau = 0.4380$.

Figure 4 shows that the pattern observed with substitutable private and public consumption is qualitatively robust. In normal times, output is more stimulated after a public spending shock when the initial level of debt is low rather than high, while the opposite holds true when the ZLB is binding. The major difference with the previous case lies in the response of private consumption, that is more strongly crowded in (or less crowded out) when private and public goods are complements, therefore producing much larger government spending output multipliers. The fact that complementary between public and private goods leads to larger spending multipliers in regular economic environments (*i.e.* out of the ZLB and without sovereign risk) has already been clearly identified in the literature (see Bouakez and Rebei (2007) or Leeper, Traum, and Walker (2015)). We show that this result carries over more complex environments, and that our main result is robust to considering complementary public and private goods.

5 Government spending multipliers

We now investigate more systematically the extent to which initial debt-to-GDP ratios affect the value of spending multipliers at different horizons. To do so, we define the present-value multiplier as the discounted cumulative increase in output T periods after the shock that results from the discounted cumulative increase in public spending after a spending shock in period 1:

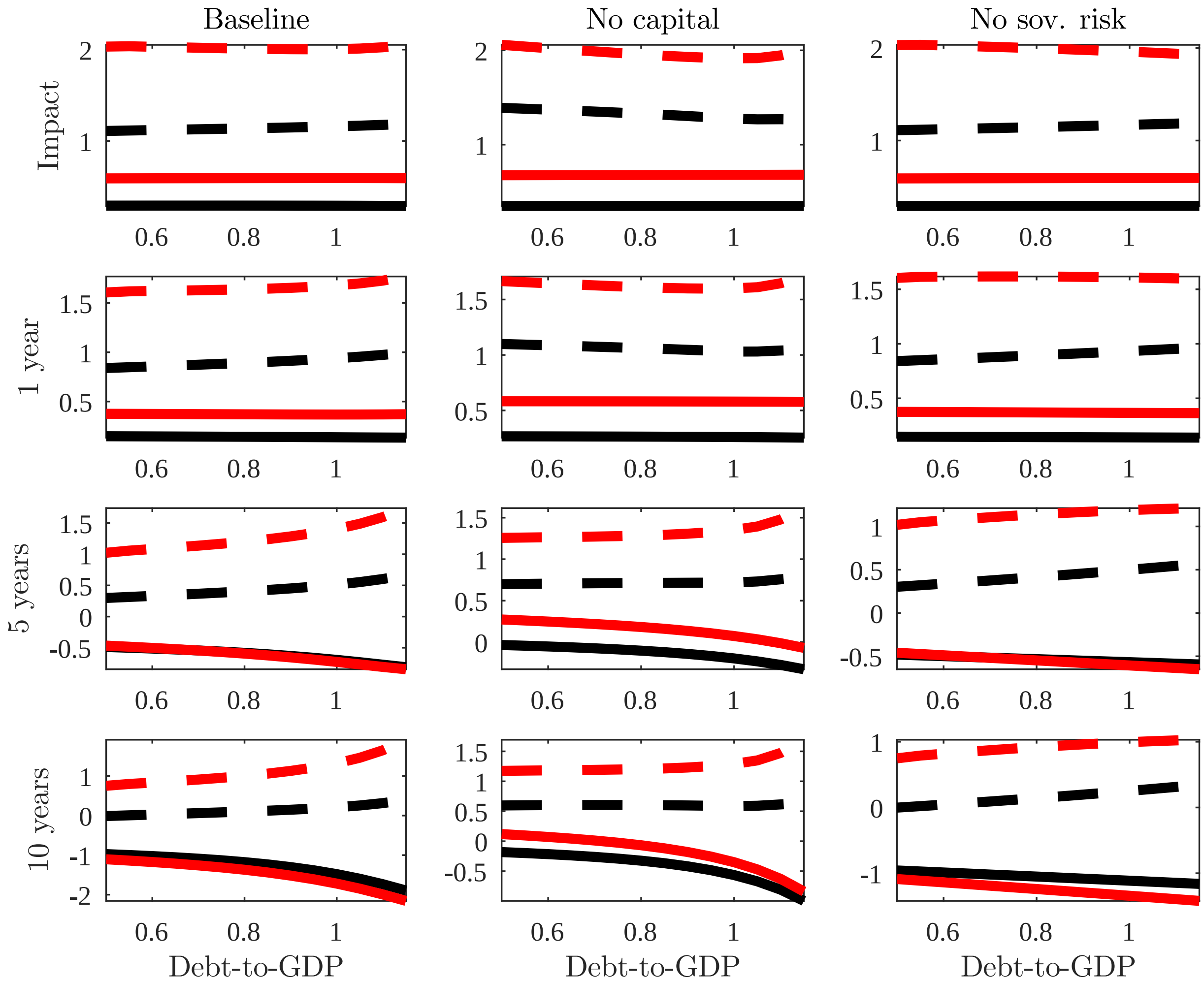
$$PVM_T = \frac{\sum_{j=1}^{T} \beta^j \left(y_{t+j} - y_t\right)}{\sum_{j=1}^{T} \beta^j \left(g_{t+j} - g_t\right)}. \tag{30}$$

