Self-Enforcing Trade Policy and Exchange Rate Adjustment*

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Abstract

We explore the cyclical pattern of trade protection in a simple New Keynesian open economy macro model. Tariff rates are determined endogenously in a sustainable equilibrium of a two country trade policy game. The incentive to levy tariffs is greater when the exchange rate is floating, since a fixed exchange rate removes the ability to manipulate the terms of trade. If price are fully flexible, we find that protectionism is basically a-cyclical. By contrast, with pre-set prices, tariffs respond to both monetary and productivity shocks. But the degree of protection may be pro-cyclical or counter-cyclical, depending on the pattern of shocks and parameter values.

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1 Introduction

A large literature in economics suggests that trade protectionism varies over the business cycle.[1] The most common view is that protectionism is counter-cyclical – a period of low growth and high unemployment will lead to greater calls for trade restrictions to protect domestic jobs. Empirically however, the evidence on the cyclicality of trade policy is mixed.[2] From a theoretical viewpoint, it is not clear why trade restrictions should be increased during an economic downturn. If trade policy is aimed at domestic employment, an increase in tariffs during a recession would help domestic import competitors, but hurt exporters. It is not obvious that the balance of influence between import competition and exporters would shift in a clear pattern over the cycle. On the other hand, if trade restrictions are designed to improve the terms of trade, it is unclear why the pressure to exploit market power would be greater in periods of low economic activity.

Bagwell and Staiger (2003) construct a model in which protectionism is counter-cyclical in some circumstances. In their model, trade protection is determined in a repeated game between two countries, and equilibrium sustainable protection satisfies an incentive constraint which balances the benefits of predatory tariffs against the future costs of a trade war. They show that a persistent fall in trade volume during a recession reduces the benefits of cooperation, and leads to an increase in equilibrium tariffs. In a related paper, Bagwell and Staiger (1999) show that tariffs will increase during periods of high import growth, but tariffs will be lower, the greater is the volatility of imports.[3]

The Bagwell and Staiger papers provide key insights into the forces determining trade protectionism. However their theoretical framework is not directly built along the lines of standard open economy macro models, which may include the presence of nominal price rigidities, business cycles driven by monetary or technology shocks, and different degrees of exchange rate flexibility. The last point is particularly important, since a substantial empirical literature suggests that movements in exchange rates, and uncompetitive real exchange rates may lead to increasing forces for protectionism.[4]

This paper revisits the question of the cyclicality of trade protection within a standard New Keynesian open economy macro model which exhibits sticky prices and exchange rate fluctuations in response to monetary and productivity shocks. We show how the basic architecture of open

[1] For instance see Bagwell and Staiger (1999), Bagwell and Staiger (2003), Irwin (2005), Bown and Crowley (2013c), Knetter and Prusa (2003).

[2] Irwin (2005) finds that US anti-dumping filings are increasing in the US unemployment rate. Bown and Crowley (2013c) find that emerging market imposition of temporary trade barrier become increasingly counter-cyclical over the 1989-2010 period. Bown and Crowley (2013b) find strong evidence for counter-cyclical trade disputes among advanced economies prior to the Great Recession, but not afterwards. On the other and, Rose (2012) argues that there is little evidence for counter-cyclical trade policy in data over the 20th century.

[3] Bown and Crowley (2013a) construct an empirical test of the Bagwell and Staiger (1999) model. They find that, empirically, surges in imports tend to precipitate more trade restrictions, but increases in the volatility of imports tend to reduce average tariff rates.

[4] See for instance Knetter and Prusa (2003), Irwin (2005).

macro models leads naturally to cyclical responses of protectionism when trade policy is determined along the lines of Bagwell and Staiger (2003). Moreover, the endogenous response of the exchange rate is an important channel for the effects of protectionism. We find that trade protection is higher in a regime of flexible exchange rates than when the exchange rate is fixed.

Our results are based on a simple two-country, two good open macro model, in which countries may levy tariffs on imported goods. For a given sequence of tariffs, the model is very simple. Each country specializes in its own category of goods. Prices are set one period in advance and adjust fully after one period. Money and monetary policy are introduced *via* a cash-in-advance constraint, and each country is subject to technology and monetary shocks. We follow Bagwell and Staiger (2003) in the assumption that tariffs are determined by a balance between costs and benefits of deviating in a repeated game between home and foreign tariff setters. The equilibrium tariff sequence can be thought of as an implicit international trade agreement between the countries. On the one hand, there is a gain to any one country on cheating on the agreement and setting a high tariff to improve its terms of trade. On the other hand, the cost of cheating is a reversion to a 'trade war' in the future. The sustainable tariffs just balance the benefits and costs for both countries of cheating on the agreement itself.

Despite its relative simplicity, we show that this model can address some of the key issues regarding the cyclicality of protectionism and the relationship between exchange rate flexibility and trade policy, as discussed above. The first question is why should protection be cyclical at all? Why would the temptation to cheat on a trade agreement vary over the business cycle? The New Keynesian model has a natural answer to this. We show that if prices are fully flexible in our model, the equilibrium tariff sequence is basically a-cyclical. While the degree of protection depends on structural features on the economy as well as the policy-maker's discount factor, tariffs do not vary over time.

When prices are sticky however, we show that equilibrium sustainable tariffs are time-varying, and move in response to monetary and productivity shocks. The essential intuition for this is that with pre-set prices, the strategic objective of the tariff setter at any time period is conditioned on given prices, but takes account of endogenous adjustment of prices in the future. This implies that aggregate shocks have different impacts on the benefits and costs of cheating on the trade agreement, and hence the equilibrium sustainable tariff rates themselves must respond to shocks.[5]

Since the incentive to levy tariffs in our model is based on gaining a terms-of-trade advantage, the exchange rate regime becomes an important factor in the determination of equilibrium sustainable tariffs. In our baseline model, we show that if exchange rates are fixed and prices sticky,

[5]Since the focus of the paper is on the incentives behind trade policy in the presence of nominal rigidities, we take monetary policy as exogenous. Appendix D discusses the implications of the model under a welfare-maximizing determination of monetary policy.

then neither tariff setter can affect the terms of trade by deviating from a trade agreement. Consequently, equilibrium protection – the average level of tariffs – are higher under flexible than under fixed exchange rates.

Further, as noted above, while the conventional wisdom is that trade protection is counter-cyclical, this is not a consensus view. In our model, we find that protection may or may not be counter-cyclical, depending both on the pattern of shocks and underlying model parameters. In particular, in our baseline model, when the intertemporal elasticity of substitution (IES) is high, we find that sustainable tariffs increase in response to positive monetary policy shocks, while sustainable tariffs fall in response to productivity shocks. Given sticky prices, there is a natural channel through which monetary policy shocks increase the gains from cheating on a trade agreement, while productivity shocks increase the consequences of cheating. As a result, depending on the preponderance of shocks, tariff rates may be pro-cyclical or counter-cyclical. In addition, when the IES is lower, tariffs may become pro-cyclical (counter-cyclical) upon realization of local productivity (money) growth shocks.

Finally, the analysis provides insights into the link between trade protectionism and the volatility of shocks. Indeed, we also find that a rise in the volatility of productivity shocks can have a dramatic effect in reducing the mean level of tariffs over the business cycle. But this arises from a very different mechanism than that of Bagwell and Staiger (1999). In particular, the linkage between the volatility of productivity shocks and equilibrium tariff rates holds only when prices are sticky.

While the above results are derived in closed-form within a simplified model where the trade elasticity is unitary and the consumption bundle a Cobb-Douglas function of imported and local goods, we also show that they hold qualitatively for a more general model where the trade elasticity is non-unitary and the consumption bundles takes a more general CES functional form.

There has been an increasing interest in investigating the effects of trade restrictions in open economy macro models. Barattieri, Cacciatore, and Ghironi (2021) investigate empirically the impact of exogenous changes in tariffs in an SVAR framework, and show that they act as negative supply shocks, depressing GDP and raising inflation with little effects on the trade balance. They propose a small open economy model with firm entry and endogenous tradability that successfully rationalizes the empirical evidence. We adopt a mirror perspective, considering tariffs as endogenous and ask how governments react to economic conditions to determine trade policies over the business cycle. Another recent paper by Erceg, Prestipino, and Raffo (2018) studies the effects of trade policies in the form of import tariffs and export subsidies. They show that the macroeconomic effects of these policies critically depend on the response of the real exchange rate, and that in turn depends on the expectations about future policies and potential retaliation from trade partners. Finally, a recent paper by Furceri, Hannan, Ostry, and Rose

(2018) examines the macroeconomic consequences of tariff shocks, and shows that these shocks are generally contractionary.

Focusing more closely on the endogenous determination of trade policies, we noted above that there is a large empirical literature investigating the link between trade restrictions and the economic cycle, and separately, the effect of real exchange rate undervaluation on trade policy (*e.g.* Oatley (2010), Gunnar and Francois (2006), Bown and Crowley (2013b), among others). In a theoretical model Eaton and Grossman (1985) study optimal tariffs when international asset markets are incomplete and show that they can be used to partly compensate the lack of consumption insurance. Bergin and Corsetti (2020) also consider tariffs as policy instruments in addition to monetary policy but their focus is not specifically on tariffs, rather on the implications of monetary policy on the building of comparative advantages. As discussed above Bagwell and Staiger (2003) propose a trade model featuring potential terms-of-trade manipulation by governments, and trade agreements as means to restrict this policy option. Our paper is complementary to theirs. Most importantly, we incorporate endogenous tariff formation within a standard open economy macro model, showing the importance of the types of shocks, price stickiness, and the exchange rate regime for the equilibrium degree of trade protection. Beshkar and Shourideh (2020) offer an analytical determination of unilaterally optimal tariffs in a Ricardian economy that features trade imbalances, and shows that those tariffs are counter-cyclical.[6] Finally, while terms-of-trade manipulation is not the sole motive to justify the existence of incentive to apply tariffs and mechanisms to cope with these incentives (such as trade agreements), they are by far the most important and the most relevant from an empirical perspective, as shown by Bagwell and Staiger (2010).[7]

The paper is structured as follows. Section 2 develops a tractable open economy model where tariffs play a role in affecting the exchange rate and the terms of trade. Section 3 looks at the welfare effects of tariffs in this model, shows that the incentive to set tariffs depends on the exchange rate regime and that incentives are lower under fixed exchange rate regimes. Section 4 then extends the analysis to allow for endogenous tariffs, where tariffs are determined as a sustainable equilibrium in a dynamic game between governments. This section shows that tariffs are a-cyclical under flexible prices, but with sticky prices (where prices are preset one period in advance), tariffs vary over the business cycle. In particular, tariffs respond negatively (respectively positively) to expansionary supply (resp. demand) shocks when the IES is high.

[6] Both Beshkar and Shourideh (2020) and Bagwell and Staiger (2003) assume that the planner maximizes national income or consumption expenditure, while we use a micro-founded utility function that depends on consumption and labor, which explains why tariffs are counter-cyclical under flexible prices in their set-up and a-cyclical in our set-up.

[7] In particular, Bagwell and Staiger (2010) mention profit-shifting or firm relocation motives in imperfect competitive environments as alternative sources of international externalities that can justify the existence of incentives to impose unilateral tariffs. In an important empirical study, Broda, Limao, and Weinstein (2008) support the terms of trade motivation for tariff setting. They show that countries systematically set higher tariffs of imports with more inelastic supply schedules.

Section 5 extends the baseline model to allow for a non-unitary trade elasticity and shows that our results continue to hold in this more complex environment. Section 6 presents some conclusions.

2 A tractable model with pre-set prices

To explore the ideas discussed in the introduction, we develop a two country open economy framework with endogenous trade policy that may depend on macroeconomic conditions. The model is extremely simple; it can be solved with pen and paper in its simplest version. Despite this, it carries quite a rich set of implications for the relationship between exchange rate regimes, business cycle shocks, and protectionism.

2.1 Households

The two countries are home and foreign. In each country, households earn wages and profits in each period, supply labor, and consume home and foreign goods. We assume there is no capital mobility across countries. The home country utility is

$$U = \frac{C^{1-\sigma}}{1-\sigma} - \frac{H^{1+\psi}}{1+\psi}$$

where C is the consumption aggregator and H is labor supply. The consumption aggregator is Cobb Douglas, and depends on home goods and foreign goods in the following way:

$$C = \left(\frac{C_x}{\omega}\right)^{\omega} \left(\frac{C_m}{1-\omega}\right)^{1-\omega}$$

Households in the home country face the budget constraint:

$$M + P_x C_x + (1+\tau) S P_m C_m = WH + \Pi + T + M_0 \tag{1}$$

where τ is a tariff levied on the imported good, M represents money holdings, S is the nominal exchange rate, P_x is the home currency price of home goods and P_m is the foreign currency price of foreign goods. W is the nominal wage rate, and T is a lump-sum rebate of the tariff revenue, so that:

$$T = \tau S P_m C_m \tag{2}$$

We assume that there is a binding cash in advance constraint for households, which acts so as to pin down nominal magnitudes:

$$M \geq P_x C_x + S P_m C_m$$

where M is home country money supply, set exogenously by the home monetary authority. For simplicity, we make the assumption that the cash in advance requirement exempts tariff payments. The consumer's first-order conditions are standard, and optimal consumption implies

$$P_x C_x = \frac{\omega}{1-\omega} S P_m (1+\tau) C_m \tag{3}$$

Optimal labor supply and the cash in advance constraint imply

$$W = H^{\psi} C^{\sigma-1} \frac{P_x C_x}{\omega} \tag{4}$$
$$M = P_x C_x + S P_m C_m \tag{5}$$

2.2 Firms

Firms choose prices to maximize expected discounted profits. Sticky prices are a key element of the model. To avoid intrinsic dynamics in the model, we assume that prices have to be set in advance of the within-period (monetary and productivity) shocks. But once the shocks are realized, prices can fully adjust before the next period. Firms operate with a linear technology, given by $Y_x = \theta H$, and maximize expected discounted profits, expressed as:

$$E\mu\Pi = E\mu(P_x Y_x - WH)$$

where E is the expectation operator, and $\mu = \frac{1}{C^{\sigma} P}$ is the households' Lagrange multiplier for nominal income.

The optimal price satisfies:[8]

$$P_x = \frac{E \frac{W}{\theta} Y_x \mu}{E Y_x \mu}$$

where

$$P = P_x^{\omega} ((1+\tau) S P_m)^{1-\omega} \tag{6}$$

The following further conditions define the equilibrium market clearing in goods and money:

$$\begin{aligned} Y_x &= C_x + C_x^* \\ Y_m &= C_m + C_m^* \\ M &= M_0 \\ M^* &= M_0 \end{aligned}$$

Details on on the model solution are given in Appendix A. We make use of the implications of the CIA constraints:

$$Y_x = \frac{M}{P_x} \tag{7}$$

[8]We implicitly assume differentiate products within each country's good, where firms are monopolistic competitors and subsidies offset the monopoly markups.

$$Y_m = \frac{M^*}{P_m} \tag{8}$$

along with the above equations.