Figure 5 reports the present-value multipliers of output on impact ($T = 1$), one year after the shock ($T = 4$), 5 years after the shock ($T = 20$) and 10 years after the shock ($T = 40$). The baseline model, the model without capital accumulation and the model without sovereign risk are considered.

Let us start with the baseline model. First, Figure 5 confirms the previous results in the sense that multipliers are clearly larger when the ZLB is binding (dashed lines) than when shocks occur at the steady state (solid lines). The latter are 0.5 or less on impact in normal times while they reach 1.2 to 2 during ZLB episodes. Second, complementarity between public and private goods (red lines) does produce much higher multipliers compared to the case where private and public goods are substitutable (black lines). Third, the initial debt-to-GDP ratio has a negative impact on multipliers during normal times and a positive or weakly negative impact on multipliers during ZLB episodes. In addition, the impact is magnified when the horizon is longer, which relates to the fact that our result depends on the dynamics of the labor income tax rate, that moves along with debt, and therefore displays quite a lot of persistence over time.

Now looking at the model without capital accumulation, we can see that abstracting from capital accumulation tends to increase the absolute size of public spending multipliers and mostly preserves the patterns uncovered in the baseline model. Finally, comparing the multipliers produced by the model without sovereign risk with those produced by the baseline model confirms that sovereign risk is not the main driver of the effects uncovered by our previous analysis: the global pattern of multipliers and the impact of the initial debt-to-GDP ratio are preserved. Hence, the steady-state level of the labor income tax rate combined with the surge in the tax base produced by the spending shock when the ZLB binds are the key to our main effect. However, sovereign risk clearly acts as a quantitative amplifier, especially for relatively high initial debt-to-GDP ratios. For those high levels of initial debt, the impact of the level of debt in the steady state

Figure 5: Present value output multipliers.



Solid: normal times. Dashed: conditional on a discount factor shock. Black: $\nu = 1$. Red: $\nu = 0.45$.

with sovereign risk is more negative when shocks are computed around the steady state and more positive when the ZLB binds.

6 Optimized spending rule after a discount factor shock

We now consider endogenous variations of public spending. Our goal is to assess the extent to which higher levels of debt imply different optimized responses of public spending to a discount factor shock. For instance, do higher debt levels imply that public spending should fall in response to a crisis? Or on the contrary do they imply larger increases in public spending to help fasten the recovery from the recession produced by the discount factor shock? To anwser these questions, we consider that the government commits to the following simple spending rule by which government spending respond to lagged deviations of output from its steady-state value

$$g_t = d_g \left(y_{t-1} - y \right) \tag{31}$$

and abstract from spending shocks *per se*. The optimized reaction coefficient d_g is chosen so as to maximize the Hicksian consumption equivalent ϵ that solves:

$$E_t \left\{ \sum_{s=t}^{\infty} \left(e_s \beta^{s-t} \right) u \left(c_s, g_s, \ell_s \right) - \sum_{s=t}^{\infty} \left(\beta^{s-t} \right) u \left(c \left(1 + \epsilon/100 \right) \right), g, \ell \right) \right\} = 0 \tag{32}$$