2.3 Price solutions

The analysis of tariff-setting in the model below depends critically on the way in which pre-set prices depend on money and productivity shocks. We can express the terms of trade as:

$$Q = \frac{Y_x}{Y_m}\left(\frac{1-\omega}{\omega^*}\frac{(1-\omega^*)(1+\tau^*)+\omega^*}{\omega(1+\tau)+1-\omega}\right) \tag{9}$$

The nominal exchange rate expression is:

$$S = \frac{1-\omega}{\omega^*}\frac{(1-\omega^*)(1+\tau^*)+\omega^*}{\omega(1+\tau)+(1-\omega)}\frac{M}{M^*} \tag{10}$$

Finally, we can express the home and foreign goods price as a function of the underlying shocks:

$$P_x = \left(\frac{E\left(\frac{M}{\theta}\right)^{1+\psi}}{E\left(\frac{M^{\omega(1-\sigma)}}{M^{*(1-\omega)(\sigma-1)}}\Lambda^{-1}(\tau,\tau^*)\right)}\right)^{\frac{\psi+\sigma+\omega^*(1-\sigma)}{\Delta}}\left(\frac{E\left(\frac{M^*}{\theta^*}\right)^{1+\psi}}{E\left(\frac{M^{*(1-\omega^*)(1-\sigma)}}{M^{\omega^*(\sigma-1)}}\Lambda^{*-1}(\tau,\tau^*)\right)}\right)^{\frac{(1-\omega)(1-\sigma)}{\Delta}} \tag{11}$$

$$P_m = \left(\frac{E\left(\frac{M^*}{\theta^*}\right)^{1+\psi}}{E\left(\frac{M^{*(1-\omega^*)(1-\sigma)}}{M^{\omega^*(\sigma-1)}}\Lambda^{*-1}(\tau,\tau^*)\right)}\right)^{\frac{\psi+\sigma+(1-\omega)(1-\sigma)}{\Delta}}\left(\frac{E\left(\frac{M}{\theta}\right)^{1+\psi}}{E\left(\frac{M^{\omega(1-\sigma)}}{M^{*(1-\omega)(\sigma-1)}}\Lambda^{-1}(\tau,\tau^*)\right)}\right)^{\frac{\omega^*(1-\sigma)}{\Delta}} \tag{12}$$

where $\Delta = (\psi+\sigma)\delta$, $\delta = (1+\psi+(\omega^*-\omega)(1-\sigma))$, and the functions Λ and Λ^* are defined as:

$$\Lambda(\tau,\tau^*) = \left(\frac{1+\tau}{\omega(1+\tau)+1-\omega}\right)^{(1-\omega)(1-\sigma)}\left(\frac{1}{(1-\omega^*)(1+\tau^*)+\omega^*}\right)^{-(1-\omega)(1-\sigma)}\left(\frac{1-\omega}{\omega^*}\right)^{(1-\omega)(1-\sigma)} \tag{13}$$

$$\Lambda^*(\tau,\tau^*) = \left(\frac{1+\tau^*}{(1-\omega^*)(1+\tau^*)+\omega^*}\right)^{1-(1-\omega^*)(1-\sigma)}\left(\frac{1}{\omega(1+\tau)+1-\omega}\right)^{-\omega^*(1-\sigma)}\left(\frac{1-\omega}{\omega^*}\right)^{-\omega^*(1-\sigma)} \tag{14}$$

Hence, the solutions (11) and (12) make clear that *ex-ante* pre-set prices depend on the distribution of money shocks, productivity shocks and home and foreign tariffs. We use (11) and (12) along with ($Y_x = M/P_x$) and ($Y_m = M^*/P_m$) to compute expected home and foreign output and expected utility below. First note that if prices were fully flexible, and could adjust to money, productivity, or tariff shocks, we would have the solutions:

$$\frac{P_x}{M} = \left[\frac{\Lambda}{\theta^{1+\psi}}\right]^{\frac{\psi+\sigma+\omega^*(1-\sigma))}{\Delta}}\left[\frac{\Lambda^*}{\theta^{*(1+\psi)}}\right]^{\frac{(1-\omega)(1-\sigma))}{\Delta}},\quad \frac{P_m}{M^*} = \left[\frac{\Lambda^*}{\theta^{*(1+\psi)}}\right]^{\frac{\psi+\sigma+(1-\omega)(1-\sigma))}{\Delta}}\left[\frac{\Lambda}{\theta^{1+\psi}}\right]^{\frac{\omega^*(1-\sigma))}{\Delta}} \tag{15}$$

With fully flexible prices money is neutral, but normalized prices – prices divided by money supply – are negatively related to domestic productivity shocks, and positively related to own country tariffs.

2.4 Utility measures

In order to determine the path of tariffs, it is necessary to construct welfare measures for benevolent governments in each country (see Appendix A for calculation details). The equilibrium period expected utility expression can be written as:

$$EU = E\frac{\Gamma(\tau)}{1-\sigma}\left(\left(\frac{M}{P_x}\right)^{\omega}\left(\frac{M^*}{P_m}\right)^{1-\omega}\zeta(\tau,\tau^*)\right)^{1-\sigma} \tag{16}$$

where the prices are expressed as the above solutions (11) and (12), and depend on expected productivity and money shocks. Expression (16) indicates that expected utility depends on the tariff rates set by the home and foreign governments. In particular, it is easily seen that for expected output levels, beginning at a zero home tariff, expected utility is increasing in the home tariff rate and (always) decreasing in the foreign tariff rate.

In the case of fully flexible prices, we may combine (15) with (16) to express expected utility solely as a function of productivity and tariff shocks:

$$U(\tau,\tau^*) = \frac{\Gamma(\tau_t)}{1-\sigma}\left(\mathcal{F}(\theta_t,\theta_t^*)\mathcal{H}(\tau_t,\tau_t^*)\right)^{1-\sigma} \tag{17}$$

where we define the following functions;

$$\mathcal{F}(\theta_t,\theta_t^*) = \theta_t^{\frac{(1+\psi)(\omega(\sigma+\psi)+(1-\sigma)\omega^*)}{(\sigma+\psi)\delta}}\,\theta_t^{*\,\frac{(1+\psi)^2(1-\omega)}{(\sigma+\psi)\delta}}$$

$$\mathcal{H}(\tau_t,\tau_t^*) = \Lambda(\tau_t,\tau_t^*)^{-\frac{\omega(\sigma+\psi)+(1-\sigma)\omega^*}{(\sigma+\psi)\delta}}\,\Lambda^*(\tau_t,\tau_t^*)^{-\frac{(1+\psi)(1-\omega)}{(\sigma+\psi)\delta}}\,\zeta(\tau,\tau^*)$$

In Section 4 below, we will use (17) to construct equilibrium value functions in the tariff game between countries.

3 Tariff setting

We first look at the motives for setting tariffs among non-cooperative, benevolent governments. The incentive to employ tariffs is critically dependent upon the timing of tariff setting, and the degree of price rigidity. Let us first assume that prices are fully flexible and tariffs are set by governments that internalize the price setting activities of firms.

3.1 Tariff setting with flexible prices

When prices are fully flexible, tariff setting must take account of both the direct effect on the country terms of trade and the indirect effect on home and foreign output through endogenous labor supply. From (9) and (16) above, we can easily show that holding home and foreign output constant, a home country tariff improves the home terms of trade, and increases home welfare. But the tariff will also affect domestic and foreign output. Note that from (15) above, home output with flexible prices may be written as:

$$Y_x = \left[\frac{\theta^{1+\psi}}{\Lambda(\tau,\tau^*)}\right]^{\frac{\psi+\sigma+\omega^*(1-\sigma))}{\Delta}} \left[\frac{\theta^{*(1+\psi)}}{\Lambda^*(\tau,\tau^*)}\right]^{\frac{(1-\omega)(1-\sigma))}{\Delta}} \tag{18}$$

The impact of a home tariff on home output depends on a mix of income and substitution effects. The tariff raises the domestic terms of trade, which increases the real wage and increases labor supply. But the rise in the terms of trade also increases consumption which reduces labor supply through an income effect. When $\sigma = 1$, we see from (13) and (18) that a home tariff reduces domestic output, but has no impact on foreign output. More generally, we can evaluate the impact of a tariff on Y_x, in the special case where home bias is symmetric across the two countries, so that $\omega^* = 1-\omega$, and evaluated at zero initial tariffs. We obtain:

$$\frac{dY_x}{d\tau}|_{\{\tau=\tau^*=0\}} = -Y_x \frac{(1-\omega)(\omega\psi(\sigma-1)+\omega(\sigma^2-1)+\psi+1)}{(\sigma+\psi)((2\omega-1)\sigma+\psi+2(1-\omega)))}$$

This may be positive or negative. In the case where $\sigma < 1$, it is possible that substitution effects are strong enough that the tariff increases home country's output. The effect of a home tariff on foreign output may be expressed as

$$\frac{dY_m}{d\tau}|_{\{\tau=\tau^*=0\}} = Y_m \frac{(1-\omega)(\sigma-1)(1+\omega\psi+\omega(\sigma-1))}{(\sigma+\psi)((2\omega-1)\sigma+\psi+2(1-\omega)))}$$

When $\sigma < 1$ this is negative, as the fall in the foreign terms of trade generates substitution effects in the opposite direction to those in the home country.

Whether the tariff increases or decreases home or foreign output, it is easy to show that the direct welfare benefit from terms of trade improvement always outweighs the indirect effects on output. Even under flexible prices and endogenous output, a country gains from imposing a small tariff, conditional on the zero tariff of the foreign country. Using (17) above, we can derive the impact of a tariff on home welfare, evaluated at $\tau = \tau^* = 0$, as:

$$\frac{dU(\tau,\tau^*)}{d\tau}|_{\{\tau=\tau^*=0\}} = \Lambda \frac{(1-\omega)(1+\omega\psi+\omega(\sigma-1))}{(2\omega-1)\sigma+\psi+2(1-\omega)}$$

where $\Lambda > 0$.

3.2 Tariff setting with pre-set prices

What are the incentives to levy tariffs in the economy with sticky prices? For this we need to be careful about the timing of tariffs. We assume that tariffs are levied at the end of a period, after prices have been set by firms. In addition, we assume that there is no commitment in tariff setting. Tariffs are set in the current period for this period only, assuming that the future trade authority sets its own tariffs. So the trade authority this period faces a static problem, setting tariffs once the prices have been set.

Here we have to distinguish between fixed and flexible exchange rates.[9] Under flexible exchange rates, we have M and M^* exogenous, and the exchange rate is, from (10)

$$S = \frac{1-\omega}{\omega^*}\frac{(1-\omega^*)(1+\tau^*)+\omega^*}{\omega(1+\tau)+(1-\omega)}\frac{M}{M^*}$$

From the point of view of the tariff authority, Y_x is taken as given, since P_x is fixed and M is outside of its control. But a tariff can tilt the terms of trade in its favour under flexible exchange rates. Utility, given output, is just captured by C, which from the authority's perspective, when exchange rates adjust to change the terms of trade, is

$$C = \left(\frac{M}{P_x}\frac{\omega(1+\tau)}{\delta_\omega}\right)^{\omega}\left(\frac{1-\omega}{\delta_\omega}\frac{\frac{M}{P_x}}{\frac{SP_m}{P_x}}\right)^{1-\omega} = \left(\frac{M}{P_x}\right)^{\omega}\left(\frac{M^*}{P_m}\right)^{1-\omega}\zeta(\tau,\tau^*) \tag{19}$$

This is increasing in τ, so the authority has an incentive to levy tariffs starting from a point of zero tariffs $\tau = 0$. In fact, it would want an infinite tariff, given the assumption of a unit elasticity of substitution across home and foreign goods, and no production of importable. To prevent this from happening in the sustainable tariff game, we will assume a maximum possible tariff rate of τ^H.

Now look at the same situation with fixed exchange rates. Here, we must be specific about the mechanism through which the exchange rate is pegged. This amounts to the question of which country adjusts its monetary policy to keep the exchange rate fixed. In both cases, we find that there under a fixed exchange rate, there is no incentive to levy a tariff, but the reasoning is different if we are considering a tariff for one country when the other country's monetary authority maintains the peg than if it is the domestic authority keeping the peg.

To be concrete, look at the case of a tariff set in the home country, and assume for now that the foreign monetary authority maintains the peg. Then from the point of view of the home tariff setter the movements in foreign money necessary to maintain an exchange rate peg have no consequences for home consumption, as can be seen from the middle equality in Equation

[9] An important assumption is that monetary policy is not set optimally to undo the impact of price stickiness. If this was the case, then Appendix D shows that the impact of tariffs would be the same as under fully flexible prices.

(19), when S is fixed. In addition, home output is pinned down by the home money stock and the preset prices, so neither home aggregate consumption or home labor supply is affected by endogenous movements in the foreign money stock.

Given this assumption, let us look at the incentive of the home monetary authority to levy a tariff under fixed exchange rates. When the exchange rate is fixed by the foreign country (where the foreign monetary authority adjusts M^* to keep S fixed), then from the definition of utility, we have:

$$C_x^{\omega} C_m^{1-\omega} = Y_x \left(\frac{(1+\tau)^{\omega}}{\omega(1+\tau)+1-\omega} \right) \left(\frac{P_x}{SP_m} \right)^{1-\omega} \omega^{\omega}(1-\omega)^{1-\omega} \tag{20}$$

This is decreasing in τ, starting at $\tau = 0$, given that S and Y_x are both fixed from the point of view of the tariff setter. Also, given fixed prices and domestic monetary policy set independently of the tariff M, home labor will be independent of τ. So there is no incentive to levy a tariff under fixed exchange rates, when the foreign monetary authority maintains the peg. Intuitively, the trade authority cannot affect its terms of trade under a fixed exchange rate regime, so the tariff only reduces its own welfare.

Alternatively, when the home authority maintains the peg, the tariff setter must take into account the endogenous movement in money supply in response to a tariff. Using (10), the home country must adjust M so as to keep S fixed. This will affect home consumption through (19). But it must also increase home output and employment, through (7). We show in Appendix A that, beginning at $\tau = 0$, the gains in the utility of consumption from a rise in the tariff rate are exactly offset by the rise in the disutility of employment, under a fixed exchange rate, when the home country takes the responsibility for the exchange rate peg. Thus, even in this case, with a fixed exchange rate, there is no incentive to levy a tariff for terms of trade gain.

From this discussion, even before we analyze the endogenous determination of tariffs, we conclude that the incentives to levy tariffs should be higher under a regime of freely floating exchange rates.

4 Self Enforcing Cooperative Tariffs

We now follow the procedure of Bagwell and Staiger (2003) in deriving a sequence of tariffs that are self-enforcing, and satisfy a set of incentive constraints in a repeated game between countries. The approach is to conjecture a sequence of sustainable tariffs $\tilde{\tau}_t$, $\tilde{\tau}_t^*$ which are no greater than the maximum possible tariff rate τ^H, and which give each authority a valuation $V(\tilde{\tau}_t, \tilde{\tau}_t^*)$ and $V^*(\tilde{\tau}_t, \tilde{\tau}_t^*)$. This tariff sequence is known to each authority, and each authority knows the full history of tariffs chosen in the past.

If any authority deviates from the sustainable path of tariffs in any period, and sets the maximum tariff, it gets utility $V^{cheat}(\tau^H, \tilde{\tau}_t^*)$ (for the home authority) or $V^{cheat}(\tau^H, \tilde{\tau}_t)$ (for the foreign authority). But following a deviation, then each authority reverts to a punishment phase, and sets the maximum possible tariff in all future periods. Since $\tau = \tau^* = \tau^H$ is a Nash equilibrium of the one-period tariff game, this punishment is credible. In general, as we show below, there may be a large number of sustainable tariffs in which neither authority has an incentive to cheat. As in Bagwell and Staiger (2003), we identify the lowest tariff rates which just offset the incentive for each country to defect on the self-enforcing cooperative tariff sequence.

However, given the timing of decision-making for the tariff authority, there is an extra complication. In the case of pre-set prices, the authorities set tariffs conditional on the existing prices. But they must take account of the way in which prices in the future are set. This means that, given sticky prices, the functional representation of the gains from cheating appear in different form than the future costs of cheating. Simply speaking, the authority internalizes the future price adjustment process, whereas in the current time period, prices are taken as given.[10]

4.1 Notation

The model has four exogenous state variables, the two states of monetary policy, and the two states of productivity. We define these as $z_t = \{M_t, M_t^*, \theta_t, \theta_t^*\}$. We define $z^t = \{z_0...z_t\}$ as a state history. In addition, from the perspective of the tariff authority at any time t, the state includes the preset prices P_{xt}, P_{mt}. Thus, sustainable tariffs will be conditioned on the state z_t and preset prices. Thus, the expanded state is defined as $\tilde{z}_t = \{z_t, P_{xt}, P_{mt}\}$.

We first characterize a sequence of *sustainable tariffs* $\tilde{\tau}(\tilde{z}_t)$, $\tilde{\tau}^*(\tilde{z}_t)$. This sequence is defined indirectly by a set of incentive constraints. First, for the home country tariff setter, define the one-period payoff from cheating on the sustainable tariff sequence at time t. Let $C\left(\tilde{z}_t, \tau^H, \tilde{\tau}_t^*(\tilde{z}_t)\right)$ and $Y\left(\tilde{z}_t, \tau^H, \tilde{\tau}_t^*(\tilde{z}_t)\right)$ respectively denote the consumption and output levels that are realized if the home tariff setter cheats and applies τ^H (the highest level of tariffs) while the foreign tariff setter remains on a sustainable tariff path $\tilde{\tau}_t^*(\tilde{z}_t)$. Then the current-period value of cheating is:

$$v^{CH}\left(\tilde{z}_t, \tau^H, \tilde{\tau}_t^*(\tilde{z}_t)\right) = \frac{C\left(\tilde{z}_t, \tau^H, \tilde{\tau}_t^*(\tilde{z}_t)\right)^{1-\sigma}}{1-\sigma} - \frac{Y_x\left(\tilde{z}_t, \tau^H, \tilde{\tau}_t^*(\tilde{z}_t)\right)^{1+\psi}}{\theta_t^{1+\psi}(1+\psi)}$$

and the value of cheating at time t in state $\tilde{z}_t$ for the home tariff-setter is

$$V^{CH}(\tilde{z}_t) = v^{CH}\left(\tilde{z}_t, \tau^H, \tilde{\tau}_t^*(\tilde{z}_t)\right) + \beta E_t V^N(\tilde{z}_{t+1}) \tag{21}$$

[10]In our main setup monetary policy does not perfectly undo the existing price rigidity. However, as described in Section D of the Appendix we show that self enforcing cooperative tariffs under optimal monetary policy simply replicate the flexible price outcome and tariffs are a-cyclical. Similarly, adopting an alternative timing protocol by which prices would be set before shocks realize but after tariffs are set would deliver identical results. Indeed, in this case, changes in tariffs would be taken into account by price setters in the very same way they would do under flexible prices, and the resulting sustainable tariffs would be a-cyclical.

where β is the tariff setter's discount factor, and $V^N(\tilde{z}_{t+1})$ is the continuation value, given that cheating has happened in the past. We define $V^N(\tilde{z}_t)$ as follows. The one-period payoff in the worst Nash equilibrium is defined as

$$v^N\left(\tilde{z}_t, \tau^H, \tau^{*H}\right) = \frac{C\left(\tilde{z}_t, \tau^H, \tau^{*H}\right)^{1-\sigma}}{1-\sigma} - \frac{Y_x\left(\tilde{z}_t, \tau^H, \tau^{*H}\right)^{1+\psi}}{\theta_t^{1+\psi}\left(1+\psi\right)}$$

Given this, the recursive form of $V^N(\tilde{z}_t)$ is written as

$$V^N(\tilde{z}_t) = v^N\left(\tilde{z}_t, \tau^H, \tau^H\right) + \beta E_t V^N(\tilde{z}_{t+1}) \tag{22}$$

That is, $V^N(\tilde{z}_t)$ is the value of being in the worst Nash equilibrium forever. The current-period value of being on the sustainable path of tariffs is defined as

$$v^C\left(\tilde{z}_t, \tilde{\tau}_t(\tilde{z}_t), \tilde{\tau}_t^*(\tilde{z}_t)\right) = \frac{C\left(\tilde{z}_t, \tilde{\tau}_t(\tilde{z}_t), \tilde{\tau}_t^*(\tilde{z}_t)\right)^{1-\sigma}}{1-\sigma} - \frac{Y_x\left(\tilde{z}_t, \tilde{\tau}_t(\tilde{z}_t), \tilde{\tau}_t^*(\tilde{z}_t)\right)^{1+\psi}}{\theta_t^{1+\psi}\left(1+\psi\right)}$$

Using this, we define the continuation value of being on the sustainable tariff path as

$$V^C(\tilde{z}_t) = v^C(\tilde{z}_t, \tilde{\tau}_t(\tilde{z}_t), \tau_t^*(\tilde{z}_t)) + \beta E_t V^C(\tilde{z}_{t+1}) \tag{23}$$

Equivalent definitions apply to the foreign tariff setters decision. Given the above definitions, a pair of sustainable tariff sequences $\tilde{\tau}(\tilde{z}_t)$, $\tilde{\tau}^*(\tilde{z}_t)$ is defined by the following conditions:

$$V^{CH}(\tilde{z}_t) \leq V^C(\tilde{z}_t) \tag{24}$$

$$V^{*CH}(\tilde{z}_t) \leq V^{*C}(\tilde{z}_t) \tag{25}$$

If (24) and (25) are continually satisfied, it is apparent that the sustainable path of tariffs $\tilde{\tau}(\tilde{z}_t)$, $\tilde{\tau}^*(\tilde{z}_t)$ is self-enforcing, since neither authority has an incentive to deviate. However, it apparent also that $\tilde{\tau}(\tilde{z}_t)$, $\tilde{\tau}^*(\tilde{z}_t)$ may not be unique. There may be many values of the sustainable path of tariffs which satisfy (24) and (25). In what follows, we adopt the approach of Bagwell and Staiger (2003) in choosing the lowest tariff sequence for which (24) and (25) are satisfied with equality and tariffs are positive. The exact procedure for obtaining this sequence is outlined below.

4.2 Self-enforcing cooperative tariffs with flexible prices

We first illustrate the result stated in the introduction; the cyclical nature of cooperative tariffs appears only when prices are sticky. To see this, we make the following assumptions regarding the money and productivity shocks. Specifically, we assume that

$$M_t = M_{t-1}(1+\mu_t), \;\; M_t^* = M_{t-1}^*(1+\mu_t^*), \;\; \{\mu_t, \; \mu_t^*\} \sim \text{i.i.d.}(0, \sigma_\mu^2) \tag{26}$$

$$\theta_t = \theta_{t-1}(1+\nu_t), \quad \theta_t^* = \theta_{t-1}^*(1+\nu_t^*), \quad \{\nu_t, \ \nu_t^*\} \sim \text{i.i.d.}(0, \sigma_\nu^2) \tag{27}$$

by which both productivity and money growth are i.i.d. processes. With flexible prices, Section 2 above showed that home utility may be written as

$$U(\tau_t, \tau_t^*) = \frac{\Gamma(\tau_t)}{1-\sigma}\left(\mathcal{F}(\theta_t, \theta_t^*)\mathcal{H}(\tau_t, \tau_t^*)\right)^{1-\sigma} \tag{28}$$

where $\Gamma(\tau_t)$, $\mathcal{F}(\theta_t, \theta_t^*)$, and $\mathcal{H}(\tau_t, \tau_t^*)$ are as defined above.

We first describe the set of *sustainable* tariffs under flexible prices. A sustainable tariff sequence is defined as $\{\tilde{\tau}_t, \tilde{\tau}_t^*\}$, $t = 0...,\infty$. Using the value function definition (21) along with (28), the expected utility from cheating along a sustainable path may be written as

$$EV^{CH}(\tilde{z}_t) = \frac{\Gamma(\tau^H)}{1-\sigma}\left(\mathcal{F}(\theta_t, \theta_t^*)\mathcal{H}(\tau^H, \tilde{\tau}_t^*)\right)^{1-\sigma} + \frac{\beta}{1-\beta E{\Xi_1}^{1-\sigma}}\frac{\Gamma(\tau^H)}{1-\sigma}\left(\mathcal{F}(\theta_t, \theta_t^*)\mathcal{H}(\tau^H, \tau^H)\right)^{1-\sigma} \tag{29}$$

where $\Xi_1 = (1+\nu_{t+1})^{\frac{(1+\psi)(\omega(\sigma+\psi)+(1-\sigma)\omega^*)}{(\sigma+\psi)\delta}}(1+\nu_{t+1}^*)^{\frac{(1+\psi)^2(1-\omega)}{(\sigma+\psi)\delta}}$. This expression uses the property that expected utility is homogeneous in productivity, and productivity shocks are i.i.d., so that $E_t(\Xi_{1t+1})^{1-\sigma}$ is constant.[11]

By contrast, the expected utility from remaining on the sustainable path is

$$EV^{S}(\tilde{\tau}_t, \tilde{\tau}_t^*) = \frac{\Gamma(\tilde{\tau}_t)}{1-\sigma}\left(\mathcal{F}(\theta_t, \theta_t^*)\mathcal{H}(\tilde{\tau}_t, \tilde{\tau}_t^*)\right)^{1-\sigma} + \frac{\beta}{1-\beta E{\Xi_1}^{1-\sigma}}\frac{\Gamma(\tilde{\tau}_{t+1})}{1-\sigma}\left(\mathcal{F}(\theta_t, \theta_t^*)\mathcal{H}(\tilde{\tau}_{t+1}, \tilde{\tau}_{t+1}^*)\right)^{1-\sigma} \tag{30}$$

Following (24) and (25), a sustainable path $\{\tilde{\tau}_{t+1}, \ \tilde{\tau}_{t+1}^*\}$ is self identified by the condition that (29) is no greater than (30). Canceling the term $\mathcal{F}(\theta_t, \theta_t^*)$ then gives us the condition

$$\frac{\Gamma(\tau^H)}{1-\sigma}\left(\mathcal{H}(\tau^H, \tilde{\tau}_t^*)\right)^{1-\sigma} + \frac{\beta}{1-\beta E{\Xi_1}^{1-\sigma}}\frac{\Gamma(\tau^H)}{1-\sigma}\left(\mathcal{H}(\tau^H, \tau^H)\right)^{1-\sigma}$$

$$\leq \frac{\Gamma(\tilde{\tau}_t)}{1-\sigma}\left(\mathcal{H}(\tilde{\tau}_t, \tilde{\tau}_t^*)\right)^{1-\sigma} + \frac{\beta}{1-\beta E{\Xi_1}^{1-\sigma}}\frac{\Gamma(\tilde{\tau}_{t+1})}{1-\sigma}\left(\mathcal{H}(\tilde{\tau}_{t+1}, \tilde{\tau}_{t+1}^*)\right)^{1-\sigma} \tag{31}$$

As noted above however, there may be many values of the sequences $(\tilde{\tau}_{t+1}, \tilde{\tau}_{t+1}^*)$ that satisfy (31). To make progress on this, we follow the selection procedure in Bagwell and Staiger (2003) in choosing the lowest values of $(\tilde{\tau}_{t+1}, \tilde{\tau}_{t+1}^*)$ which satisfy (31) and the analogous condition for the foreign country (conditional on both tariffs to be positive, see below).

To characterize the lowest sustainable tariff sequence, we establish first the following Lemma

Lemma 1 The tariff pair $(\tilde{\tau} = \tau^H, \tilde{\tau}^* = \tau^H)$ satisfies the incentive constraints (31) with equality.

Proof: When $\tilde{\tau} = \tau^H, \tilde{\tau}^* = \tau^H$, the right-hand and left-hand side of (31) are identical.

[11] We must also assume that expected utility converges, which requires $\beta E{\Xi_1}^{1-\sigma} < 1$.

Assumption 1. The discount factor ($\beta E {\Xi_1}^{1-\sigma}$) is high enough so that there exists a tariff solution ($\tilde{\tau},\ \tilde{\tau}^*$) strictly less than ($\tau^H,\ \tau^H$) that satisfies (31).

Since Assumption 1 is always satisfied as $\beta E {\Xi_1}^{1-\sigma} \to 1$, this ensures that there exists a discount factor that satisfies the Assumption.

We can then state the following:

Proposition 1. Given Assumption 1, along with Lemma 1, there exist values $\tilde{\tau}_t$ and $\tilde{\tau}_t^*$ strictly less than τ^H with the following properties.

a. If (31) is satisfied at a strict inequality at $\tilde{\tau}_t = \tilde{\tau}_t^* = 0$, then $\tilde{\tau}_t = \tilde{\tau}_t^* = 0$, is an equilibrium.

b. If (31) is violated at $\tilde{\tau}_t = \tilde{\tau}_t^* = 0$, then

(*i*) There exist values $\tilde{\tau}_t > 0$ and $\tilde{\tau}_t^* > 0$ strictly less than τ^H which satisfy (31) with strict equality.

(*ii*) $\tilde{\tau}_t = \tilde{\tau}_t^*$, so that countries chose identical tariff rates

(*iii*) $\tilde{\tau}_t = \tilde{\tau} = \tilde{\tau}^*$, so that tariffs are time invariant.

Proof: By Assumption 1, there exist tariffs less than τ^H which satisfy (31). Part a) follows immediately because it is always satisfied as $\beta E {\Xi_1}^{1-\sigma} \to 1$.

Part b) (*i*) follows because both the left-hand side and right-hand side of (31) are continuous in τ and τ^*, and if (31) is violated at $\tilde{\tau}_t = \tilde{\tau}_t^* = 0$, and given Lemma 1, it follows that there exist positive values of τ and τ^* strictly less than τ^H which satisfy (31) with equality. Condition (*ii*) holds because countries have identical discount factors (by assumption), home bias in preferences is symmetric, and productivity draws follow a random walk, so that productivity levels in each country cancel out on either side of the incentive constraints. Condition (*iii*) is implied by the fact that monetary shocks play no role in tariff setting with flexible prices, and again that productivity shocks impact equally on the two sides of the incentive constraint.

From Proposition 1, the sustainable tariff pair $\tilde{\tau} = \tilde{\tau}^* < \tau^H$ is described by the condition[12]

$$\frac{\Gamma(\tau^H)}{1-\sigma}\left(\mathcal{H}(\tau^H,\tilde{\tau})\right)^{1-\sigma} + \frac{\beta}{1-\beta E {\Xi_1}^{1-\sigma}}\frac{\Gamma(\tau^H)}{1-\sigma}\left(\mathcal{H}(\tau^H,\tau^H)\right)^{1-\sigma}$$

$$\leq \frac{\Gamma(\tilde{\tau})}{1-\sigma}\left(\mathcal{H}(\tilde{\tau},\tilde{\tau})\right)^{1-\sigma} + \frac{\beta}{1-\beta E {\Xi_1}^{1-\sigma}}\frac{\Gamma(\tilde{\tau}_{t+1})}{1-\sigma}\left(\mathcal{H}(\tilde{\tau},\tilde{\tau})\right)^{1-\sigma} \quad \text{with equality when } \tilde{\tau} > 0 \tag{32}$$

[12] Note also that because $\tilde{\tau}$ is equal across countries and countries are symmetric, it achieves the highest level of world utility among the set of sustainable tariffs.