Variable ϵ measures the steady-state percentage of consumption loss associated with experiencing macroeconomic fluctuations, induced in our case by a discount factor shock. As seen in Figure 2, a large discount factor shock implies a very large drop in private consumption, inducing large current-period utility losses. In addition, these short-run utility losses are over-weighted because the discount factor rises in the first periods.

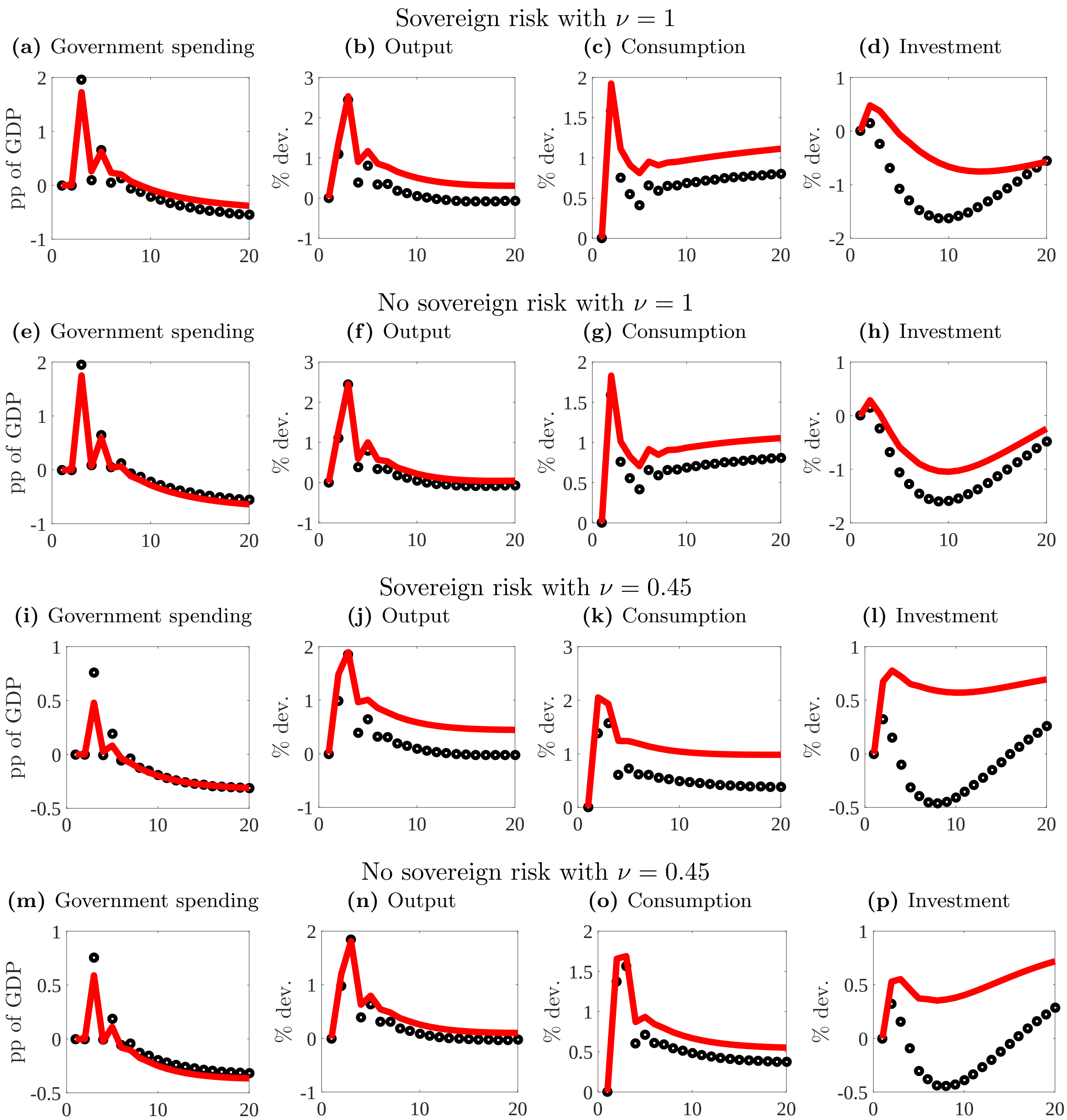
Figure 6 reports the optimized dynamic response of government spending after a 1.2 percent discount factor shock, as well as the net effects of this spending policy on macroeconomic aggregates.[9]

First, Figure 6 shows that the optimized response of the government is to increase government spending under all configurations. Intuitively, raising public spending provides direct utility gains, since the amount of public goods enters the utility function of households. It also provides indirect utility gains as raising public spending at the ZLB produces a crowding-in effect of private consumption, and lowers the private investment crowding-out effect with respect to what happens during normal times. As such, raising public spending raises the welfare of households.[10]

[9] The latter are the differences between the responses with an optimized policy and the responses with constant public spending.

[10] Our results, although derived in a different environment, are qualitatively and quantitatively consistent with those reported by Bilbiie, Monacelli, and Perotti (2017) and Nakata (2017).

Figure 6: Optimized response of government spending after a discount factor shock and net effect on macroeconomic aggregates.



Black circles: $b^g/(4y) = 0.6$ and $\tau = 0.3118$. Red: $b^g/(4y) = 1.15$ and $\tau = 0.4380$.

Second, the size of the government intervention is critically affected by the initial debt-to-GDP ratio, and by the elasticity of substitution between public and private goods. We have already seen that government spending policies were more effective (in the sense of a larger output multiplier) at the ZLB when the initial debt-to-GDP ratio was high. Our analysis confirms this, as the optimized response of public spending is slightly smaller in this case, but produces larger positive effects on output, consumption and investment. This is true as well when public and private goods are complements instead of substitutes: public spending react less in this case—less than 1 pp of GDP—while the increase is much larger—around 2 pp of GDP—when public and private goods are substitutes.

The presence of sovereign risk has less visible effects on the design of optimized spending policies, at least graphically. Table 2 makes the role of sovereign risk more apparent. It reports the optimized coefficients d_g, the marginal welfare gains from an active spending policy, and the size of the associated output multiplier at the horizon of 4 quarters. Table 2 suggests that the presence of sovereign risk tends to make fiscal policy more effective at the ZLB when the initial debt-to-GDP ratio is high. In the case of high initial debt levels, sovereign risk is associated with less active but more effective policies both in terms of output and welfare. The impact of sovereign risk is even larger when public and private goods are complements.

Table 2: Optimized policy rules.

	With sovereign risk			Without sovereign risk		
	d_g	$\Delta\epsilon$	PVM_4	d_g	$\Delta\epsilon$	PVM_4
			$\nu = 1$			
$by = 0.6$	-0.5908	0.1634	1.7511	-0.5909	0.1565	1.7648
$by = 1.15$	-0.5231	0.6220	2.2960	-0.5983	0.2194	2.2138
			$\nu = 0.45$			
$by = 0.6$	-0.3396	0.0983	4.1275	-0.3381	0.0942	4.1342
$by = 1.15$	-0.2753	0.4073	9.1323	-0.3268	0.1396	6.5431

Note: d_g is the reaction of public spending to lagged output deviations. $\Delta\epsilon$ is the welfare gain from active spending policies, computed as the welfare loss with active spending minus the welfare loss with passive spending, in percent of steady-state consumption. PVM_4 is the present-value multiplier of output, at the horizon of 4 quarters.