Hence, with flexible prices, protectionism is a-cyclical in our baseline model. Even though countries are subject to random productivity shocks, the productivity shocks affect the costs and benefits of deviating from the sustainable path in the same way. Thus, the incentive towards increased protectionism is unaffected. Note that our result contrasts with Bagwell and Staiger (2003), who obtain counter-cyclical tariffs under flexible prices, because we consider a micro-founded utility function that depends positively on consumption and negatively on labor, while the objective function of tariff setters in Bagwell and Staiger (2003) is national income.

4.3 Sustainable tariffs with pre-set prices

When prices are pre-set, the set of sustainable tariffs is characterized in a different manner, and critically, the selected sustainable tariff sequence will be time-varying, depending on the outcome of productivity and money shocks (again we are assuming that monetary policy is not set optimally to replicate the flexible price equilibrium). A key feature of the determination of tariffs is that the policy-maker in any time period takes the prices as pre-set, so that output is independent of tariffs, within the period of pre-set prices. Hence, while monetary policy shocks will affect output and the gains from cheating by affecting demand, productivity shocks will not. And although technology shocks affect the disutility of labor in the one-period utility of cheating, they affect the one-period utility of remaining on a sustainable path in exactly equivalent ways, so the two effects cancel out. However, productivity shocks have a permanent effect on productivity levels, as described by (27) above, so these shocks will impact on future expected utility, and as such affect the costs of cheating on any sustainable tariff path.

Again, as in the previous section, we focus on the lowest tariff sequence among all possible sustainable tariff sequences. The key difference in this case however is that the selected self-enforcing tariffs will generally be time-varying.

In order to explore this trade-off, we compute the value functions faced by the tariff setters in each country when tariffs are set conditional on pre-set prices. To begin, we have the value of being in the worst Nash equilibrium as described by (22). This may be written more explicitly as

$$V^N(\tilde{z}_t) = \frac{1}{1-\sigma}\left(\left(\frac{M_t}{P_{xt}}\right)^{\omega}\left(\frac{M^*}{P_{mt}}\right)^{1-\omega}\zeta(\tau^H,\tau^H)\right)^{1-\sigma} - \frac{1}{1+\psi}\left(\frac{M_t}{\theta_t P_{xt}}\right)^{1+\psi} + \beta E_{t-1}V^N(\tilde{z}_{t+1}) \tag{33}$$

where the term $\zeta(\tau^H,\tau^H)$ indicates that both countries set the highest possible tariff τ^H.

Taking expectations of (33) in $t-1$, we can use the property (17) in combination with the equilibrium home and foreign prices (11) and (12), to express (33) as

$$E_{t-1}V_t^N(\tilde{z}_t) = \frac{\Gamma(\tau^H)}{1-\sigma}\left(\mathcal{F}(\theta_{t-1},\theta^*_{t-1})\mathcal{H}_1(\tau^H,\tau^H)\right)^{1-\sigma} E_{t-1}\left(\Xi_t\right)^{1-\sigma} + \beta E_{t-1}V^N(\tilde{z}_{t+1}) \tag{34}$$

where $\mathcal{F}$ is defined as before, and the function $\mathcal{H}_1(\tau^H, \tau^H)$ is a function of tariffs and the distribution of money growth and productivity shocks, given in the Appendix, and the expression Ξ_t represents a term in expected monetary shocks, defined as $\Xi_t = (1+\mu_t)^{\omega}(1+\mu_t^*)^{1-\omega}$.

Given the form of (33), we conjecture that $E_{t-1}V(\tilde{z}_t) = A_N \mathcal{F}(\theta_{t-1}, \theta_{t-1}^*)$, where A_N is a constant. We can show that

$$A_N = \frac{\frac{\Gamma(\tau^H)}{1-\sigma}\mathcal{H}_1(\tau^H, \tau^H)^{1-\sigma} E_{t-1}\left(\Xi_t\right)^{1-\sigma}}{1-\beta E_{t-1}\Xi_1^{1-\sigma}} \tag{35}$$

where Ξ_1 is as defined above.

Note that given the assumption of i.i.d. shocks to money and productivity growth, both $E_{t-1}(\Xi_t)^{1-\sigma}$ and $E_{t-1}(\Xi_{1t})^{1-\sigma}$ are constant, and can thus be represented as unconditional expectations $E(\Xi)^{1-\sigma}$ and $E(\Xi_1)^{1-\sigma}$. It follows that A_N is constant, as conjectured.

Now, conjecture the existence of a sequence of time-varying sustainable tariffs $\tilde{\tau}_t, \tilde{\tau}_t^*$. If the home country were to cheat on this sustainable path, it would obtain the value at time t as:

$$V^{CH}(\tilde{z}_t) = \left(\left(\frac{M_t}{P_{xt}}\right)^{\omega}\left(\frac{M_t^*}{P_{mt}}\right)^{1-\omega}\zeta(\tau^H, \tilde{\tau}_t^*)\right)^{1-\sigma} - \frac{\left(\frac{M_t}{P_{xt}}\right)^{1+\psi}}{1+\psi} + \beta A_N\left(\mathcal{F}(\theta_t, \theta_t^*)\right)^{1-\sigma} \tag{36}$$

where implicitly we are assuming that P_{xt} and P_{mt} are pre-set by firms on the assumption that the conjectured sustainable sequence of tariffs $\tilde{\tau}_t, \tilde{\tau}_t^*$ is in place.

Using the same logic, we may derive the full evaluation of remaining on a sustainable path as:

$$V^{S}(\tilde{z}_t) = \left(\left(\frac{M_t}{P_{xt}}\right)^{\omega}\left(\frac{M_t^*}{P_{mt}}\right)^{1-\omega}\zeta(\tilde{\tau}_t, \tilde{\tau}_t^*)\right)^{1-\sigma} - \frac{\left(\frac{M_t}{P_{xt}}\right)^{1+\psi}}{1+\psi} + \beta A_S\left(\mathcal{F}(\theta_t, \theta_t^*)\right)^{1-\sigma} \tag{37}$$

where it can be shown that

$$A_S = \frac{E\left(\frac{\Gamma(\tilde{\tau}_t)}{1-\sigma}\mathcal{H}_1(\tilde{\tau}_t, \tilde{\tau}_t)^{1-\sigma}\Xi_t^{1-\sigma}\right)}{1-\beta E\Xi_1^{1-\sigma}} \tag{38}$$

The key difference between (35) and (38) is that the conjectured sequence of sustainable tariffs $\{\tilde{\tau}_t, \tilde{\tau}_t^*\}$ is now stochastic. But the value function conjecture is only verified if A_S is constant, which requires that each sequence of possible sustainable tariffs represent time-invariant functions of the shocks z_t. Since shocks are i.i.d. this would ensure that $\{\tilde{\tau}_t, \tilde{\tau}_t^*\}$ are also i.i.d., verifying the conjecture. We will show below that if the incentive constraint for the existence of a sustainable sequence of tariffs is satisfied, this in fact ensures that the existence of a sustainable path of tariffs $\{\tilde{\tau}_t, \tilde{\tau}_t^*\}$ which is i.i.d.

Both (36) and (37) represent value functions pertaining to the home country, but equivalent functions taking analogous forms may be derived for the foreign country.

4.4 The dynamic sustainability conditions

The conditions (24) and (25) may now be applied using (36) and (37) to characterize the set of equilibrium sequences of sustainable tariffs. In particular, the incentive constraint for the home country is

$$V(\tilde{z}_t)^N \leq V(\tilde{z}_t)^S$$

or

$$\frac{1}{1-\sigma}\left(\left(\frac{M_t}{P_{xt}}\right)^{\omega}\left(\frac{M_t^*}{P_{mt}}\right)^{1-\omega}\zeta(\tau^H,\tilde{\tau}_t^*)\right)^{1-\sigma}+\beta A_N\left(\mathcal{F}(\theta_t,\theta_t^*)\right)^{1-\sigma}$$

$$\leq\frac{1}{1-\sigma}\left(\left(\frac{M_t}{P_{xt}}\right)^{\omega}\left(\frac{M_t^*}{P_{mt}}\right)^{1-\omega}\zeta(\tilde{\tau}_t,\tilde{\tau}_t^*)\right)^{1-\sigma}+\beta A_s\left(\mathcal{F}(\theta_t,\theta_t^*)\right)^{1-\sigma} \tag{39}$$

Note that we have dropped the disutility of labor terms in the one-period utility for both sides of the incentive constraint because they cancel out. This is because $H=\frac{M}{P_x}$ and the price is set on the assumption that the sustainable path is maintained, so employment and the disutility term for the current-period utility is the same for $V^N(\tilde{z}_t)$ and $V^S(\tilde{z}_t)$.

Now, we can use the homogeneity of the value function to cancel out the term $\mathcal{F}(\theta_{t-1},\theta_{t-1}^*)$ from both sides of (39), and again use the properties of the pricing equations (11) and (12) to restate (39) as:

$$\frac{1}{1-\sigma}\Xi_t^{1-\sigma}(E_{t-1}\mathcal{J}_1(\tilde{\tau}_t,\tilde{\tau}_t^*))^{1-\sigma}\zeta(\tau^H,\tilde{\tau}_t^*)^{1-\sigma}+\beta A_N\Xi_{1t}^{1-\sigma}$$

$$\leq\frac{1}{1-\sigma}\Xi_t^{1-\sigma}(E_{t-1}\mathcal{J}_1(\tilde{\tau}_t,\tilde{\tau}_t^*))^{1-\sigma}\zeta(\tilde{\tau}_t,\tilde{\tau}_t^*)^{1-\sigma}+\beta A_S\Xi_{1t}^{1-\sigma} \tag{40}$$

where the function $\mathcal{J}_1$ is defined in the Appendix. An analogous condition holds for the foreign country, representing (25), and may be written as

$$\frac{1}{1-\sigma}\Xi_t^{*1-\sigma}(E_{t-1}\mathcal{J}_1^*(\tilde{\tau}_t,\tilde{\tau}_t^*))^{1-\sigma}\zeta^*(\tau^H,\tilde{\tau}_t^*)^{1-\sigma}+\beta A_N^*\Xi_{1t}^{*1-\sigma}$$

$$\leq\frac{1}{1-\sigma}\Xi_t^{*1-\sigma}(E_{t-1}\mathcal{J}_1^*(\tilde{\tau}_t,\tilde{\tau}_t^*))^{1-\sigma}\zeta^*(\tilde{\tau}_t,\tilde{\tau}_t^*)^{1-\sigma}+\beta A_S^*\Xi_{1t}^{*1-\sigma} \tag{41}$$

These expressions differ from (31) in two key ways. First, technology shocks no longer cancel out on both sides of the incentive constraints, and secondly, monetary policy shocks now affect the one period gains from cheating. In addition, while Proposition 1 showed that tariffs were identical in the symmetric equilibrium, now the two types of shocks will affect (41) differs from (40) differently, due to home bias in preferences, since $\omega\geq\omega^*$.

Any pair of tariffs functions $\tilde{\tau}_t(\tilde{z}_t), \tilde{\tau}_t^*(\tilde{z}_t)$ that satisfies conditions (40) and (41) represents a sequence of sustainable tariffs. The equations indicate that the incentive constraints will depend on shocks to money growth Ξ_t and to productivity growth Ξ_{1t}. Moreover, we can now confirm that there exists a sustainable tariff sequence that is i.i.d., since, self-evidently, (40) and (41) depend only on current valued shocks, and since the shocks themselves are i.i.d.

Again, as in the case of fully flexible prices, we follow Bagwell and Staiger (2003) in focusing on the lowest sustainable tariff sequence. To identify this, we follow the same procedure as in sub-section 4.2, except now the tariffs will be time-varying.

We first note that Lemma 1 applies equally to (40) and (41), since the two conditions must hold with equality when $\tilde{\tau}_t = \tilde{\tau}_t^* = \tau^H$. Hence, the static Nash equilibrium tariff rates remain sustainable in the sticky price case. In order to characterize the selected tariff functions $\tilde{\tau}(z_t)$ and $\tilde{\tau}^*(z_t)$ we extend Assumption 1.

Assumption 2 The discount factor $\beta E {\Xi_1}^{1-\sigma}$ is sufficiently high so, that for each state of the world z_t, there exists a set of time-invariant tariff functions strictly less than τ^H that satisfy (40) and (41) with strict inequality.

Assumption 2 ensures that in the case of sticky prices, the distribution of z_t is such that there always exists sustainable tariffs below the static Nash equilibrium. As before, Assumption 2 is guaranteed to exist for some discount factor as it is always satisfied as $\beta E {\Xi_1}^{1-\sigma} \to 1$.
We now state:

Proposition 2. Given Assumption 2, there exists a pair of tariff functions that satisfy

a. $\tilde{\tau}(z_t) = 0$ (respectively $\tilde{\tau}^*(z_t) = 0$) if (40) (resp. (41)) are satisfied with strict inequality at $\tilde{\tau}(z_t) = 0$ (resp. $\tilde{\tau}^*(z_t) = 0$).

otherwise:

b. $0 < \tilde{\tau}(z_t) < \tau^H$, $0 < \tilde{\tau}^*(z_t) < \tau_H$ satisfy (40) and (41) with equality.

Proof:

Part a. follows immediately given Assumption 2 since there is always a discount factor that will sustain free trade for one or both countries for any realization of shocks. Part b. follows because for any realization of shocks, (40) and (41) are continuous in $\tilde{\tau}(z_t)$ and $\tilde{\tau}^*(z_t)$ so, given Assumption 2, if the incentive constraints are violated at $\tilde{\tau}(z_t) = 0$ ($\tilde{\tau}^*(z_t) = 0$) there must exist functions which satisfy (40) and (41) with equality.