In addition, Table 2 confirms the results given by the IRFs: high initial levels of debt and complementarity between public and private goods are associated with less aggressive spending policies, given that the latter are more effective at stabilizing output. Indeed, output multipliers are larger at the horizon of 4 quarters when debt levels are initially high or when public and private goods are complements in utility.[11] In terms of welfare however, potential stabilization

[11]Notice that the values of output multipliers differ quite substantially from those reported in the first sections of the paper. These differences are attributed to the fact that the persistence and size of increases in public spending both depend on the rule, and are no longer determined exogenously. In addition, the optimized path of government spending features a fall below steady state after 10-12 quarters, and the latter is fully anticipated, which impacts multipliers even in the short-run. Given this, one should not seek to compare the values reported in Table 2 with those reported in Figure 5.

gains are lower when goods are complements, but substantially larger when debt levels are initially high. The potential welfare gains are further magnified when sovereign risk is present.

Closest to the analysis in this last Section are Bilbiie, Monacelli, and Perotti (2017) and Nakata (2017), who derive Ramsey policies subject to ZLB episodes. Some of our results are in perfect accordance with theirs. In particular, the fact that government interventions imply an increase in public spending. Further, the size of government interventions are broadly comparable, as well as the size of welfare gains from active policies at the ZLB. Our results differ however along crucial dimensions. First, Bilbiie, Monacelli, and Perotti (2017) do not consider sovereign risk and assume lump-sum taxation. As such, the initial level of debt is irrelevant to the analysis. Second, Nakata (2017) finds that higher initial levels of debt should imply more aggressive spending policies while we find the opposite. In our view, this is due to the fact that the labor income tax follows a systematic feedback rule while this variable is an optimized policy instrument in Nakata (2017). In any case the design of optimal policies is not the main focus of our paper. We simply derive optimized rules to contrast the potential impact of the initial level of debt on the design of government interventions and show that it has non-trivial consequences on the latter.

7 Conclusion

This paper investigates the relation between the initial level of debt and the effectiveness of public spending shocks in a New-Keynesian model with sovereign risk, capital accumulation and a lower bound constraint on the nominal interest rate. We find that countries with high debt are more fragile in the event of a crisis, as they experience larger economic downturns. In line with the literature, we also find that the spending multipliers are lower with a high initial debt-to-GDP ratio during normal times, and larger than during normal times when the ZLB is binding.

The novel result of the paper is that economies with a high initial debt-to-GDP ratio feature larger spending multipliers at the ZLB. The result is driven by the massive rise in the tax base induced by the spending shock, and by the fact that fiscal revenues increase more in the indebted country because the latter has a larger steady-state labor tax rate. This effect is magnified by the presence of sovereign risk, that makes the economy more sensitive to fluctuations of the debt-to-GDP ratio. Finally, this result implies that optimized spending policies at the ZLB are affected by the initial level of debt: a lower increase in public spending is required with high initial levels of public debt, as fiscal policy is more effective at stimulating output and provides larger reductions in the welfare losses from the crisis.

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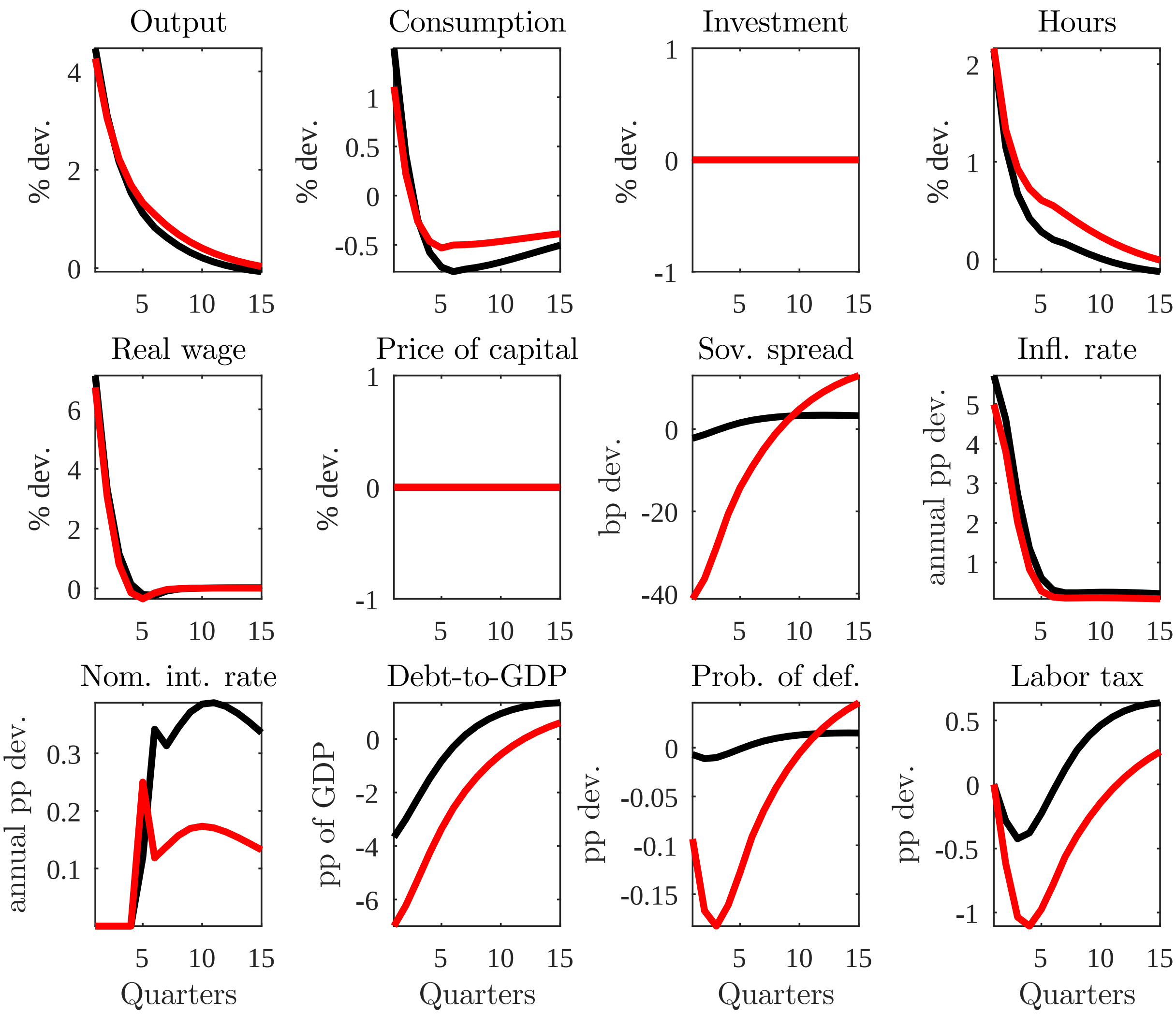