Proposition 2 may be summarized by the conditions

$$\frac{1}{1-\sigma}\Xi_t^{1-\sigma}(E_{t-1}\mathcal{J}_1(\tilde{\tau}_t, \tilde{\tau}_t^*))^{1-\sigma}\zeta(\tau^H, \tilde{\tau}_t^*)^{1-\sigma} + \beta A_N \Xi_{1t}^{1-\sigma}$$

$$\leq \frac{1}{1-\sigma}\Xi_t^{1-\sigma}(E_{t-1}\mathcal{J}_1(\tilde{\tau}_t,\tilde{\tau}_t^*))^{1-\sigma}\zeta(\tilde{\tau}_t,\tilde{\tau}_t^*)^{1-\sigma}+\beta A_S\Xi_{1t}^{1-\sigma} \text{ with equality when } \tilde{\tau}_t > 0 \tag{42}$$

$$\frac{1}{1-\sigma}\Xi_t^{*1-\sigma}(E_{t-1}\mathcal{J}_1^*(\tilde{\tau}_t,\tilde{\tau}_t^*))^{1-\sigma}\zeta^*(\tau^H,\tilde{\tau}_t^*)^{1-\sigma}+\beta A_N^*\Xi_{1t}^{*1-\sigma}$$

$$= \frac{1}{1-\sigma}\Xi_t^{*1-\sigma}(E_{t-1}\mathcal{J}_1^*(\tilde{\tau}_t,\tilde{\tau}_t^*))^{1-\sigma}\zeta^*(\tilde{\tau}_t,\tilde{\tau}_t^*)^{1-\sigma}+\beta A_S^*\Xi_{1t}^{*1-\sigma} \text{ with equality when } \tilde{\tau}_t^* > 0 \tag{43}$$

These conditions implicitly define the equilibrium, time-invariant tariff functions $\tilde{\tau}(\tilde{z}_t),\ \tilde{\tau}(\tilde{z}_t)^*)$. Unlike Proposition 1, we note that the tariffs are in general time-varying, and will differ across countries. Moreover, there may exist a realization of shocks where tariffs are zero in one or both countries. In our calibration below however, we find that this never occurs for the chosen distribution of shocks. Accordingly, in the following discussion we assume that tariffs satisfy part b) of Proposition 2. [13]

Although (42) and (43) are affected by both money growth and productivity growth, these shocks have very different effects on the equilibrium tariff responses $\tilde{\tau}_t(\tilde{z}_t), \tilde{\tau}_t^*(\tilde{z}_t)$. A money growth shock affects the immediate benefits from cheating, affecting current-period utility but not expected future utility. By contrast, a productivity shock has no immediate effects on the current benefits from cheating, since it impacts only the disutility of labor supply, and it does so in an equal way for both the value of cheating and the value of remaining on the sustainable path. But a productivity growth shock affects the expected future path of utility, both for the Nash "punishment" path, and the expected future utility along the sustainable path.

How will this difference in the time dimension of shocks affect the response of equilibrium sustainable tariffs? The critical feature of (42) and (43) is that there is a current benefit from cheating, but this brings future costs of cheating. As a result, we have:

$$\zeta(\tau^H,\tilde{\tau}_t^*) > \zeta(\tilde{\tau}_t,\tilde{\tau}_t^*)$$

and

$$A_S > A_N$$

This implies that a money growth shock will lead the response of equilibrium tariffs to move in a different direction than will a productivity growth shock. To make this concrete, assume that $\sigma < 1$. Then a money growth shock will raise the first expression on both the left and right-hand side of (40), but will also raise the incentive to cheat, since it raises the left-hand expression more

[13] Appendix E describes a case where the equilibrium involves one country with a zero tariff while the other country has a positive tariff.

than the right-hand expression. As a result, given a higher incentive to cheat on the sustainable tariff sequence, equilibrium sustainable tariffs must rise to offset this incentive.

By the same logic, when $\sigma < 1$, a productivity shock will raise the future cost of cheating more than it raises the future benefits of remaining on the sustainable path. Hence, there is a reduced incentive to cheat, and equilibrium sustainable tariffs have to fall to reflect this.

When $\sigma > 1$, this logic is reversed, and monetary growth shocks will lead to a fall in the sustainable equilibrium tariff rates, while productivity growth shocks will lead to a rise in sustainable tariffs.

Given this discussion, we conclude:

Proposition 3. (*i*) When $\sigma < 1$ (resp. > 1), a positive shock to the growth rate of M or M^* raises (reduces) the benefits of cheating on the sustainable tariff policy, leaving the costs of cheating unchanged, leading to an increase (decrease) in the equilibrium sustainable tariff. (*ii*) When $\sigma < 1$ (resp. > 1), a rise in home or foreign productivity θ or θ^* growth raises (reduces) the costs of cheating on the sustainable tariff equilibrium, leaving the benefits of cheating unchanged. As a result, the equilibrium sustainable tariff falls (rises).

Proof: follows from the discussion above.

Thus, we find that the cyclical pattern of tariffs depend on the source of shocks and the value of σ. We consider the case $\sigma < 1$ as a baseline. While macro and asset pricing models typically assume $\sigma > 1$, it is typical in trade models, that abstract from intertemporal asset trade, to assume $\sigma = 0$. In any case, we present the $\sigma > 1$ case below, following our main discussion. Thus, in the case $\sigma < 1$, protectionism is pro-cyclical when the business cycle is driven by monetary (or demand) shocks, but counter-cyclical when productivity shocks are the main sources of business cycle variation. The pattern is reversed when $\sigma > 1$.

4.5 Calibration and simulations

We now calibrate the baseline model with the following parameters. We set $\sigma = 0.5$ and $\psi = 2$ and assume a moderate degree of home bias in preferences, so that $\omega = 1 - \omega^* = 0.7$. We also set the maximum feasible tariff rate at sixty-two percent, so that $\tau^H = .62$. This is the average tariff rate estimated by Ossa (2014) that would apply in a full scale world "tariff war", and hence represents the appropriate limit for the static Nash equilibrium tariff rate within our model – the implications of varying τ^H are also explored below.

Given this, we choose a discount factor $\beta = \beta^*$ so that the mean tariff rate in the sustainable equilibrium in the baseline case is 10 percent, which is approximately the average degree of trade restriction (including both tariff and non-tariff barriers), reported by UNCTAD (2013) for advanced economies. This leads to a value of $\beta = \beta^* = 0.6$. We then choose independent money and productivity shocks in the home and foreign country, assuming a standard deviation of 2 percent for each shock.[14]

Figure 1 illustrates the relationship between tariffs and *ex-post* productivity and monetary policy shocks, under various scenarios. Figure 1a illustrates the variation in home and foreign tariffs as a function of home country productivity. The blue line shows the baseline case, and the red line shows the impact of an unanticipated home money growth shock. Absent the money shock, tariff rates are effectively equal in the two countries, so that tariffs respond in the same way to productivity shocks in either country. After a home money growth shock, the tariff schedule shifts up in both countries, but to a greater extent in the home country, since the money growth shock gives a greater incentive for the home country to deviate from the sustainable tariff equilibrium.

Figure 1b shows the effect of differential discounting among the two countries. For this Figure, we set $\beta = 0.5$ and $\beta^* = 0.6$. Thus, the home country is more impatient than the foreign country. As expected, the home country sets a higher tariff rate than the foreign country, for any pattern of monetary and productivity shocks, which reflects the higher relative valuation of the current benefits from cheating, compared to the patient country. However, the response to productivity and money growth shocks is qualitatively the same as in Figure 1a.

Figure 1c illustrates an opposite parametrization for the elasticity of intertemporal substitution (IES), where $\sigma = 1.5$. As explained above, in this case, the cyclical pattern of tariffs is the reverse of that described in the previous Figures. Tariffs are *increasing* in productivity shocks, and *decreasing* in money shocks.

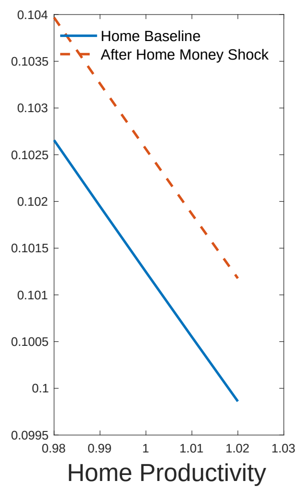
Table 1 reports the mean and coefficient of variation of tariff rates in the baseline case and under various alternative scenarios. The most important implication of the Table is the large impact of productivity growth shocks on the average tariff levels. In the absence of variation in productivity growth, average tariff rates would be 56 percent, close to the maximum Nash tariff levels. By contrast, variance in monetary shocks has almost no effect on the mean tariff levels.

What explains the large impact of productivity variance on tariff levels? The key intuition relates to the impact of productivity uncertainty on sustainable tariffs, and through this channel, on equilibrium nominal prices. The intuition stems from Equations (11) and (12) above, which give the pre-set prices in the presence of money and productivity growth shocks. With i.i.d money and productivity shocks across countries, an increase in the volatility of either shock leads to a lower price set by firms in each country. This is particularly more important for productivity shocks. A fall in the level of pre-set prices then increases the continuation value of the game for each country, whether on the sustainable path or in the Nash punishment equilibrium. However,

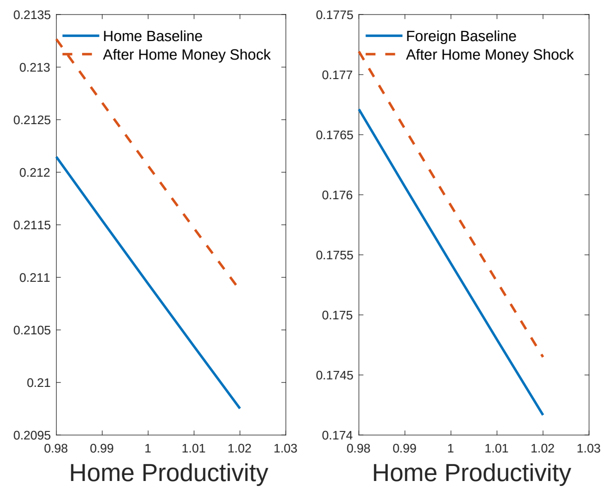
[14] The model is solved assuming each shock takes on a five point distribution with equal probabilities, with a standard deviation of 2 percent.

Figure 1: Tariff schedules under alternative calibrations

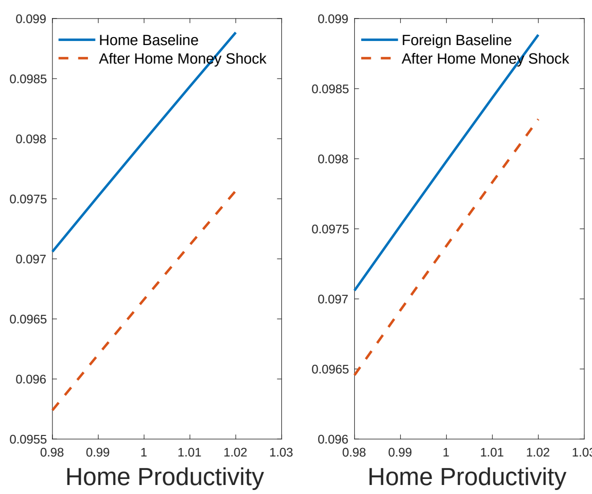
(a) Baseline case



(b) Home impatient



(c) $\sigma = 1.5$



Note: Baseline calibration is as follows: $\omega = 1 - \omega^* = 0.7$, $\tau^H = 0.62$, $\beta = \beta^* = 0.6$, $\sigma = 0.5$, $\psi = 2$. In home impatient case, $\beta = .5$.

Table 1: Tariffs in the simulated model

		Mean	Coefficient of variation	
Baseline		10.2%	3%	
Flexible prices		62%	0	
Money shocks		56%	0.6%	
Productivity shocks		10.2%	2.3%	
No shocks		56%	0	
High punishment $\tau^H = 1$		6.2%	7.1%	
$\sigma = 1.5$		9.7%	2.8%	
	Home mean	Foreign mean	Home CV	Foreign CV
Large economy	6.7%	3.9%	4.8%	5.7%
Home impatient	21.2%	17.6%	1.2%	1.6%%

Calibration is as follows. $\omega = 1 - \omega^* = 0.7$, $\tau^H = 0.62$, $\beta = \beta^* = 0.6$, $\sigma = 0.5$, $\psi = 2$. In large economy case $\omega = 0.725, \omega^* = 0.325$. In home impatient case, $\beta = .5$.

there is a critical difference between the sustainable path and the Nash punishment equilibrium in that, under the sustainable path, tariff rates in the future are uncertain, since they respond to realized productivity shocks. This implies that the effect of uncertain productivity on price levels is much stronger for the continuation values under the sustainable path than the analogous effect under the Nash punishment path. As a result, a rise in the variance of productivity shocks makes the continuation value in the sustainable path more attractive, allowing for a lower mean level of sustainable tariffs required to offset the incentive to cheat in any period. Hence, a higher variance of productivity shocks reduces the mean level of tariffs in the sustainable equilibrium.

However, the above result is critically dependent on the price setting assumption. With fully flexible prices, as captured by (30), tariff rates are constant, but the level of tariffs depends on the distribution of productivity shocks due to effect of this distribution on the discount factor. In fact in this case, uncertainty in productivity has the *opposite* effect on the level of tariffs in a sustainable equilibrium: a higher variance of productivity shocks reduces the term $E{\Xi_1}^{1-\sigma}$ in the effective discount factor. This reduces the expected benefit from the continuation game, and reduces the cost of cheating. As a result, with flexible prices, productivity uncertainty *raises* the mean tariff rate in a sustainable equilibrium.[15]

Table 1 also illustrates the effect of differences in the discount factor between countries. When the home country discount factor falls from 0.6 to 0.5, the mean sustainable tariff rate rises substantially for the home country. The mean tariff rate rises as well for the foreign country, although not as much as in the home country, even though the foreign country's discount factor is unchanged.

Finally, country size plays an interesting role. Country size may be captured by variations in ω and ω^*. In particular, allowing for a rise in both ω and ω^*, implies that the home country

[15]Note that, in the baseline calibration, sustainable tariffs under fully flexible prices are equal to the maximum Nash tariff rates. With a higher discount factor, sustainable tariff rates would be lower. In that case, it is easy to see from (30) that a rise in the variance of productivity shocks raises the level of sustainable tariffs.

produces a larger share of the world goods than the foreign country, and thus is the larger country.[16] Here we set $\omega = 0.725$ and $\omega^* = 0.325$. In this case we find that the mean tariff rates falls for both countries, although the home country (the larger country) tariff rate remains higher than that of the foreign country.

5 Non-unit trade elasticity

5.1 Model summary

The results in the paper so far pertain to the case of a unitary elasticity of substitution between home and foreign goods. While this parameter assumption is common in many open economy models, it is admittedly a special case. Here we extend the model to allow for a non-unitary trade elasticity. We describe the essential features of the more general model, and illustrate how the main results on the degree of protection and its cyclical pattern differ from the baseline case.

We assume that preferences remain identical except that the consumption aggregator takes a more general CES form:

$$C = \left(\omega^{\frac{1}{\lambda}} C_x^{1-\frac{1}{\lambda}} + (1-\omega)^{\frac{1}{\lambda}} C_m^{1-\frac{1}{\lambda}} \right)^{\frac{1}{1-\frac{1}{\lambda}}}$$

which implies the true price index:

$$P = \left(\omega P_x^{1-\lambda} + (1-\omega) \left((1+\tau) S P_m \right)^{1-\lambda} \right)^{\frac{1}{1-\lambda}}$$

A similar function applies to the foreign country, with the same elasticity λ but with weights ω^* and $1-\omega^*$. Labor supply, the production technology, and the cash-in-advance specification are all the same as before. In the case of preset prices, again, firms set prices one period ahead.

First, we can represent expected utility as:

$$E_{t-1} \frac{1}{1-\sigma} C_t^{1-\sigma} \Gamma_t \tag{44}$$

where $\Gamma_t = \left(1 - \frac{(1-\sigma)(1-\tau_t \left(\frac{S_t P_{m,t}}{P_t} \right)^{(1-\lambda)} (1-\omega)}{(1+\psi)(1+\tau_t)^{\lambda}} \right)$. As in the previous section, a sustainable sequence of tariff rates $\hat{\tau}_t,\ \hat{\tau}_t{}^*$ is defined as any sequence in which neither tariff setter has an incentive to cheat and set the maximum tariff in any period to exploit its terms-of-trade advantage, when this will be followed by each country setting the maximum Nash tariff rate forever in the future.[17] As

[16] Obstfeld and Rogoff (1995) develop a model in which country size (population) has the same measure of a country's share of differentiated traded goods. We follow that interpretation here.

[17] Unlike the case of the previous section, when $\lambda > 1$, the one-period optimal unrestricted tariff rate will not be unbounded in general. But in our quantitative analysis, we maintain the assumption that the maximum possible tariff is τ^N. We check in each case that the optimal unrestricted tariff rate exceeds τ^N.

before, we seek to select the lowest tariffs that satisfy the sustainability conditions with equality.

5.2 Tariffs with flexible prices and non-unitary trade elasticity

It is convenient to begin first with the fully flexible price model. In this case we can represent the partial solution for the consumption of the home country as:

$$C_t = \theta^{\frac{1+\psi}{\sigma+\psi}} P_t^{\frac{1+\lambda\psi}{\sigma+\psi}} \left(\omega + \omega^* \left(\frac{S_t P_{m,t}}{P_{x,t}} \right)^{1-\lambda} (1+\tau_t)^{-\lambda} \right)^{\frac{-\psi}{\sigma+\psi}} \tag{45}$$

An analogous equation holds for C_t^*. In addition, with flexible price case, the terms of trade, defined as $\mathcal{S}_t = \frac{S_t P_{m,t}}{P_{x,t}}$, are

$$\mathcal{S}_t = \left(\frac{\theta_t}{\theta_t^*} \right)^{\frac{1+\psi}{\Delta_1}} \left(\frac{P_t}{P_t^*} \right)^{-\frac{1-\lambda\sigma}{\Delta_1}} \left(\frac{1+\tau_t^*}{1+\tau_t} \right)^{\frac{\lambda\sigma}{\Delta_1}} \Phi_t^{-\frac{\psi}{\Delta_1}} \tag{46}$$

where $\Delta_1 = 1 - \sigma + \lambda(\sigma + \psi)$ and $\Phi_t = \frac{\omega(1+\tau_t)^\lambda + \omega^* \mathcal{S}_t^{1-\lambda}}{(1-\omega)+(1-\omega^*)\mathcal{S}_t^{1-\lambda}(1+\tau_t^*)^\lambda}$. Equations (45) and (46) give implicit solutions for $C_t(\theta_t, \theta_t^*, \tau_t, \tau_t^*)$ and $\mathcal{S}_t(\theta_t, \theta_t^*, \tau_t, \tau_t^*)$, given realizations for productivity and home and foreign tariff rates. We may express the valuation for the home country following the sustainable sequence of tariff rates as:

$$v^s(\theta_t, \theta_t^*) = \frac{1}{1-\sigma} C_t(\theta_t, \theta_t^*, \hat{\tau}_t, \hat{\tau}_t^*)^{1-\sigma} \Gamma((\theta_t, \theta_t^*, \hat{\tau}_t, \hat{\tau}_t^*)) + E_t \beta v^s(\theta_{t+1}, \theta_{t+1}^*) \tag{47}$$

In like manner, we may define the value of cheating for the home country as:

$$v^{ch}(\theta_t, \theta_t^*) = \frac{1}{1-\sigma} C_t(\theta_t, \theta_t^*, \hat{\tau}_N, \hat{\tau}_t^*)^{1-\sigma} \Gamma((\theta_t, \theta_t^*, \hat{\tau}_t, \hat{\tau}_t^*)) + E_t \beta v^N(\theta_{t+1}, \theta_{t+1}^*) \tag{48}$$

where $v^N(\theta_t, \theta_t^*)$ represents the the value of being in the maximum tariff Nash equilibrium forever.

The tariff sequence is then defined by:

$$v^{ch}(\theta_t, \theta_t^*) = v^s(\theta_t, \theta_t^*) \tag{49}$$

Unlike before, there are no closed form expressions that solve (49), and the analogous equation for the foreign country. We therefore solve the system numerically, using the calibration of Section 4, except for the value of the trade elasticity λ.

We show numerically that the following results hold:

1. the higher λ, the lower the selected sustainable tariffs.
2. when $\theta_t = \theta_t^*$, (*i.e.* when all productivity shocks are global), the selected sustainable tariff rates are constant.

3. the selected sustainable tariff rates are increasing (decreasing) in own (foreign) country productivity shocks.

The intuition behind these results can be developed as follows. First, with a higher trade elasticity, the response of the terms of trade to an increase in tariff is lessened, thus reducing the incentive to deviate, and as a result lowering the required tariff rates that prevent a deviation from the sustainable path. Figure 2a illustrates the relationship between the sustainable tariff rate and λ in the case of equal and constant productivity values in the two countries. When λ is close to – but slightly larger than – one, under flexible prices and symmetric productivity levels, sustainable tariff rates very high, around 53 percent.[18] As λ rises to 1.5, tariffs fall to 11 percent under the same assumptions of flexible prices and symmetric productivity levels.

The two next results can be gleaned form conditions (45) and (46). In response to global shocks, the terms of trade is unchanged and home and foreign utility move in unison with one another. Moreover, because productivity growth shocks are permanent, future utility moves in the same proportion as current utility, and the argument of Result 1 in the previous section carries over. But when responding to country-specific shocks, the terms of trade are affected. A home productivity growth shock leads to a terms-of-trade deterioration for the home country. When $\lambda > 1$, the response of the terms of trade to a home country tariff is greater, the higher (*i.e.* the more depreciated) are the terms of trade themselves. Figure 2b plots the elasticity of the home terms of trade to a home tariff shock, for different values of the initial terms of trade, and for a trade elasticity λ equal to unity (as in the previous section), and for $\lambda = 1.5$.

Figure 2b shows that the elasticity is independent of the level of the terms of trade when $\lambda = 1$, but is increasing in absolute terms, when $\lambda > 1$. Thus, when a country experiences a local productivity growth shock, the incentive to deviate from the sustainable tariff is elevated. The argument holds in reverse for the other country. Its terms of trade have improved, which reduces the elasticity of the terms of trade to a foreign tariff increase and reduces its incentive to deviate from the sustainable tariff sequence. As a result, while a global productivity shock has no impact on the equilibrium degree of protection with fully flexible prices, a country-specific positive productivity growth shock makes the country experiencing the shock more protectionist and the other country less protectionist.

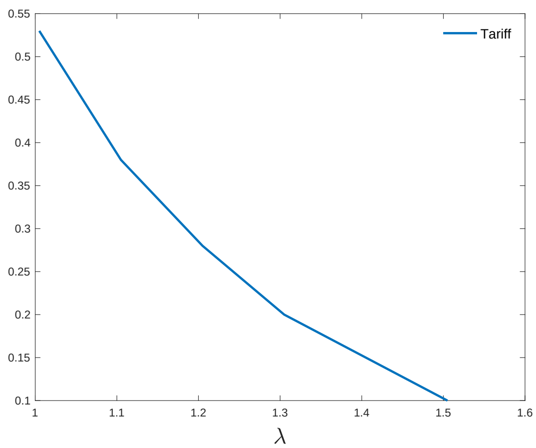
Figure 2c illustrates the relationship between home productivity θ and the sustainable equilibrium tariff rates for the home and foreign country, assuming that $\theta^* = 1$. With equal productivity, the tariff rates are the same, at roughly 10 percent. But as θ rises, the home tariff rate rises and the foreign tariff rate falls.

Nevertheless, as shown by Figure 2c, the impact of a home productivity shock on the equilibrium tariff rates is very small. As θ rises from 1 to 1.25, a 25 percent rise in productivity, the tariff rates

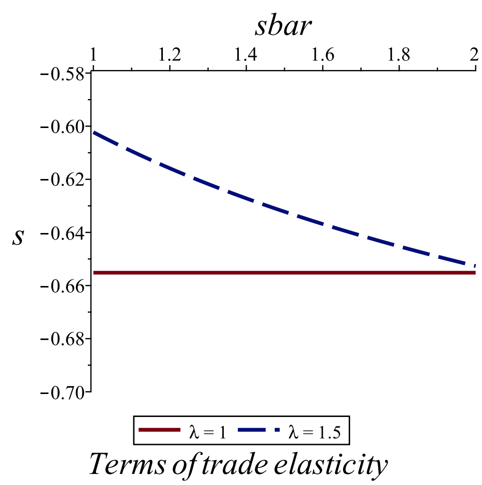
[18] The corresponding tariff rates were 62 percent with $\lambda = 1$ as shown in Table 1

Figure 2: Tariff under flexible prices with non-unitary trade elasticity

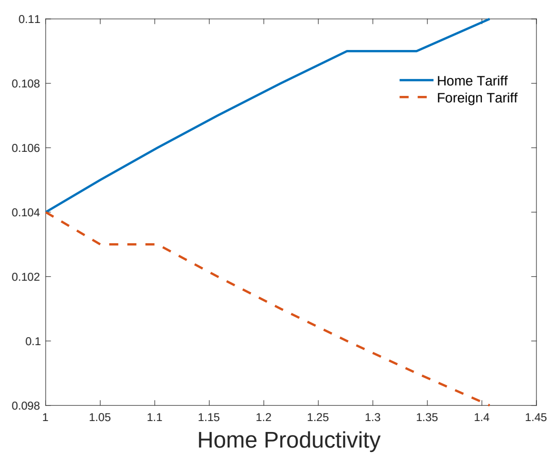
(a) Tariffs and Trade Elasticity



(b) Elast. of terms of trade to tariffs



(c) Tariffs and Productivity



Note: In Figure 2a, productivity levels are symmetric. In Figure 2b, S denotes the steady-state level of terms of trade. In Figure 2c, productivity varies in the Home country while $\theta^* = 1$.

diverge by only by one percentage point. Hence, we find that, in the non-unit trade elasticity case with flexible prices, while protectionism is not theoretically a-cyclical as before, tariffs are *almost constant* over time. This is quantitatively consistent with the results of the previous section.

5.3 Tariffs with pre-set prices with non-unitary trade elasticity

When prices are preset, as in the previous section, the binding cash-in-advance constraint determines output in each country, so that the discretionary tariff setter takes output as given when choosing whether to deviate from the sustainable path of tariffs. Therefore, taking Y_x and Y_m as given, we can represent aggregate consumption for the home country as

$$C_t = Y_{x,t} \frac{P_t^{-\lambda}(1+\tau)^{\lambda}}{(1+\tau_t)^{\lambda}\omega + \omega^* \mathcal{S}_t^{1-\lambda}} \tag{50}$$

where the terms of trade $\mathcal{S}_t$ is determined implicitly by

$$\mathcal{S}_t = \left(\frac{Y_{x,t}}{Y_{m,t}}\right)^{\frac{1}{\lambda}} \left(\frac{\omega(1+\tau_t)^{\lambda} + \omega^* \mathcal{S}_t^{1-\lambda}}{1-\omega+(1-\omega^*)(1+\tau_t^*)^{\lambda}\mathcal{S}_t^{1-\lambda}}\right)^{\frac{1}{\lambda}} \tag{51}$$

Figures 3a-3c describe the results analogous to Section 4 in the case of a non-unit trade elasticity. Again, we assume that $\lambda = 1.5$. Figure 3a describes the impact of a home productivity shock on home and foreign tariffs.

The sustainable tariff rates are defined as before. For the home country, the value from cheating is equal to the value of continuing on the sustainable path, so that:

$$v^{ch}(\tilde{z}_t) = v^s(\tilde{z}_t) \tag{52}$$

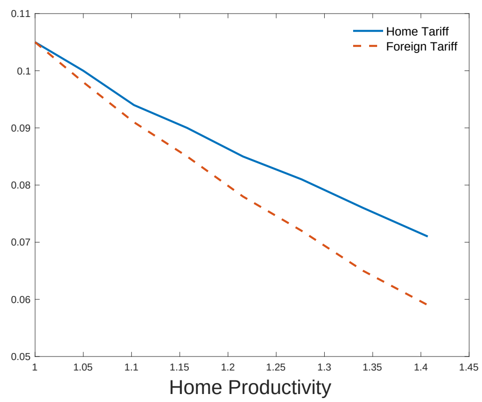
where $\tilde{z}_t = \{M_t, M_t^*, \theta_t, \theta_t^*\}$.

As in the previous section, tariff rates are declining in productivity when prices are pre-set, for the same reason as before; a higher productivity increases the future costs of deviating more than it does the benefits. But because of the differential incentives created by productivity differences, as described in the previous paragraphs, the home tariff falls by less than the foreign tariff rate. As stated above, with a terms of trade disadvantage driven by the productivity shock itself, the home country has a greater incentive to deviate. This must be balanced by a relatively higher home country sustainable tariff rate.

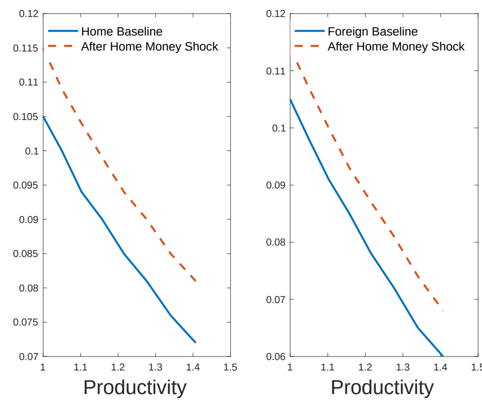
Figure 3b shows the impact of a home country monetary policy shock on the home and foreign tariff rates. As in the Figures of Section 4, for a given productivity shock, a home monetary policy shock increases the sustainable tariff rate in both countries. The logic behind the result is the same as in the previous discussion: a monetary shock raises current output and the gains from

Figure 3: Tariff schedules under alternative calibrations with $\lambda = 1.5$

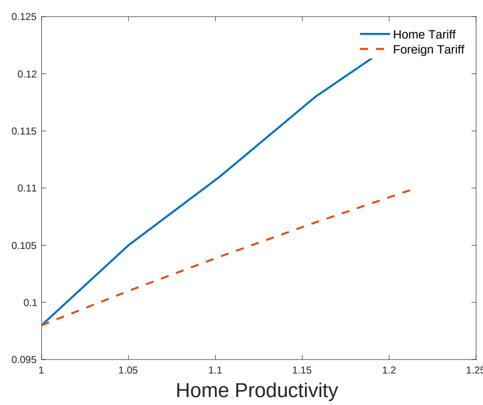
(a) Home productivity shocks



(b) Home productivity and money shocks



(c) Productivity shocks with $\sigma = 1.5$



Note: Baseline calibration is as follows: $\lambda = 1.5$, $\omega = 1 - \omega^* = 0.7$, $\tau^H = 0.62$, $\beta = \beta^* = 0.6$, $\sigma = 0.5$, $\psi = 2$.

deviation, while leaving the future costs of deviation unchanged. Finally, Figure 3c illustrates the case where $\sigma = 1.5$. As before, tariff rates are now increasing with home productivity, and with $\lambda = 1.5$, the home country tariff increases at a higher rate than that of the foreign country.