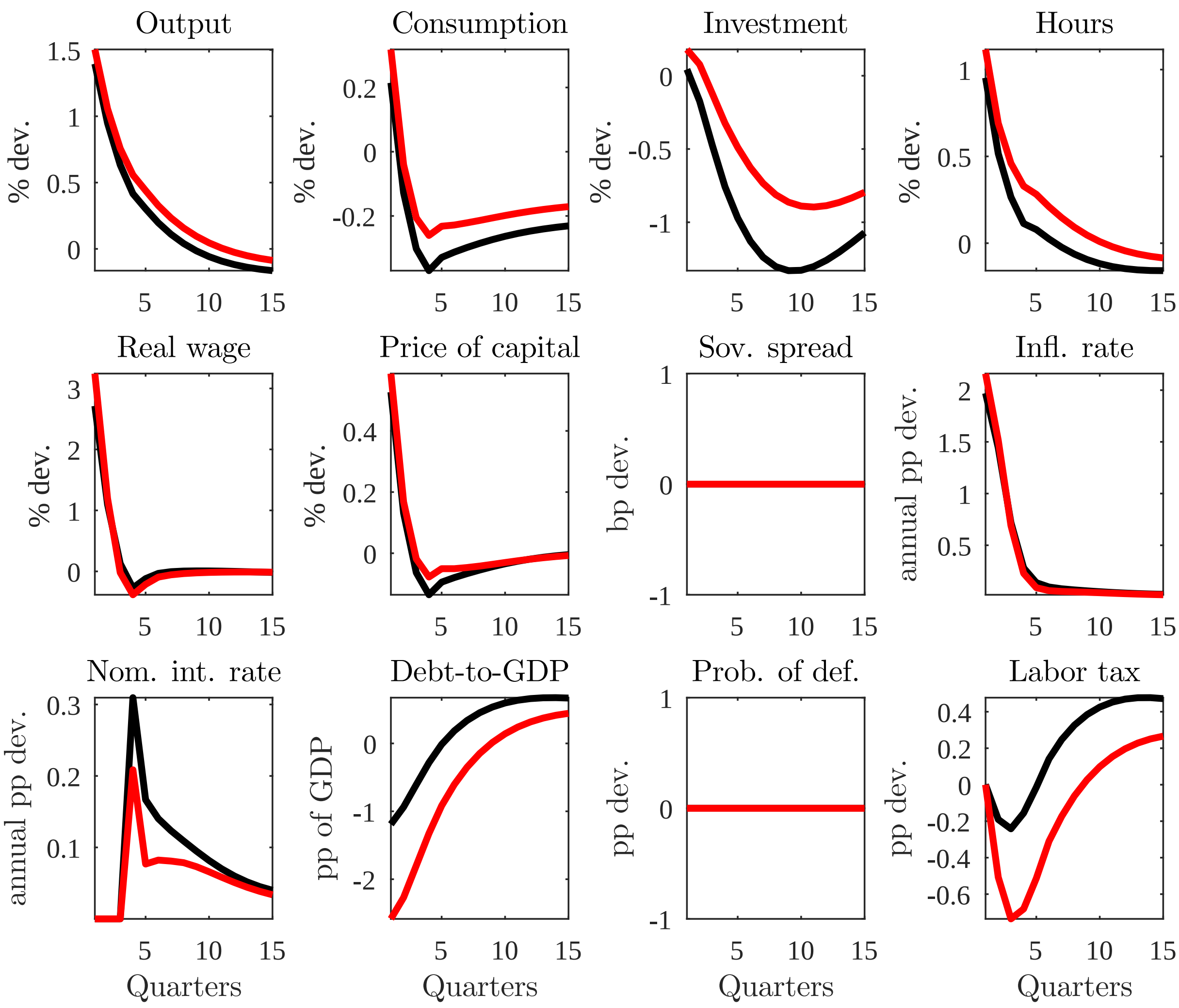
A Robustness checks

Figure 7: IRFs after a 1% public spending shock conditional on a discount factor shock with $\nu = 1$ - No capital accumulation $\iota = 0$



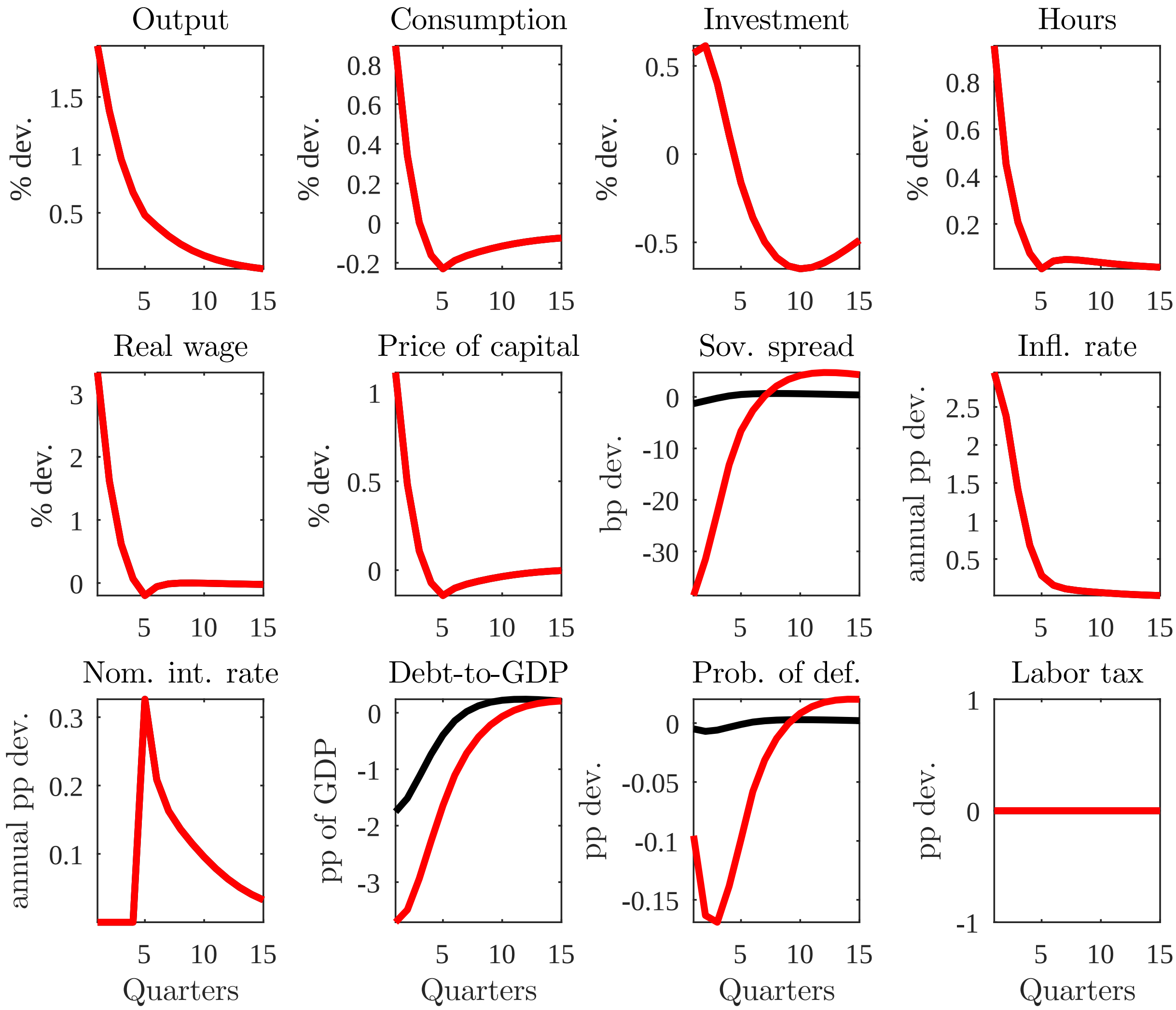
Black: $b^g/(4y) = 0.6$ and $\tau = 0.2839$. Red: $b^g/(4y) = 1.15$ and $\tau = 0.3719$.

Figure 8: IRFs after a 1% public spending shock conditional on a discount factor shock with $\nu = 1$ - No sovereign risk $pd_t = 0$



Black: $b^g/(4y) = 0.6$ and $\tau = 0.3074$. Red: $b^g/(4y) = 1.15$ and $\tau = 0.3505$.

Figure 9: IRFs after a 1% public spending shock conditional on a discount factor shock with $\nu = 1$ - Lump-sum taxes



Black: $b^g/(4y) = 0.6$ and $\tau = 0.3402$. Red: $b^g/(4y) = 1.15$ and $\tau = 0.3402$.