6 Conclusion

We have analyzed the cyclical pattern of trade protection in a stylized open-economy model with sticky prices, productivity and money growth shocks. Despite its relative simplicity and our ability to derive most results in closed form, the model was able to deliver a surprisingly large number of results. We found that tariff setters faced higher incentive to set tariffs in flexible exchange rate regimes than in fixed exchange rate regimes. We also showed that, within this model, slow price adjustment was a critical element in determining the cyclicality of trade protection. We explored a number of additional determinants of sustainable equilibrium tariffs, such as the nature of the shock and other key parameters of the model. In the dynamic tariff game with sticky prices, with a low intertemporal elasticity of substitution (IES), tariffs will tend to rise in response to money growth shocks, and fall in response to productivity growth shocks. The opposite logic applies with a a high IES. A general message of the paper is that there is a close and complex relationship between features of business cycle dynamics and the equilibrium degree of trade protection.

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Appendix

A The Model

A.1 Solving the model

For given tariffs, using (1), (2), and (5) with $M = M_0$ we can derive:

$$C_x = \frac{\omega(1+\tau)}{\omega(1+\tau)+1-\omega} Y_x, \quad C_x^* = \frac{\omega^*}{(1-\omega^*)(1+\tau^*)+\omega^*} \frac{SP_m Y_m}{P_x}$$

$$C_m = \frac{(1-\omega)}{\omega(1+\tau)+1-\omega} \frac{P_x Y_x}{SP_m}, \quad C_m^* = \frac{(1-\omega^*)(1+\tau^*)}{(1-\omega^*)(1+\tau^*)+\omega^*} Y_m$$

We can also show that $PC = \frac{P_x C_x}{\omega}$, so that the wage W above is

$$W = H^{\psi} C^{\sigma} P$$

and therefore the expression for the pre-set price may be written as

$$P_x = \frac{E \frac{H^{\psi} Y_x}{\theta}}{E \frac{Y_x}{C^{\sigma} P}} \tag{53}$$

We write out the equilibrium in a simple form as

$$Y_x = \frac{\omega(1+\tau)}{\omega(1+\tau)+1-\omega} Y_x + \frac{\omega^*}{(1-\omega^*)(1+\tau^*)+\omega^*} Q Y_m \tag{54}$$

$$Y_m = \frac{1-\omega}{\omega(1+\tau)+1-\omega} \frac{Y_x}{Q} + \frac{(1-\omega^*)(1+\tau^*)}{(1-\omega^*)(1+\tau^*)+\omega^*} Y_m \tag{55}$$

where $Q = SP_m/P_x$ is the terms of trade, with

$$Y_x = \frac{M}{P_x} \tag{56}$$

$$Y_m = \frac{M^*}{P_m} \tag{57}$$

A.2 Price solutions

Expanding (53) using (6), (56) and (57), we get the expression for the optimal price of home goods as

$$P_x^{\psi+\sigma} = \frac{E\left(\frac{M}{\theta}\right)^{1+\psi}}{E\left(M^{1-\sigma}((1+\tau)Q)^{(1-\omega)(\sigma-1)}\left(\frac{1+\tau}{\omega(1+\tau)+1-\omega}\right)^{-\sigma}\right)} \tag{58}$$

and for foreign goods:

$$P_m^{\psi+\sigma} = \frac{E\left(\frac{M^*}{\theta^*}\right)^{1+\psi}}{E\left(M^{*(1-\sigma)}\left(\frac{(1+\tau^*)}{Q}\right)^{\omega^*(\sigma-1)}\left(\frac{1+\tau^*}{(1-\omega^*)(1+\tau^*)+\omega^*}\right)^{-\sigma}\right)} \tag{59}$$

where we have used $P = P_x((1+\tau)Q)^{1-\omega}$ and $P^* = P_m((1+\tau^*)/Q)^{\omega^*}$. Equations (58) and (59) also use the property that

$$C = \frac{P_x C_x}{\omega P}, \text{ and } C^* = \frac{P_m C_m^*}{(1-\omega^*)\, P^*}$$

The analysis of tariff-setting in the model below depends critically on the way in which pre-set prices depend on money and productivity shocks. We can illustrate this linkage by explicitly solving (58) and (59). First, starting from (54), we can express the terms of trade as:

$$Q = \frac{Y_x}{Y_m}\left(\frac{1-\omega}{\omega^*}\frac{(1-\omega^*)\,(1+\tau^*)+\omega^*}{\omega(1+\tau)+1-\omega}\right) \tag{60}$$

Then, using (56) and (57) we get the following nominal exchange rate equation:

$$S = \frac{1-\omega}{\omega^*}\frac{(1-\omega^*)(1+\tau^*)+\omega^*}{\omega(1+\tau)+(1-\omega)}\frac{M}{M^*} \tag{61}$$

Finally, using (61) along with the price equations (58) and (59), we can express the home and foreign goods price as a function of the underlying shocks:

$$P_x = \left(\frac{E\left(\frac{M}{\theta}\right)^{1+\psi}}{E\left(\frac{M^{\omega(1-\sigma)}}{M^{*(1-\omega)(\sigma-1)}}\Lambda^{-1}(\tau,\tau^*)\right)}\right)^{\frac{\psi+\sigma+\omega^*(1-\sigma)}{\Delta}}\left(\frac{E\left(\frac{M^*}{\theta^*}\right)^{1+\psi}}{E\left(\frac{M^{*(1-\omega^*)(1-\sigma)}}{M^{\omega^*(\sigma-1)}}\Lambda^{*-1}(\tau,\tau^*)\right)}\right)^{\frac{(1-\omega)(1-\sigma)}{\Delta}} \tag{62}$$

$$P_m = \left(\frac{E\left(\frac{M^*}{\theta^*}\right)^{1+\psi}}{E\left(\frac{M^{*(1-\omega^*)(1-\sigma)}}{M^{\omega^*(\sigma-1)}}\Lambda^{*-1}(\tau,\tau^*)\right)}\right)^{\frac{\psi+\sigma+(1-\omega)(1-\sigma)}{\Delta}}\left(\frac{E\left(\frac{M}{\theta}\right)^{1+\psi}}{E\left(\frac{M^{\omega(1-\sigma)}}{M^{*(1-\omega)(\sigma-1)}}\Lambda^{-1}(\tau,\tau^*)\right)}\right)^{\frac{\omega^*(1-\sigma)}{\Delta}} \tag{63}$$

where $\Delta = (\psi+\sigma)\delta$, $\delta = (1+\psi+(\omega^*-\omega)(1-\sigma))$, and the functions Λ and Λ^* are defined as:

$$\Lambda(\tau,\tau^*) = \left(\frac{1+\tau}{\omega(1+\tau)+1-\omega}\right)^{(1-\omega)(1-\sigma)}\left(\frac{1}{(1-\omega^*)(1+\tau^*)+\omega^*}\right)^{-(1-\omega)(1-\sigma)}\left(\frac{1-\omega}{\omega^*}\right)^{(1-\omega)(1-\sigma)} \tag{64}$$

$$\Lambda^*(\tau,\tau^*) = \left(\frac{1+\tau^*}{(1-\omega^*)(1+\tau^*)+\omega^*}\right)^{1-(1-\omega^*)(1-\sigma)}\left(\frac{1}{\omega(1+\tau)+1-\omega}\right)^{-\omega^*(1-\sigma)}\left(\frac{1-\omega}{\omega^*}\right)^{-\omega^*(1-\sigma)} \tag{65}$$

Hence, the solutions (62) and (63) make clear that *ex-ante* pre-set prices depend on the distribution of money shocks, productivity shocks and home and foreign tariffs. We use (62) and (63) along with (56) and (57) to compute expected home and foreign output and expected utility below. First note that if prices were fully flexible, and could adjust to money, productivity, or tariff shocks, we would have the solutions:

$$\frac{P_x}{M} = \left[\frac{\Lambda}{\theta^{1+\psi}}\right]^{\frac{\psi+\sigma+\omega^*(1-\sigma))}{\Delta}} \left[\frac{\Lambda^*}{\theta^{*(1+\psi)}}\right]^{\frac{(1-\omega)(1-\sigma))}{\Delta}}, \quad \frac{P_m}{M^*} = \left[\frac{\Lambda^*}{\theta^{*(1+\psi)}}\right]^{\frac{\psi+\sigma+(1-\omega)(1-\sigma))}{\Delta}} \left[\frac{\Lambda}{\theta^{1+\psi}}\right]^{\frac{\omega^*(1-\sigma))}{\Delta}} \tag{66}$$

With fully flexible prices money is neutral, but normalized prices are negatively related to domestic productivity shocks, and positively related to own country tariffs.

A.3 Utility measures

In order to determine the path of tariffs, it is necessary to construct welfare measures for benevolent governments in each country. Given optimal price-setting, we can express expected period utility for the home country as:

$$EU = E\left(\frac{C^{1-\sigma}}{1-\sigma} - \frac{H^{1+\psi}}{1+\psi}\right) = E\Gamma\frac{C^{1-\sigma}}{1-\sigma} \tag{67}$$

where

$$\Gamma(\tau) \equiv 1 - \frac{(1-\sigma)(\omega(1+\tau)+1-\omega)}{(1+\tau)(1+\psi)} \tag{68}$$

From the equilibrium terms of trade in the previous section, we can express the consumption aggregator as:

$$C = C_x^{\omega}C_m^{1-\omega} = Y_x^{\omega}Y_m^{1-\omega}\zeta(\tau,\tau^*)$$

where $\zeta = \frac{(1+\tau)^{\omega}}{\delta_{\omega}(\tau)^{\omega}\delta_{\omega^*}(\tau^*)^{1-\omega}}(\frac{\omega^*}{1-\omega})^{1-\omega}$, $\delta_{\omega} = \omega(1+\tau)+1-\omega$ and $\delta_{\omega^*} = (1-\omega^*)(1+\tau^*)+\omega^*$.

Hence, the equilibrium period expected utility expression can be written as:

$$EU = E\frac{\Gamma(\tau)}{1-\sigma}\left(\left(\frac{M}{P_x}\right)^{\omega}\left(\frac{M^*}{P_m}\right)^{1-\omega}\zeta(\tau,\tau^*)\right)^{1-\sigma} \tag{69}$$

where the prices are expressed as the above solutions (62) and (63), and depend on expected productivity and money shocks. Expression (69) indicates that expected utility depends on the tariff rates set by the home and foreign governments. In particular, it is easily seen that for expected output levels, beginning at a zero home tariff, expected utility is increasing in the home tariff rate and (always) decreasing in the foreign tariff rate.

B Tariffs under fixed exchange rates

Here, we show the effect of a tariff under a fixed exchange rate when the home authority maintains the peg, and show that in this case, there is no gain to the home country in imposing a tariff. We may write out the welfare expression facing the home country as:

$$U = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{H_t^{1+\psi}}{1+\psi} \tag{70}$$

From (19) in the text, we have:

$$\begin{aligned} C_t &= \frac{M_t}{P_{xt}} \left(\frac{(1+\tau)^{\omega}}{\omega(1+\tau)+1-\omega} \right) \left(\frac{P_{xt}}{S_t P_{mt}} \right)^{1-\omega} \omega^{\omega}(1-\omega)^{1-\omega} \\ &= \frac{\left(\frac{\omega^*}{1-\omega}\right) \frac{(1+\tau)^{\omega}}{(1-\omega^*)(1+\tau^*)+\omega^*} S_t M_t^*}{P_{xt}} \left(\frac{P_{xt}}{S_t P_{mt}} \right)^{1-\omega} \omega^{\omega}(1-\omega)^{1-\omega} \end{aligned} \tag{71}$$

where the second equality holds due to the endogenous home money supply implied by (10). From (7) and (10) in the text, we have

$$H_t = \frac{M_t}{P_{xt}} = \frac{1}{P_{xt}} \left(\frac{\omega^*}{1-\omega} \right) \frac{\omega(1+\tau)+(1-\omega)}{(1-\omega^*)(1+\tau^*)+\omega^*} S_t M_t^*$$

Now taking the derivative of (70) with respect to τ, we have

$$\frac{dU}{d\tau} = \frac{\omega C_t^{1-\sigma}}{1+\tau} - \frac{\omega H_t^{1+\psi}}{\omega(1+\tau)+(1-\omega)} \tag{72}$$

Since as shown in (67) in the text

$$H_t^{1+\psi} = \frac{(\omega(1+\tau)+1-\omega)}{(1+\tau)} C_t^{1-\sigma}$$

it must be that (72) is zero, so there is no gain for the home country in imposing a tariff under fixed exchange rates.

C Conditions for existence of sustainable tariffs

Here we establish the conditions for the existence of a sustainable tariff sequence and show the determinants of the selected self-enforcing tariffs. We start with the case of fully flexible prices. From (31) of the text, a sustainable tariff sequence $\tilde{\tau}_t,\ \tilde{\tau}_t^*$ satisfies the condition

$$\frac{\Gamma(\tau^H)}{1-\sigma} \mathcal{H}(\tau^H, \tilde{\tau}_t^*)^{1-\sigma} + \frac{\beta}{1-\beta E \Xi_1{}^{1-\sigma}} \frac{\Gamma(\tau^H)}{1-\sigma} \mathcal{H}(\tau^H, \tau^H)^{1-\sigma}$$

$$\leq \frac{\Gamma(\tilde{\tau}_t)}{1-\sigma}\mathcal{H}(\tilde{\tau}_t, \tilde{\tau}_t^*)^{1-\sigma} + \frac{\beta}{1-\beta E\Xi_1{}^{1-\sigma}}\frac{\Gamma(\tilde{\tau}_{t+1})}{1-\sigma}\mathcal{H}(\tilde{\tau}_{t+1}, \tilde{\tau}_{t+1}^*)^{1-\sigma} \tag{73}$$

We first note that $\tilde{\tau} = \tilde{\tau}^* = \tau^H$ satisfies (73) with equality, since the value of cheating and the continuation value are the same if tariffs are at their maximum value τ^H for both countries. In addition, since countries share the same preferences, the analogous condition to (73) for the foreign country must be identical. So, it must be that in the flexible price case, if there exists a sustainable tariff sequence strictly less than τ^H, then there is a solution where $\tilde{\tau}_t = \tilde{\tau}_t^*$. We focus therefore on such a symmetric solution. Finally, since (73) has no time varying parameters, a stationary solution to (73) must be time-invariant [19]

To determine the conditions under which a tariff rate $\tilde{\tau} < \tau^H$ satisfies (73), we can rearrange the equation in the following way.

$$\frac{1}{1-\sigma}\left(\Gamma(\tau^H)\mathcal{H}(\tau^H, \tilde{\tau})^{1-\sigma} - \Gamma(\tilde{\tau})\mathcal{H}(\tilde{\tau}, \tilde{\tau})^{1-\sigma}\right)$$

$$\leq \frac{\beta}{(1-\beta E\Xi_1{}^{1-\sigma})(1-\sigma)}\left(\Gamma(\tilde{\tau})\mathcal{H}(\tilde{\tau}, \tilde{\tau})^{1-\sigma} - \Gamma(\tau^H)\mathcal{H}(\tau^H, \tau^H)^{1-\sigma}\right) \tag{74}$$

The left-hand side of (74) represents the one period gain from cheating on the sustainable tariff $\tilde{\tau}$, while the right-hand side represent the costs in terms of foregone utility following a period of cheating. Both the left-hand side and right-hand side are positive, when $\tilde{\tau} < \tau^H$. The right-hand side is positive since utility is strictly decreasing in tariffs for equal tariff rates across countries. The left-hand side is positive since based on our previous assumptions, $\tilde{\tau}$ is less than the optimal one-period Nash tariff rate for any country, so either country can gain by deviating from the sustainable tariff $\tilde{\tau} < \tau^H$ and setting $\tau = \tau^H$. But following standard results from repeated games, for a high enough effective discount factor $\beta E\Xi_1{}^{1-\sigma}$, there must exist a value of $\tilde{\tau} < \tau^H$ which satisfies (74) with strict inequality. In our calibration from Section 4 of the paper, for all values of β and distribution of productivity shocks, we find values of $\tilde{\tau} < \tau^H$ that satisfy (74).

To determine the lowest sustainable tariff rates, we begin by asking whether (74) is satisfied under fully free trade, $\tilde{\tau} = 0$. That is, we ask whether

$$\frac{1}{1-\sigma}\left(\Gamma(\tau^H)\mathcal{H}(\tau^H, 0)^{1-\sigma} - \Gamma(0)\mathcal{H}(0,0)^{1-\sigma}\right)$$

$$< \frac{\beta}{(1-\beta E\Xi_1{}^{1-\sigma})(1-\sigma)}\left(\Gamma(0)\mathcal{H}(0,0)^{1-\sigma} - \Gamma(\tau^H)\mathcal{H}(\tau^H, \tau^H)^{1-\sigma}\right) \tag{75}$$

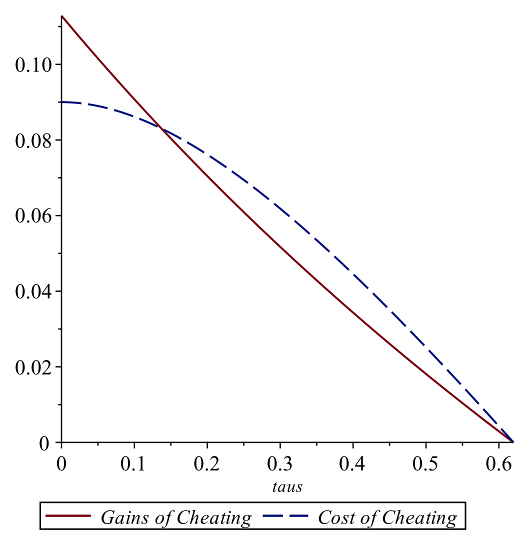
If condition (75) is satisfied, then the lowest sustainable tariff is zero; *i.e.* complete free trade. But if (75) is violated, and simultaneously (74) is satisfied for $\tilde{\tau} < \tau^H$, then there must exist a strictly

[19] It may be possible to construct arbitrary cycles in tariffs rates which satisfy (73). However, we ignore such solutions.

positive value of $\tilde{\tau} < \tau^H$ which satisfies (74) with equality.

We may illustrate these conditions in Figure 4. The Figure illustrates the right-hand side and left-hand side of condition (74) for the calibration used in the text of the paper. The maximum tariff is set at $\tau^H = 0.62$. In this case, we see that tariff rates below τ^H are sustainable. Zero tariffs however are not sustainable. As a result, the selected tariff rate is illustrated as the value where the gains and costs to cheating are equivalent, which occurs at a tariff rate of approximately 15 percent.

Figure 4: Gains and costs of cheating



In general, there is no theoretical proof that there is a unique $\tilde{\tau} < \tau^H$ which satisfies (74) with equality. While both the left and right-hand side of (74) are monotonic in $\tilde{\tau}$ it is not necessarily the case that the left-hand side is strictly convex, and the right-hand side strictly concave in $\tilde{\tau}$. However, for all simulations carried out in the text, we find only one solution for the selected tariff.

Finally, since tariffs are the same for both countries, it follows immediately that the selected tariff maximizes the sum of home and foreign welfare among all sustainable tariffs.

In the case of sticky prices, a similar argument to that above can be followed, except in each case, the tariff rates are conditional on the realizations of productivity and money growth shocks. Also, due to home bias in preferences, tariff rates will no longer be identical across countries. But given the analogous restrictions on discount factors, for each set of realizations of productivity and money growth shocks, there will exist unique levels of the selected tariff rates $\tilde{\tau}(z_t),\ \tilde{\tau}(z_t)^*$. To show this, here we repeat the conditions describing a sustainable tariff sequence for the home and foreign country as:

$$\frac{1}{1-\sigma}\Xi_t^{1-\sigma}\left(\left(E_{t-1}\mathcal{J}_1(\tilde{\tau}_t,\tilde{\tau}_t^*)^{1-\sigma}\right)\zeta(\tau^H,\tilde{\tau}_t^*)^{1-\sigma}-\left(E_{t-1}\mathcal{J}_1(\tilde{\tau}_t,\tilde{\tau}_t^*)^{1-\sigma}\right)\zeta(\tilde{\tau}_t,\tilde{\tau}_t^*)^{1-\sigma}\right)$$

$$\leq\frac{\beta\left(E_{t-1}\left(\Gamma(\tilde{\tau}_t)\mathcal{H}_1(\tilde{\tau}_t,\tilde{\tau}_t)^{1-\sigma}\Xi_t^{1-\sigma}\right)-\Gamma(\tau^H)\mathcal{H}_1(\tau^H,\tau^H)^{1-\sigma}E_{t-1}\left(\Xi_t\right)^{1-\sigma}\right)}{(1-\sigma)(1-\beta E_{t-1}\Xi_1^{1-\sigma})}\Xi_{1t}^{1-\sigma} \tag{76}$$

$$\frac{1}{1-\sigma}\Xi_t^{*1-\sigma}\left(\left(E_{t-1}\mathcal{J}_1^*(\tilde{\tau}_t^*,\tilde{\tau}_t)^{1-\sigma}\right)\zeta^*(\tau^H,\tilde{\tau}_t)^{1-\sigma}-\left(E_{t-1}\mathcal{J}_1^*(\tilde{\tau}_t^*,\tilde{\tau}_t)^{1-\sigma}\right)\zeta^*(\tilde{\tau}_t^*,\tilde{\tau}_t)^{1-\sigma}\right)$$

$$\leq\frac{\beta\left(E_{t-1}\left(\Gamma^*(\tilde{\tau}_t^*)\mathcal{H}_1^*(\tilde{\tau}_t,\tilde{\tau}_t^*)^{1-\sigma}\Xi_t^{*1-\sigma}\right)-\Gamma(\tau^H)\mathcal{H}_1^*(\tau^H,\tau^H)^{1-\sigma}E_{t-1}\left(\Xi_t^*\right)^{1-\sigma}\right)}{(1-\sigma)(1-\beta E_{t-1}\Xi_1^{*1-\sigma})}\Xi_{1t}^{*1-\sigma} \tag{77}$$

where we define

$$\mathcal{H}_1(\tau_t,\tau_t^*)=(\Lambda(\tau_t,\tau_t^*)\Delta_1)^{-\frac{\omega(\sigma+\psi)+(1-\sigma)\omega^*}{(\sigma+\psi)\delta}}(\Lambda^*(\tau_t,\tau_t^*)\Delta_2)^{-\frac{(1+\psi)(1-\omega)}{(\sigma+\psi)\delta}}\zeta(\tau_t,\tau_t^*)$$

$$\mathcal{H}_1^*(\tau_t,\tau_t^*)=(\Lambda^*(\tau_t,\tau_t^*)\Delta_1)^{-\frac{(1-\omega^*)(\sigma+\psi)+(1-\sigma)(1-\omega}{(\sigma+\psi)\delta}}(\Lambda^*(\tau_t,\tau_t^*)\Delta_2)^{-\frac{(1+\psi)\omega^*}{(\sigma+\psi)\delta}}\zeta(\tau_t,\tau_t^*)$$

In addition, the expressions Δ_1 and Δ_2 are defined as

$$\Delta_1=\frac{E(\frac{1+\mu}{1+\nu})^{1+\psi}}{E\frac{(1+\mu)^{\omega(1-\sigma)}}{(1+\mu^*)^{-(1-\omega)(1-\sigma)}}},\qquad\Delta_2=\frac{E(\frac{1+\mu^*}{1+\nu^*})^{1+\psi}}{E\frac{(1+\mu^*)^{(1-\omega^*)(1-\sigma)}}{(1+\mu)^{-\omega^*(1-\sigma)}}}$$

and

$$\mathcal{J}_1(\tilde{\tau}_t,\tilde{\tau}_t^*)=(\Lambda(\tilde{\tau}_t,\tilde{\tau}_t^*)\Delta_1)^{-\frac{\omega(\sigma+\psi)+(1-\sigma)\omega^*}{(\sigma+\psi)\delta}}(\Lambda^*(\tilde{\tau}_t,\tilde{\tau}_t^*)\Delta_2)^{-\frac{(1+\psi)(1-\omega)}{(\sigma+\psi)\delta}}$$

$$\mathcal{J}_1(\tilde{\tau}_t,\tilde{\tau}_t^*)^*=(\Lambda(\tilde{\tau}_t,\tilde{\tau}_t^*)\Delta_1)^{-\frac{\omega^*(1+\psi)}{(\sigma+\psi)\delta}}(\Lambda^*(\tilde{\tau}_t,\tilde{\tau}_t^*)\Delta_2)^{-\frac{(1-\omega^*)(\sigma+\psi)+(1-\sigma)(1-\omega)}{(\sigma+\psi)\delta}}$$

As before the left-hand side of (76) and (77) represent the gains from cheating on a sustainable tariff sequence $\tilde{\tau}_t$, $\tilde{\tau}_t^*$, while the right-hand side of each condition represents the continuation losses from cheating. As before, it is apparent that when $\tilde{\tau}_t = \tilde{\tau}_t^* = \tau^H$ both conditions are satisfied with strict equality, since both the left and right-hand sides of both conditions are zero. Reducing sustainable tariffs below the maximum value τ^H will again increase both the left-hand and right-hand side of both conditions (76) and (77). But again from high enough values of the discount factors $\beta E_{t-1}\Xi_1^{1-\sigma}$ and $\beta E_{t-1}\Xi_1^{*1-\sigma}$, the conditions will be satisfied with strict inequality for values of $\tilde{\tau}_t < \tau^H$, $\tilde{\tau}_t^* < \tau^H$. As stated in Proposition 2 part a), for some realizations of shocks, one or both conditions may be satisfied with zero tariffs. Otherwise there must exist non-zero values for $\tilde{\tau}(z_t)$ and $\tilde{\tau}^*(z_t)$ that satisfy (76) and (77) with strict equality. The lowest pair of such values represents the selected sustainable tariff functions.

Again, in our simulations based on the calibration of the text, we always find unique values for $\tilde{\tau}(z_t) < \tau^H$ and $\tilde{\tau}^*(z_t) < \tau^H$ that satisfy (76) and (77) with equality.

The key difference between (76) and (77) and the case of flexible prices, represented by (74) is that now the tariff rates will be functions of the realized shocks to productivity and money growth. Moreover, tariffs will in general differ across countries, given home bias in preferences. Finally, changes in tariffs are unpredictable, since the selected sustainable tariffs are time invariant functions of the realized shocks to productivity and money growth, which themselves are i.i.d.

In general, the lowest tariff rates which satisfy both the home and foreign incentive constraints (76) and (77) with equality are not guaranteed to be the tariff rates which maximize joint welfare among all sustainable tariff rates. Since tariffs generate existing distortions it is not always the case that, for a given tariff rate in one country, that a reduction in the other country's tariff will raise world welfare evaluated as the unweighted sum of both countries utilities. However, if the equilibrium levels of $\tilde{\tau}(z_t)$ and $\tilde{\tau}^*(z_t)$ are sufficiently close to one another, it is possible to show that a small rise in either tariff (above the level of the selected tariff) must reduce world welfare. Hence, if countries are not too different, then it follows that the selected sustainable tariff rates must maximize world welfare among all possible sustainable tariffs.

D Extension: Optimal Monetary Policy

An important assumption for our results on cyclical tariffs is that monetary policy does not perfectly undo the existing price rigidity. So we assumed that monetary policy is passive, unless it is targeted to maintain an exchange rate peg. The polar opposite assumption would be that monetary authorities choose an optimal policy to maximize expected utility, taking into account the nature of the productivity shocks affecting the home and foreign countries. Here we show that a cooperative optimal monetary policy response followed by each government, taking tariffs as given, would support the full flexible price equilibrium. In that case, optimal tariffs would again be a-cyclical, just as in the flexible price case.

To see this, we now make the additional assumption that home and foreign productivity innovations follow a log-normal distribution, so that

$$\theta_{t+1} = \theta_t \exp(v_t), \theta^*_{t+1} = \theta^*_t \exp(v^*_t)$$

where $v_t \sim N(0, \sigma_v^2)$ and $v^*_t \sim N(0, \sigma_v^{*2})$. We further assume that in each period, the home and foreign governments follow a monetary feed-back rule (in logs) given by:

$$m_t = \bar{m} + \phi_1 v_t + \phi_2 v^*_t m^*_t = \bar{m}^* + \phi^*_1 v_t + \phi^*_2 v^*_t \tag{78}$$

Hence, monetary policy is chosen to adjust in response to realized productivity shocks.[20] The parameters ϕ_i and ϕ^*_i, $i = 1, 2$ are chosen to maximize expected utility, evaluated as the sum of home and foreign expected utility. We assume that monetary authorities choose these parameters taking tariff rates as given. In equilibrium, it will be the case that tariff rates are independent of the productivity shocks.

Assuming symmetric home bias, so that $\omega^* = 1 - \omega$, it can be verified that the following optimal monetary feedback rules support the full flexible price equilibrium

$$\phi_1 = \frac{(1+\psi)(1+\psi+\omega(\sigma-1))}{(\sigma+\psi)(1+\psi+(1-\sigma)(1-2\omega))}, \quad \phi_2 = \frac{-(\sigma-1)(1+\psi)(1-\omega)}{(\sigma+\psi)(1+\psi+(1-\sigma)(1-2\omega))} \tag{79}$$

with $\phi^*_1 = \phi_2$, $\phi^*_2 = \phi_1$.

Because these optimal monetary responses support the full flexible price equilibrium, the solution for the lowest sustainable tariff sequence follows exactly as in section 4.2 and implies that home and foreign tariffs are equal and constant over time. This also implies that the conjecture that tariffs are independent of productivity shocks is verified, in equilibrium.

E Free trade as an equilibrium outcome

Proposition 2 allows for the possibility that one or both countries may be pushed down to a zero tariff outcome as part of the equilibrium response of tariffs to shocks. As noted, in our calibrated model, we found that this never occurred for the shocks drawn from the distribution of productivity and money shocks. Figure 5 shows a case for a simplified version of the model where the home country may have a zero tariff while the foreign country has a positive tariff for very high levels of relative productivity shocks. This example takes a one time unexpected shock (an 'MIT shock') to home relative productivity, assuming no future uncertainty, and abstracts

[20]Since an optimal monetary policy would just offset monetary policy shocks, we omit such shocks.

from money shocks. Given home bias in preferences, the productivity shock raises the costs of deviating from the lowest equilibrium tariff rates more for the home country than for the foreign country, leading the home country to have a lower tariff. Due to the absence of uncertainty, the equilibrium tariff rate at equal productivities is substantially higher than in our baseline model, consistent with the results in Table 1. But as the home productivity rises to more than 6 percent above foreign productivity, the home country is pushed to a zero tariff, while the foreign country still has a positive Tariff. This provides an example of the outcome in Proposition 2, part a.

Figure 5: Zero tariff as an equilibrium outcome